Discretization Order Influences on Extended Kalman Filter Estimation for Doubly-Fed Induction Generator

Abstract: The main objective of this paper is to analyze the influence of the discretization step on the estimated states of the Doubly-Fed Induction Generator (DFIG). Although the Extended Kalman Filter (EKF) has been widely used for such systems, the discretization process is conventionally ensured by the first-order Forward Euler method. Therefore, the effects of the discretization order of the discrete state-space representation on the Extended Kalman Filter estimation have not been studied before. In this paper, we combine the Extended Kalman Filter with two second-order discretization methods: Central Difference and Adams-Bashforth methods, to estimate the states of a Doubly-Fed Induction Generator and improve the estimation precision of the rotor speed and the flux of the generator. A comparative study has been conducted to analyze the qualitative and quantitative responses of the estimator for different cases. The obtained results have demonstrated the significance of the discretization order on the estimation process of the two states of the DFIG.


Keywords: Discretization, Doubly-Fed Induction Generator (DFIG), Extended Kalman Filter (EKF), Second-Order Method.

Słowa kluczowe: Dyskretyzacja, Dwubiegowy Generator Indukcyjny (DFIG), Rozszerzony Filtr Kalmana (EKF), Metody drugiego rzędu,

I. Introduction

State estimation is a very broad subject, as it is involved in many research areas such as control, system identification, and telecommunications, among others. It consists of inferring the states of a system based on the information provided by the process model and the taken measurements. In general, the model is not an exact replica of the behavior of the process, but it provides a good approximation with a certain level of accuracy, while the measurements are affected by noises that can reduce their accuracy. To address these imperfections, various estimation techniques have been developed, such as Maximum Likelihood Estimator (MLE) and Kalman Filter (KF).

The Kalman Filter (KF) is widely used in state estimation problems for linear dynamical systems and is known to be one of the most optimal estimators under predefined conditions. Some applications of using the KF for estimation are presented in [1–3]. However, most industrial systems exhibit nonlinear dynamic behavior, which can render the standard Kalman filter less effective. To address this limitation, modifications have been introduced to improve the accuracy of the Kalman filter for nonlinear systems, leading to the development of new algorithms such as the Extended Kalman Filter (EKF) and the Unscented Kalman Filter (UKF) [4]. The EKF is based on the linearization of the nonlinear model at each iteration around an operating point. It is widely used due to its simplicity and ease of implementation. However, in strongly nonlinear systems, the EKF often provides poor estimation results. On the other hand, the UKF is known to have better accuracy than the EKF for nonlinear systems, but it is more complex and requires more computational time. Researchers have made attempts to improve the EKF, as demonstrated in the work presented in [5], where three modified EKF algorithms have been compared in terms of their performances. These methods involve changing the integration step length using the Gauss-Newton method with Quasi-Newton technique and the Levenberg-Marquardt method. The latter introduces a damping factor to the Gauss-Newton method. Another strategy proposed in the literature involves approximating the nonlinear functions of the model using Taylor series up to the second derivative [6–7]. However, the use of second derivatives results in the computation of the Hessian matrix, making the EKF more complex and less efficient in terms of required computational time. To address this issue, Michael Roth and Frederick Gustafsson proposed a new contribution in [8] to enhance the required computational time and reduce complexity. Another modification to tackle the problem of calculating the Hessian matrix is presented in [9], wherein the algorithm used is called the Second Order Extended Particle Filter, where the EKF is used to obtain an approximation of the posterior probability density needed in the particle filter algorithm. Additional modifications to improve the accuracy of the EKF are presented in [10–13]. Another algorithm known as the "Cubature Kalman Filter" (CKF) was developed by Lenkanar Arasarathnam and Simon Haykin in 2009 [14]. This algorithm is quite similar to UKF, but it differs in the set of rules used to calculate the Kalman filter weights. The CKF uses a spherical radial curvature rule to generate the weights instead of the Sigma-points set. The CKF was designed to tackle the problems of divergence in high-dimensional nonlinear systems.

