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Application of superconducting resonator for energy storage

Zastosowanie rezonatora nadprzewodzącego do magazynowania energii

Abstract. *The article describes the construction and principle of operation of a system with a superconducting resonator designed to store electrical energy and cooperate with the power grid. The resonator has the shape of a rectangular cavity cooled by liquid helium. The results of parameters calculations with fundamental importance for evaluating of the resonator, such as the volumetric density and massive density of the stored energy and the efficiency of the system are given. The obtained results show that for currently available superconductors of the second type with high critical field, the calculated densities are several times higher than for known systems storing energy in superconducting coils. The massive energy density also has the advantage of increasing with increase resonator size.*

Streszczenie. *W artykule jest opisana budowa i zasada działania układu z nadprzewodzącym rezonatorem, przeznaczonego do magazynowania energii elektrycznej i współpracy z siecią energetyczną. Rezonator ma kształt prostopadłościennej wnęki chłodzonej ciekłym helem. Podano wyniki obliczeń parametrów, mających podstawowe znacznie dla oceny użyteczności rezonatora, takich jak gęstość objętościowa i gęstość masowa magazynowanej energii oraz sprawność układu. Otrzymane wyniki pokazują, że dla dostępnych obecnie nadprzewodników drugiego rodzaju o wysokim polu krytycznym obliczone gęstości są kilka razy większe, niż dla znanych układów, magazynujących energię w cewkach nadprzewodzących. Zaletą rozpatrywanego rozwiązania jest również wzrost masowej gęstości energii wraz ze wzrostem rozmiarów rezonatora.*

Keywords: electric energy, storage, density, superconductor, cavity, resonance
Słowa kluczowe: energia elektryczna, magazynowanie, gęstość, nadprzewodnik, wnęka, rezonans

Introduction

Electrical energy is increasingly being used in technology and everyday life because it has many important advantages. Primary among these advantages are the following below. Easy conversion of electrical energy into other useful types of energy, including mechanical, thermal and light energy. The ability to generate electrical energy from a variety of energy sources including renewable sources such as solar, water and wind energy. Easy transmission of electrical energy with relatively low losses over long distances of several hundred kilometers or more. Unfortunately, electrical energy also has a significant disadvantage, which is the difficulty of storing it [1, 2].

The problem of storing electrical energy is important for two reasons. The first reason is that the demand for electrical energy is out of synchronization with the ability to produce it, especially in systems that generate electricity from renewable sources [3, 4], This results in deficits and surpluses of needed or produced energy. The second reason is the low capacity of known systems for storing electrical energy, such as systems using a spinning wheel or capacitors [5]. The described situation contributes to ongoing research aimed at building systems designed to store electrical energy more efficiently.

From the state of the art there are known systems for storing electrical energy in the form of electric current energy flowing in a coil made of superconductor [6-8]. The disadvantages of these systems are the low density of stored energy and the need to use a large mass of the superconductor, since in these systems the energy is stored in its entire volume [9]. The purpose of this article is to analyze the efficiency of electrical energy storage using a standing electromagnetic wave generated in a superconducting resonator. The article also describes the construction and operating principle of the system designed to implement this concept.

Construction and principle of operation of the system

The construction of the electrical energy storage system is shown in Fig. 1 and 2 [10]. The system include resonator in the shape of a rectangular cavity, having an outer wall 1 made of metal and covered on the inside with a layer of superconductor 2. The resonator was placed in a tank of liquid helium 3, covered with thermal insulation 4. In addition, the resonator is cooled below the critical temperature of superconductor 2 and isolated from the walls of the tank 3 by supports 5. The tank 3 is set on supports 6 connected to the ground. In the center of the resonator is placed a transmitter-receiver antenna 7, having terminals 8 which are surrounded by insulating sleeves 9 and led outside the tank 3.

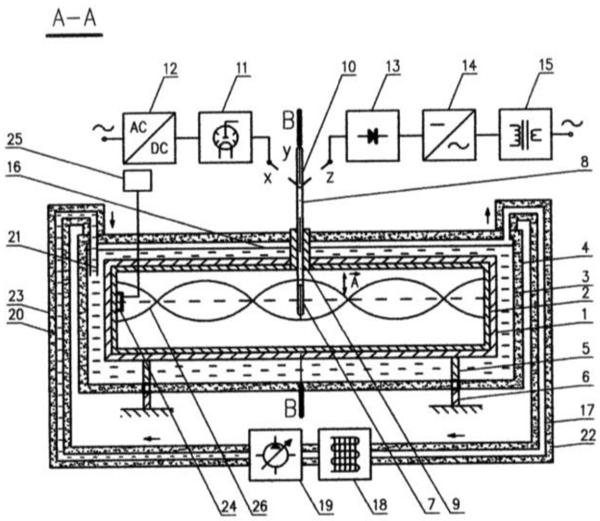

Fig. 1: Longitudinal cross section of the resonator with the A–A plane and schematic diagram of the system: 1 – outer wall of the resonator, 2 – superconducting layer, 3 – liquid helium tank, 4 – thermal insulation of the tank, 5 – resonator supports, 6 – tank supports, 7 – transmit-receive antenna, 8 – antenna terminals, 9 – insulating sleeves, 10 – three-position switch, 11 – magnetron, 12 – magnetron power supply, 13 – bridge rectifier, 14 – inverter, 15 – mains transformer, 16 – helium vapor outlet tube, 17 – liquid helium, 18 – helium condenser, 19 – liquid helium pump, 20 – helium inlet tube, 21 – inlet tube end, 22 – inlet tube thermal insulation, 23 – outlet tube thermal insulation, 24 – hallotron, 25 – magnetic field induction meter, 26 – electromagnetic microwave, X, Y, Z switch contact positions, H – magnetic field intensity

The terminals 8 are connected to a three-position switch 10, the contacts of which can be in the X, Y, Z positions, with the Y position being neutral. When the contacts of the switch 10 are in the X position, the antenna 7 is connected to the power grid through magnetron 11 and power supply 12. The contacts of the switch 10 in the Z position allow the antenna 7 to be connected to the power grid through a bridge rectifier13 connected to an inverter 14 and a power transformer 15. Above the level of liquid helium 16 are its vapors discharged through an outlet pipe 17 to a condenser 18 connected to a pump 19. After liquefaction, liquid helium is transported into tank 3 through pump 19 and inlet pipe 20, the lower end 21 of which is located in tank 3 below the level of liquid helium 16. Pipes 17 and 20 are covered with thermal insulation 22 and 23. A hallotron 24 is attached inside the resonator and, has been connected to a magnetic field induction meter 25 located outside tank 3.

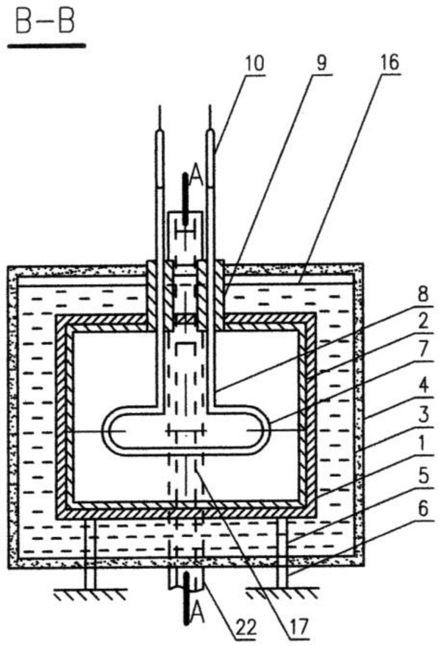

Fig. 2. Transversal cross section of the resonator with the B-B plane – the designations of the elements are the same as in Fig. 1.