In this paper, we combine an attempt to improve the accuracy of the Kalman filter for nonlinear systems without increasing the algorithm's complexity or computational inefficiency. Specifically, we propose the investigation of two second-order discretization methods (central difference and Adams-Bashforth) for the continuous nonlinear system, resulting in a second-order Extended Kalman Filter (EKF) that does not require the computation of the Hessian matrix. The rest of the manuscript is organized as follows: in section two, the central difference and Adams-Bashforth discretization methods are carefully overviewed. In section three, the DFIG nonlinear state equations are derived in the synchronous reference frame (dq) to be used for EKF estimation. Then, in section four, the EKF combined with
the different discretization methods is designed for the DFIG model. In section five, an analytical comparison is conducted for the different obtained results.

II. Overview of 1ST and 2ND Order Discretization of Ordinary Differential Equations

In this section, we demonstrate the advantage of using second-order discretization methods over first-order methods by solving a 1st-order ordinary differential equation (ODE) given by Equation (1).

\[ \dot{x}(t) = f(x(t), t) \]

The function \( f(x(t), t) \) is continuous and can be either linear or nonlinear. For linear functions with high order (greater than 2) or nonlinear functions, finding an analytical solution is either really difficult or impossible. Instead, an approximate solution is obtained by using numerical methods, which is based on replacing the first derivative by a discrete approximation using discretization methods. The discretization methods are classified as first-order methods, second-order methods, or higher-order methods. The most used first order discretization method is Forward Euler’s method because of its simplicity, where \( \dot{x}(t) \) is replaced by (2).

\[ \dot{x}(t) = \frac{x(k\Delta t) - x((k-1)\Delta t)}{\Delta t} + O(\Delta t^2) \]

This yields to the following discrete equation:

\[ x_{k+1} = x_k + \frac{3\Delta t}{2} f(x_k, t_k) - \frac{\Delta t}{2} f(x_{k-1}, t_{k-1}) + O(\Delta t^3) \]

Adams-Bashforth method is also known to have better accuracy than Forward Euler method i.e, a local truncation error of order \( O(\Delta t^3) \) and a global truncation error of order \( O(\Delta t^2) \). The method can also be easily implemented, i.e. only one function evaluation per step time is needed.

In the rest of this section, a comparison between the methods stated above is illustrated by showing the solution obtained by each method and its error graphically. For this purpose, a simple ordinary differential equation is chosen, which is given by:

\[ x(t) = -x \]

With \( x(0) = 1. \)

We can see from Fig. 2 that the error of LP method is better than FE (almost one fifth the error of the Forward Euler’s method) but it exhibits some oscillations; whereas AB2 method is the best in terms of accuracy and stability.
III. Dynamic Model of DFIG

Induction machines (IM) are very important in renewable energy domain. They are used extensively to generate electric power from wind energy. Different types of induction machine exist such as squirrel cage IM, permanent magnet IM…etc. However, Doubly-fed induction generators (DFIG) are best suited for wind turbines (WT) because they can produce a regulated electric power of constant voltage and frequency, regardless of the disturbance caused by the variation of wind speed. The ability to produce constant voltage and frequency electric power is ensured primarily by adjusting the amplitude and frequency of the voltage fed back to the rotor. Therefore, the output of the wind turbine that has a DFIG can be directly connected to the electric grid network [16-18]. A DFIG is composed of three-phase stator windings, which are the output of the machine, and a three-phase rotor winding, which is used as input to regulate the voltage and frequency of the three-phase stator voltage output. To derive a state space model of the DFIG, we express the voltage equations of both the stator and rotor referred to their natural reference frames. These equations are given by the references [19, 20].

\[
\begin{align*}
\frac{d\psi_{ar}}{dt} &= R_{i}i_{ar} + \frac{d\psi_{ar}}{dt} - \omega \psi_{br} \\
\frac{d\psi_{br}}{dt} &= R_{i}i_{br} + \frac{d\psi_{br}}{dt} + \omega \psi_{ar} \\
\frac{d\psi_{cr}}{dt} &= R_{i}i_{cr} + \frac{d\psi_{cr}}{dt} \\
\frac{d\psi_{qr}}{dt} &= R_{i}i_{qr} + \frac{d\psi_{qr}}{dt} - \omega \psi_{dr} \\
\frac{d\psi_{dr}}{dt} &= R_{i}i_{dr} + \frac{d\psi_{dr}}{dt} + \omega \psi_{qr}
\end{align*}
\]

Where $\psi_{ar}$, $\psi_{br}$, $\psi_{cr}$ are the three-phase stator fluxes, and $\psi_{qr}$, $\psi_{dr}$ are the three-phase rotor fluxes.

To transform equations (10) and (11) to the (dq) rotating reference frame (Fig.3), we first need to transform them to the stationary two-phase components (aβ) and then to the synchronous reference frame (dq). The resulting equation (12) is obtained as a result of this transformation process.

\[
\begin{align*}
\frac{dv_{a}}{dt} &= R_{i}i_{a} + \frac{dv_{a}}{dt} - \omega \psi_{b} \\
\frac{dv_{b}}{dt} &= R_{i}i_{b} + \frac{dv_{b}}{dt} + \omega \psi_{a} \\
\frac{dv_{q}}{dt} &= R_{i}i_{q} + \frac{dv_{q}}{dt} - \omega \psi_{r} \\
\frac{dv_{r}}{dt} &= R_{i}i_{r} + \frac{dv_{r}}{dt} + \omega \psi_{q}
\end{align*}
\]

With $\omega = \frac{d\theta}{dt}$ and $\omega_r = \frac{d\theta_r}{dt}$.

The mechanical torque of the DFIG machine is given by:

\[
T_m = T_\omega + \frac{d\theta}{dt} + B\omega
\]

The electromagnetic torque is then expressed in terms of stator current and rotor flux as:

\[
T_m = \frac{pL_m}{L_r} (\psi_{ar} \psi_{br} - \psi_{qr} \psi_{dr})
\]

By choosing a state vector $x = [v_{a} v_{b} v_{q} v_{r} i_{a} i_{b} i_{q} i_{r}]^T$, we obtain a state space model by combining equations (12-15), which is given by equation (16).

\[
\begin{align*}
\begin{bmatrix}
\frac{dv_{a}}{dt} \\
\frac{dv_{b}}{dt} \\
\frac{dv_{q}}{dt} \\
\frac{dv_{r}}{dt}
\end{bmatrix} &= \begin{bmatrix}
R_{i} & 0 & 0 & -L_{r} & 0 & 0 & 0 & 0 \\
0 & R_{i} & 0 & 0 & -L_{r} & 0 & 0 & 0 \\
0 & 0 & R_{i} & 0 & -L_{r} & 0 & 0 & 0 \\
0 & 0 & 0 & R_{i} & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
v_{a} \\
v_{b} \\
v_{q} \\
v_{r}
\end{bmatrix} \\
&+ \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
i_{a} \\
i_{b} \\
i_{q} \\
i_{r}
\end{bmatrix} + \begin{bmatrix}
\frac{pL_m}{L_r} (\psi_{ar} \psi_{br} - \psi_{qr} \psi_{dr}) - \frac{R_{i}i_{a}}{L_r} \\
\frac{R_{i}i_{b}}{L_r} - \frac{R_{i}i_{a}}{L_r} \frac{\omega \psi_{b}}{\omega} - \frac{L_{r} \psi_{q}}{L_{r}} \frac{\omega \psi_{q}}{\omega} + \frac{L_{r} \psi_{r}}{L_{r}} \frac{\omega \psi_{r}}{\omega}
\end{bmatrix}
\end{align*}
\]

Equation (16) can be written in the form:

\[
x = f(x) + Bu
\]

Where $u = [v_{a} v_{b} v_{q} v_{r} i_{a} i_{b} i_{q} i_{r}]^T$ and the output measurement vector is:

\[
z = [i_{a} i_{b} i_{q} i_{r}]^T = Cx
\]

With $C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$

The resulting state space model is nonlinear and will be used to estimate the speed and rotor flux of the DFIG machine.