The operating principle of the system is as follows. When the contacts of the switch 10 are in the X position, the AC energy from the power grid is transferred to the power supply 12, which gives DC voltages that allow the magnetron 11 to work, The high-frequency voltage generated by the magnetron 11 feeds the antenna 7, which radiates microwaves in the resonator, which is covered with a layer of superconductor 2. Thus, the microwaves are reflected practically without loss from the walls of the resonator and there is an increase in the energy of the oscillating electromagnetic field [11, 12]. Magnetron 11 operates at a frequency equal to the resonant frequency of the resonator, and thus standing microwave 26 is produced. The intensity of the magnetic field *H* of standing microwave 26 is controlled by hallotron 24 and meter 25. When the intensity *H* reaches a value close to the critical value for superconductor 2, then the contacts of switch 10 are moved to the Y position. This disconnects the system from the power grid and opens the terminals 8 of antenna 7. In this way, the electrical energy taken from the power grid is stored in the resonator. Thanks to the reflection of the electromagnetic wave without loss from the walls of the resonator covered with a layer of superconductor 2 and the opening of the terminals 8 of antenna 7, this energy is conserved [13, 14].

In order to recover the stored electricity, the contacts of the switch 10 should be set to the Z position. This causes the connection of the terminals 8 along with the antenna 7 to the bridge rectifier 13. The vibrating electromagnetic field of the standing microwave 26 induces a high-frequency alternating electric current in the antenna 7, which then flows through the bridge rectifier 13. After rectification, this current feeds the inverter 14 and is converted to an alternating current of 50 Hz frequency, used in the power grid. Further, the current is fed to the primary winding of transformer 15 and, after voltage adjustment, is transferred to the power grid. In this way, the energy of the electromagnetic field of the standing wave is given back to the power grid.

Calculation of resonator parameters

One of the most important parameters that characterize the efficiency of energy storage in a system is its density [9]. Two densities are used – volumetric density and massive density. Volumetric density ρ_V is defined as the amount of energy stored in a unit volume of the system. Similarly, massive density ρ_m is defined as the amount of energy stored per unit mass of the system, The density of energy in an electromagnetic field is the sum of the density ρ_{1V} of energy contained in a magnetic field of intensity *H* and the density ρ_{2V} of energy contained in an electric field of intensity *E*, and described by formula

(1) $$\rho_V = \rho_{1V} + \rho_{2V}$$

If we denote by H_0 the amplitude of the magnetic field intensity of the standing wave in the resonator, the maximum energy density of the magnetic field ρ_{1V0} is expressed by formula

(2) $$\rho_{1V0} = \frac{\mu_0 H_0^2}{2}$$

in which $\mu_0 = 4\pi \cdot 10^{-7}$ (Vs)/(Am) denotes the magnetic permeability of the vacuum. The value of the magnetic field intensity *H* in a standing microwave varies sinusoidally with distance *x*, so the average value, calculated as follows should be taken as ρ_{1V}

(3) $$\rho_{1V} = \frac{\mu_0}{2(\lambda/2)} \int_{x=0}^{x=\lambda/2} H_0^2 \sin^2\left(\frac{2\pi x}{\lambda}\right) dx = \frac{\mu_0 H_0^2}{4}$$

Similarly, they denote by E_0 the amplitude of the electric field intensity for the maximum energy density ρ_{V10} of this field can be written formula

(4) $$\rho_{2V0} = \frac{\varepsilon_0 E_0^2}{2}$$

Between the intensities *E* and *H* in an electromagnetic wave is the relation

(5) $$E = H\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

After substituting formula (5) into formula (4) and calculating the average in a similar way as before, one gets

(6) $$\rho_{2V} = \frac{\mu_0 E_0^2}{4} = \frac{\mu_0 H_0^2}{4} = \rho_{1V}$$

Substituting formulas (4) and (6) into formula (1), one gets the final formula, which expresses ρ_V by the amplitude of the magnetic field intensity H_0

$$\rho_V = \frac{\mu_0 H_0^2}{2} \tag{7}$$

A superconducting material losses its properties if the intensity of the magnetic field in which it is located exceeds the critical value H_k. Therefore, the condition $H_0 < H_k$ must be fulfilled. This condition means that the maximum volumetric energy density ρ_V in the resonator cannot exceed the value calculated from equation (7).