IV. Extended Kalman Filter

The Kalman Filter (KF) is extensively used for state estimation in stochastic linear dynamical systems. The estimation is performed based on a recursive algorithm that estimates the state from prior knowledge given by the state space model and some measurements related to the estimated state. For nonlinear systems, the Kalman Filter can still be used for state estimation, but the nonlinear state space model has to be linearized before applying the algorithm. The resulting algorithm is known as the Extended Kalman Filter (EKF).

Whether for KF or EKF algorithm, the estimation process is done in two steps: a prediction step and a correction step. In the prediction step, we estimate the states by using the prior knowledge given by the state space model, and then we correct the estimation with the measurements performed and by calculating Kalman gain to get a better estimation. The Kalman gain is calculated by applying the minimum mean square error (MMSE) criteria to
the state space model. The model is supposed to be not precise and is affected by random process noise vector \( w_{k-1} \) and the measurements are noisy and altered by a random noise vector \( v_k \). Both \( w_{k-1} \) and \( v_k \) are assumed to be independent, have a Gaussian distribution of zero means and covariance matrices \( \Sigma_w \) and \( \Sigma_v \) respectively [21].

The DFIG state space model obtained in equation (16) has to be discretized in order to be used for state estimation by Kalman Filter algorithm [22].

The discretization of DFIG nonlinear model using Forward Euler’s method gives the following discrete nonlinear model.

\[
\begin{align*}
\mathbf{x}_k &= \mathbf{x}_{k-1} + \Delta t \hat{f}(\mathbf{x}_{k-1}) + \mathbf{B}_1 \mathbf{u}_{k-1} \\
\mathbf{z}_k &= \mathbf{C} \mathbf{x}_k + \mathbf{v}_k
\end{align*}
\]

Where: \( \mathbf{B}_1 = \mathbf{B} \Delta t \), \( \Delta t \) is the discretization step.

The following steps summarizes the proposed EKF algorithm for DFIG machine:

**Step 1: state-space discretization**
\[
\begin{align*}
\mathbf{x}_k &= \mathbf{x}_{k-1} + \Delta t \hat{f}(\mathbf{x}_{k-1}) + \mathbf{B}_1 \mathbf{u}_{k-1} + \mathbf{w}_{k-1} \\
\mathbf{z}_k &= \mathbf{C} \mathbf{x}_k + \mathbf{v}_k
\end{align*}
\]

**Step 2: Initialization**
\[
\begin{align*}
\hat{\mathbf{x}}_0 &= \hat{f}(\mathbf{x}_0) \\
\hat{\mathbf{p}}_0 &= \hat{f}(\mathbf{x}_0) - \mathbf{C} \mathbf{x}_0 \Rightarrow \hat{\mathbf{p}}_0 = \mathbf{E}(\mathbf{x}_0 - \hat{\mathbf{x}}_0)^T
\end{align*}
\]

**Step 3: For \( k = 1, 2, ..., N \), calculate:**

**Linearization:**
\[
\begin{align*}
\mathbf{A}_{k-1} &= \frac{df_x}{dx}(\mathbf{x}_{k-1}) \\
\mathbf{A}_x &= \mathbf{I} + \Delta t \mathbf{A}_{k-1}
\end{align*}
\]

**Prediction:**
\[
\begin{align*}
\hat{\mathbf{x}}_k &= \mathbf{A}_{k-1} \hat{\mathbf{x}}_{k-1} + \mathbf{B}_1 \mathbf{u}_{k-1} + \mathbf{w}_{k-1} \\
\hat{\mathbf{p}}_k &= \mathbf{A}_{k-1} \hat{\mathbf{p}}_{k-1} \mathbf{A}_{k-1}^T + \Sigma_w
\end{align*}
\]

**Correction:**
\[
\begin{align*}
\mathbf{L}_k &= \Sigma_{k-1} \mathbf{C}^T \left[ \Sigma_{k-1} \mathbf{C} \Sigma_w + \Sigma_v \right]^{-1} \\
\hat{\mathbf{x}}_k &= \hat{\mathbf{x}}_k + \mathbf{L}_k (\mathbf{z}_k - \hat{\mathbf{z}}_k) \\
\Sigma_k &= \left( \mathbf{I} - \mathbf{L}_k \mathbf{C} \right) \Sigma_{k-1}
\end{align*}
\]

Now, we develop the Extended Kalman Filter equations for the central difference method [23].

**Step 1:** The nonlinear discrete-time state-space representation:
\[
\begin{align*}
\mathbf{x}_k &= \mathbf{x}_{k-1} + 2 \Delta t f(\mathbf{x}_{k-1}) + 2 \mathbf{B}_1 \mathbf{u}_{k-1} + \mathbf{w}_{k-1} \\
\mathbf{z}_k &= \mathbf{C} \mathbf{x}_k + \mathbf{v}_k
\end{align*}
\]

**Step 2:** Linearization of the equation yields to:
\[
\begin{align*}
\mathbf{A}_{k-1} &= \frac{df_x}{dx}(\mathbf{x}_{k-1}) \Rightarrow \mathbf{A}_x = 2 \Delta t \mathbf{A}_{k-1} \\
\mathbf{x}_k &= \mathbf{x}_{k-1} + \mathbf{A}_x \mathbf{x}_{k-1} + 2 \mathbf{B}_1 \mathbf{u}_{k-1} + \mathbf{w}_{k-1} \\
\mathbf{z}_k &= \mathbf{C} \mathbf{x}_k + \mathbf{v}_k
\end{align*}
\]

**Step 3:** The state prediction is:
\[
\begin{align*}
\hat{\mathbf{x}}_k &= \hat{\mathbf{x}}_{k-1} + 2 \Delta t f(\hat{\mathbf{x}}_{k-1}) + 2 \mathbf{B}_1 \mathbf{u}_{k-1} \\
\text{The estimation error is:} \\
\hat{\mathbf{x}}_k - \hat{\mathbf{x}}_{k-1} &= \mathbf{A}_x \hat{\mathbf{x}}_{k-1} + \mathbf{w}_{k-1}
\end{align*}
\]

The error covariance matrix is obtained as follows:
\[
\Sigma_{k+1} = E[(\mathbf{x}_k - \hat{\mathbf{x}}_k)(\mathbf{x}_k - \hat{\mathbf{x}}_k)^T]
\]

The extended Kalman filter for Adams-Bashforth 2nd order method:

**Step 1:** discrete-time state-space representation:
\[
\begin{align*}
\mathbf{x}_k &= \mathbf{x}_{k-1} + \frac{3}{2} \Delta t f(\mathbf{x}_{k-1}) - \frac{1}{2} \Delta t f(\mathbf{x}_{k-2}) + \mathbf{B}_1(3 \mathbf{u}_{k-1} - \mathbf{u}_{k-2}) + \mathbf{w}_{k-1} \\
\mathbf{z}_k &= \mathbf{C} \mathbf{x}_k + \mathbf{v}_k
\end{align*}
\]

**Step 2:** Linearizing the equation yields to:
\[
\begin{align*}
\mathbf{A}_{k-1} &= \frac{df_x}{dx}(\mathbf{x}_{k-1}) \Rightarrow \mathbf{A}_x = \frac{df_x}{dx}(\mathbf{x}_{k-1}) \\
\mathbf{x}_k &= \mathbf{x}_{k-1} - \mathbf{A}_2 \mathbf{x}_{k-2} + \mathbf{B}_1(3 \mathbf{u}_{k-1} - \mathbf{u}_{k-2}) + \mathbf{w}_{k-1} \\
\mathbf{z}_k &= \mathbf{C} \mathbf{x}_k + \mathbf{v}_k
\end{align*}
\]