The massive energy density ρ_m is defined by the formula

$$\rho_m = \frac{E_r}{m_r} \tag{8}$$

in which E_r is the total energy included in the resonator, while m_r is the mass of the resonator. Having the previously calculated ρ_V, one can calculate E_r from equation

$$E_r = \rho_V V_w \tag{9}$$

in which V_w is the volume of the resonator cavity. Denoting the length of the cavity as l, its width as a and its height as b, one calculates V_w from the formula

$$V_w = lab \tag{10}$$

Denoting by g_n and g_s the thicknesses of the superconductor layer and resonator wall, respectively, and by d_n and d_s the densities, respectively, of the superconductor and wall material, the following formula for m_r can be written

$$m_r = 2(la + lb + ab)(g_n d_n + g_s d_s) \tag{11}$$

In writing formula (11), a simplification was accepted by omitting the increase in the dimensions of resonator wall 1 by the thickness of superconductor layer 2. This simplification is justified because both thicknesses are small compared to the dimensions of the resonator cavity. After substituting formulas (9), (10) and (11) into formula (8), the final formula for ρ_m is obtained

$$\rho_m = \frac{\rho_V lab}{2(la + lb + ab)(g_n d_n + g_s d_s)} \tag{12}$$

The walls of the resonator are in a strong magnetic field, the maximum intensity of which reaches H_k. As a result, the material used for the walls is subjected to mechanical stresses σ, which cannot exceed its tensile stress σ_r. To calculate these stresses, the formula is used [15].

$$\sigma = \frac{\mu_0 H_k^2}{2} \tag{13}$$

The frequency of the standing wave f_r, produced inside a cuboid resonator, is expressed by the formula

$$f_r = \frac{c}{2}\sqrt{\left(\frac{n_l}{l}\right)^2 + \left(\frac{n_a}{a}\right)^2 + \left(\frac{n_b}{b}\right)^2} \tag{14}$$

in which $c = 3 \cdot 10^8$ m/s denotes the speed of light in vacuum, while n_l, n_a, n_b are natural numbers expressing the number of standing-wave lengths λ that were produced along each side of the resonator.

Results of calculations

Nine superconductors were selected for the calculations, whose basic parameters are given in Tab. 1 [16-19]. The following dimensions of the resonator cavity were also assumed: l = 4 m, a = 0.8 m, b = 0.4 m, as well as the thickness of its wall g_s = 10 mm and the thickness of the superconductor layer g_n = 5. In addition, it was assumed that walls 1 would be made of tungsten with tensile stress σ_r = 1715 MPa and density ρ_s = 19.27 kg/dm^3 [20]. The choice of tungsten for walls 1 is justified by the high tensile stress of this material and the very low relative elongation before break of 2%. Using the superconductor parameters given in Tab. 1, the assumptions made and formulas (7), (9) (12), the energy densities ρ_V, ρ_m and the total energy E_r included in the resonator were calculated. The results of these calculations are given in Tab. 2 and shown in Fig. 3-5. In turn, stresses were calculated from formula (13), obtaining values of σ = 39÷1685 MPa. In addition, n_l = 20, n_a = 4 and n_b = 2 were assumed, and from equation (14) the resonant frequency f_r = 1.308 GHz was calculated, which corresponds to the wavelength λ = 22.94 cm.

Table 1. Superconductor parameters

No	Superconductor formula	T_k [K]	H_k [MA/m]	ρ_m [kg/dm^3]
1	$C_{0.44}Mo_{0.56}$	13	76.9	6.12
2	GeV_3	9	28.0	6.06
3	La_2BaCuO_4	38	39.8	6.22
4	$Nb_{0,79}Al_{0,15}Ge_{0,06}$	23	22.0	7.38
5	Nb_3Sn	18	19.5	8.26
6	$PbMo_6S_8$	14.5	47.8	5.96
7	SiV_3	17	12.5	5.17
8	$Ti_{0.75}V_{0.25}$	5.3	15.9	4.90
9	$YBa_2Cu_3O_8$	92	51.8	6.40

T_k – critical temperature, H_k – magnetic field intensity, ρ_n – density

Table 2. Resonator parameters

No	Superconductor formula	ρ_V [kJ/dm^3]	ρ_m [kJ/kg]	E_r [MJ]
1	$C_{0.44}Mo_{0.56}$	39	22	50
2	GeV_3	492	276	630
3	La_2BaCuO_4	995	556	1274
4	$Nb_{0,79}Al_{0,15}Ge_{0,06}$	304	166	389
5	Nb_3Sn	240	128	307
6	$PbMo_6S_8$	1435	770	1896
7	SiV_3	98	56	125
8	$Ti_{0.75}V_{0.25}$	159	80	203
9	$YBa_2Cu_3O_8$	1685	937	2157

ρ_V – volumetric energy density, ρ_V – massive energy density
E_r – total energy in resonator

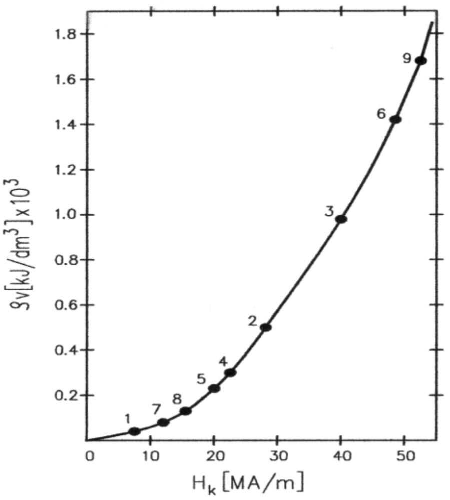

Fig. 3. Dependence of the volumetric energy density ρ_V on the critical field intensity H_k for the superconductors under consideration, the point numbers denote the numbers of lines in Tab. 1

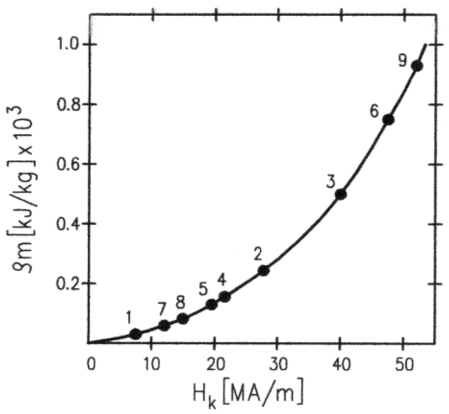

Fig. 4. Dependence of the massive energy density ρ_m on the critical field intensity H_k for the superconductors under consideration, point numbers denote numbers of lines in Tab. 1

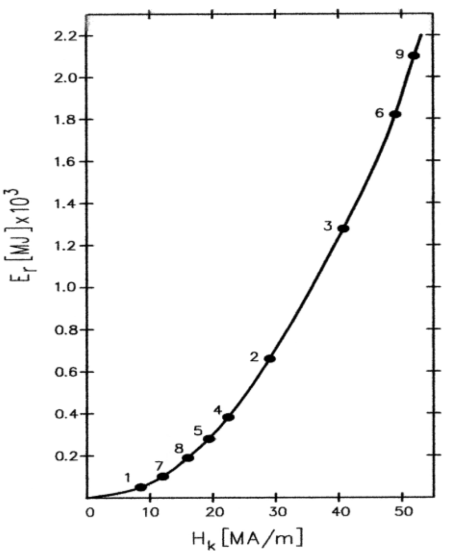

Fig. 5. Dependence of the total energy in the E_r resonator on the critical field intensity H_k of the superconductors used to make it, the numbers of points denote the numbers of lines in Tab. 1

Discussion of the results and conclusions

As mentioned in the introduction, there are known systems for storing energy in coils made of superconductor. The volumetric energy density ρ_V in these coils is about 40 kJ/dm^3, while the massive energy density ρ_m is in the range of 4÷40 kJ/kg. For almost all superconductors, the values of the corresponding energy densities calculated and given in Tab. 2 are a few times greater than those for the coils. The exception is the superconductor with the formula $C_{0.44}Mo_{0.56}$, for which the value of H_k is the smallest. In addition, superconductors with the highest critical field intensities H_k show the largest density differences.