**Step 3:** The state prediction is:
\[
\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k-1} + \frac{3}{2} \Delta t f(\hat{\mathbf{x}}_{k-1}) - \frac{1}{2} \Delta t f(\hat{\mathbf{x}}_{k-2}) + \mathbf{B}_1(3 \mathbf{u}_{k-1} - \mathbf{u}_{k-2})
\]

The estimation error is:
\[
\hat{\mathbf{x}}_k - \hat{\mathbf{x}}_{k-1} \approx \mathbf{A}_x \hat{\mathbf{x}}_{k-1} + \mathbf{w}_{k-1}
\]

The error covariance matrix is obtained as follows:
\[
\Sigma_{k+1} = E[(\mathbf{x}_k - \hat{\mathbf{x}}_k)(\mathbf{x}_k - \hat{\mathbf{x}}_k)^T]
\]

The solution to this problem is to reinitialize the algorithm with \( N \) steps after \( N \) steps [24, 25].

Lastly, we develop the extended Kalman filter for Adams-Bashforth 2nd order method:

**Step 1:** discrete-time state-space representation:
\[
\begin{align*}
\mathbf{x}_k &= \mathbf{x}_{k-1} + \frac{3}{2} \Delta t f(\mathbf{x}_{k-1}) - \frac{1}{2} \Delta t f(\mathbf{x}_{k-2}) + \mathbf{B}_1(3 \mathbf{u}_{k-1} - \mathbf{u}_{k-2}) + \mathbf{w}_{k-1} \\
\mathbf{z}_k &= \mathbf{C} \mathbf{x}_k + \mathbf{v}_k
\end{align*}
\]

**Step 2:** Linearizing the equation yields to:
\[
\begin{align*}
\mathbf{A}_{k-1} &= \frac{df_x}{dx}(\mathbf{x}_{k-1}) \Rightarrow \mathbf{A}_x = \frac{df_x}{dx}(\mathbf{x}_{k-1}) \\
\mathbf{x}_k &= \mathbf{x}_{k-1} - \mathbf{A}_2 \mathbf{x}_{k-2} + \mathbf{B}_1(3 \mathbf{u}_{k-1} - \mathbf{u}_{k-2}) + \mathbf{w}_{k-1} \\
\mathbf{z}_k &= \mathbf{C} \mathbf{x}_k + \mathbf{v}_k
\end{align*}
\]

**Step 3:** The state prediction is:
\[
\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k-1} + \frac{3}{2} \Delta t f(\hat{\mathbf{x}}_{k-1}) - \frac{1}{2} \Delta t f(\hat{\mathbf{x}}_{k-2}) + \mathbf{B}_1(3 \mathbf{u}_{k-1} - \mathbf{u}_{k-2})
\]

The estimation error is:
\[
\hat{\mathbf{x}}_k - \hat{\mathbf{x}}_{k-1} \approx \mathbf{A}_x \hat{\mathbf{x}}_{k-1} + \mathbf{w}_{k-1}
\]
Where:
\[
\Sigma_{r,(k-1,k-2)} = E[\{\dot{x}_{r,i}^* (\hat{x}_{r,j}^*)\}^T]
\]
\[
= E[(A_{r,i} \dot{x}_{r,i}^* - A_{r,i} \dot{x}_{r,j}^* + w_{r,j}) (\hat{x}_{r,j}^*)^T]
\]
\[
= A_{r,i} \Sigma_{r,(k-2)} - A_{r,j} \Sigma_{r,(k-3,k-2)}
\]
And
\[
\Sigma_{r,(k-2,k-1)} = E[\{\dot{x}_{r,i}^* (\hat{x}_{r,j}^*)\}^T]
\]
\[
= E[(A_{r,i} \dot{x}_{r,i}^* - A_{r,i} \dot{x}_{r,j}^* + w_{r,j}) (\hat{x}_{r,j}^*)^T]
\]
\[
= A_{r,i} \Sigma_{r,(k-1,k-2)} - A_{r,j} \Sigma_{r,(k-2,k-2)}
\]
As in the LP method, the new terms in these equations are: \(\Sigma_{r,(k-1,k-2)}\) and \(\Sigma_{r,(k-2,k-1)}\) which are computed recursively as shown in the equations above.

The following steps summarize the EKF algorithm for the Adams-Bashforth discretization applied for DFIG model [26].

**Step 1:** state-space discretization
\[
\begin{align*}
\dot{x}_i &= x_{i+1} + \Delta t f(x_i) + B_i u_i + w_i \\
\dot{z}_i &= C x_i + v_i
\end{align*}
\]
**Step 2:** Initialization
\[
\dot{x}_i = E[x_i], \quad \Sigma_{i,0} = E[(x_i - \dot{x}_i)(x_i - \dot{x}_i)^T]
\]
**Step 3:** For \(k = 1,2,...,N\), calculate:

**Linearization:**
\[
A_{r,i} = \frac{df(x_i)}{dx_i}, \quad A_{r,j} = \frac{df(x_j)}{dx_i}, \quad B_i = \frac{Ba}{2}
\]

**Prediction:**
\[
\dot{x}_i = \dot{x}_{i+1} + \frac{\Delta t}{2} Mf(\dot{x}_{i+1}) + \frac{1}{2} B_i (3u_{i+1} - u_i) - \frac{1}{2} \frac{df}{dx_i} (\dot{x}_{i+1} - \dot{x}_i)
\]
\[
\Sigma_{i,i} = \Sigma_{r,(i+1,i)} - \frac{1}{2} \frac{df}{dx_i} \Sigma_{r,(i+2,i)} - \frac{1}{2} \frac{df}{dx_i} \Sigma_{r,(i+1,i+2)} + A_{r,i} \Sigma_{r,(i,i+1)} A_{r,j}^T
\]
\[
\Sigma_{i,j} = C \Sigma_{r,i} C^T + \Sigma_{z_i}
\]

**Correction:**
\[
L_i = \Sigma_{i,i} C^T [C \Sigma_{i,i} C^T + \Sigma_{z}]^{-1}
\]
\[
\dot{x}_i = \dot{x}_i + L_i (z_i - \dot{x}_i)
\]
\[
\Sigma_{i,j} = (I - L_i) \Sigma_{i,j}
\]

In the following section, the developed algorithms will be implemented to estimate the speed and rotor flux of a DFIG machine, then a comparison study will be discussed [27].

### V. Results and Discussion

Table 1. shows the DFIG rating parameters used for the simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Rating values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Power</td>
<td>3 kW</td>
</tr>
<tr>
<td>(R_s)</td>
<td>2.0 (\Omega)</td>
</tr>
<tr>
<td>(R_r)</td>
<td>1.78 (\Omega)</td>
</tr>
<tr>
<td>(L_s)</td>
<td>0.2406 H</td>
</tr>
<tr>
<td>(L_r)</td>
<td>0.2406 H</td>
</tr>
<tr>
<td>(L_m)</td>
<td>0.2304 H</td>
</tr>
<tr>
<td>Pole pairs</td>
<td>2</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>0.0408 kg.m²</td>
</tr>
</tbody>
</table>

Figure 4 shows the mechanical torque input used for simulation. We use variable mechanical torque, assuming it results from a variable wind speed hitting the wind turbine blades and rotating the DFIG rotor.