According to formulas (7) and (12), the volumetric energy density ρ_V, as well as its mass density ρ_m, are directly proportional to the square of the magnetic field intensity. Therefore, it is advantageous to use superconductors of the second type, which exhibit a mixed state, limited by a high value of the critical field H_k [21-23]. It is the parameters of such superconductors were used for the calculations. Since in the system under consideration the superconductor is needed only to cover the inner surface of the resonator walls and thus its mass is much smaller than in systems, containing coils. In addition, according to equation (12), the massive energy density ρ_m increases with the volume of the resonator and the total energy E_r stored in it. This relationship can be shown even better for resonators with simple shapes presented in the two examples.

Using formulas (8) and (9), it is also possible to calculate the energy densities ρ_{ms} and ρ_{mk} contained in a resonator having the shape of a cube or a sphere, respectively. After substituting the commonly known formulas for the volumes of these solids into formulas (8) and (9), the following formula is obtained for a cube-shaped resonator with side a

$$\rho_{ms} = \rho_V \left(\frac{a}{6}\right) \frac{1}{(\rho_n g_n + \rho_s g_s)} \tag{15}$$

In turn, for a sphere-shaped resonator of radius r, the formula is of the form

$$\rho_{mk} = \rho_V \left(\frac{r}{3}\right) \frac{1}{(\rho_n g_n + \rho_s g_s)} \tag{16}$$

From formulas (15) and (16), it follows that the mass densities of stored energy grow directly proportional to the length of the side of the cube a and to the radius of the sphere r, respectively. A comparison of equations (15) and (16) also shows that if $a = r$, the mass density of energy in a cubic resonator is twice that in a spherical resonator. This result is understandable, since the sphere limits the maximum volume from all solids of different shapes having the same surface area. The obtained relations lead to the conclusion that it is more economical to build the analyzed resonators, designed to store a large quantity of energy. In addition, calculated from equation (14), the resonant frequency f_r = 1.308 GHz. This value corresponds to microwaves of 22.94 cm wave length, and therefore a magnetron is used to generate vibrations. The stresses σ calculated from equation (13) for all superconductors given in Tab. 1 are contained within 39÷1685 MPa and do not exceed the tensile stress of tungsten σ_r = 1715 MPa. This means that tungsten is a suitable material for the resonator walls.

An important parameter is also the total efficiency η of the system under consideration, defined as the ratio of energy taken from the power grid to energy given back. The efficiency of systems converting alternating current energy into microwave energy reaches 75% [11]. The efficiencies of inverters and power transformers are higher, reaching over 95%. Assuming that in the superconducting state the losses of stored energy in the resonant cavity are negligible, the upper limit of the efficiency of the considered system can be estimated as 67% based on these data.

The calculated energy densities are comparable to the energy densities stored in capacitors and lithium-ion batteries, which are 40-200 kJ/kg, and several times higher than the energy densities stored in lead-acid batteries, or in a spinning wheel [24]. However, these densities are many times less than the amount of energy possible to obtain from a unit mass of fuels, as the heat of combustion. For gaseous fuels, this heat is contained in the range of 7.3-80 MJ/kg, for liquid fuels it is 23-40 MJ/kg, and for solid fuels it is 19-32 MJ/kg. Therefore, the calculations and analyses carried out allow the general conclusion that the concept of the described system is promising, but requires further research to improve its performance. One possibility for progress in this area is to develop methods for producing new superconductors that exhibit high temperatures and high critical fields without exerting large pressures [25].

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