![Fig.4. Input mechanical torque of the DFIG](image_url)

For the purpose of simulation, the measured variables \(i_b\) and \(i_{sl}\) are obtained from solving the DFIG equations numerically by using the Runge-Kutta algorithm. Then we add some measurement noise of covariance matrix:

\[
\Sigma_i = \begin{bmatrix}
\sigma_{i_b}^2 & 0 \\
0 & \sigma_{i_{sl}}^2
\end{bmatrix} = \begin{bmatrix}
10^{-1} & 0 \\
0 & 10^{-1}
\end{bmatrix}
\]

Also, we need some process noise to the DFIG state space model with a covariance matrix given by:

\[
\Sigma_e = \begin{bmatrix}
10^{-1} & 0 & 0 & 0 & 0 \\
0 & 10^{-1} & 0 & 0 & 0 \\
0 & 0 & 10^{-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Figure 5 shows the stator currents of the DFIG machine. The current waveforms contain fluctuations resulting from the variation in mechanical torque applied to the DFIG machine [26, 27].

![Fig.5. Stator currents of the DFIG](image_url)

In what follows, the results of estimation using EKF corresponding to Forward Euler, central difference and Adams-Bashforth discretization methods will be discussed and compared. The estimated and simulated rotor speeds are shown in Fig. 6 where \(\dot{\omega}_{r,1}\) is for Forward Euler method, \(\dot{\omega}_{r,2}\) is for EKF with central Difference method and \(\dot{\omega}_{r,3}\) is for EKF with Adams-Bashforth method.

From Fig. 6, we can see that the rotor speed is better estimated with the Adams-Bashforth method than with the Forward Euler or Central Difference methods. Moreover, ABZ has better performance in both the transient regime and permanent response. This is due to the better accuracy of second-order methods over first-order methods. As shown in Fig. 6, the Central Difference discretization (LP
method) results are noisy due to the instability problems discussed earlier. Overall, the AB2 with EKF gives better results with less noise in the response than LP with EKF or Forward Euler with EKF [29]. Table II gives the maximum errors of speed estimation for the three methods in both the transient regime and permanent regime.

Figures 7 and 8 demonstrate the estimated rotor flux by Forward Euler (FE), Leap-Frog (LP), and AB2 methods. The estimation of the rotor flux is better with AB2 than with FE or LP methods. As in speed estimation, the Central Difference discretization gives noisy estimation. The noisy response in the LP method is related to its instability problems over long-term integration. Similar to speed estimation, we can also say that AB2 with EKF is less noisy and more accurate than FE or LP with EKF. Overall, the rotor flux is better estimated with the AB2 method. Table III and Table IV give the maximum errors of flux estimation for the three methods in both transient and permanent regimes.

VI. Conclusion

In this paper, a combination of second-order discretization methods with the Extended Kalman Filter (EKF) was proposed. Modified equations of the EKF have been introduced based on central difference and Adams-Bashforth discretization methods. The developed EKF algorithm is used to estimate the rotor speed and flux of a Doubly Fed Induction Generator (DFIG). From the obtained results, it was found that the EKF combined with Adams-Bashforth provides the best performance in terms of precision and stability. EKF with central difference has shown good performance, but the estimation results were a bit noisy due to instability problems, as discussed earlier. Based on these results, it is highly recommended to use the proposed EKF combined with Adams-Bashforth, as it...
provides the best accuracy with less complexity compared to other algorithms such as Unscented Kalman Filter (UKF) and Cubature Kalman Filter (CKF). For future work, it is highly recommended to implement the proposed approach in a test bench of a DFIG machine.

**ACKNOWLEDGEMENTS**

This study has been sponsored by DGRSDT (Direction Générale de la Recherche Scientifique et Développement Technologique) Algiers, Algeria.

**Conflict of Interest**

The authors declare that they have no conflicts of interest and we do not have any financial relationship with the organization that sponsored the research.

**Author:** PhD Student. Ahmad BOUSSOUFA, Applied Automation Laboratory, Faculty of Hydrocarbons, University of Mhamed Bougara of Boumerdes, 35000, Algeria, E-mail: a.boussoufa@univ-boumerdes.dz

**REFERENCES**


https://doi.org/10.1080/03772063.2020.1724834


DOI: 10.1109/TII.2016.2549940


DOI: 10.1109/TII.2016.2549940


