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Analytical and numerical evaluation of the efficiency of a low power WPT system

***Abstract**. The article presents the results of the numerical and analytical analysis of the wireless power transfer (WPT) system. The system consists of a transmitting and receiving surfaces, each consisting of flat spiral coils. Both proposed approaches reduce the size and complexity of models. Two coil systems were also considered: periodic and with variable winding direction. The difference between the proposed systems is a different way of winding the coils. In the periodic system, all coils are wound in the same direction. The influence of the type of coils, winding direction, number of turns and distance between coils on the efficiency of the WPT system was compared. The analysis covered a wide frequency range from 100 kHz to 1000 kHz. The Finite Element Method (FEM) was used for the analysis. Comparing both systems, models with variable coil winding show higher efficiency values. The proposed system allows for simultaneous charging of many sensors (located e.g. in walls).*

***Streszczenie.** W artykule przedstawiono wyniki analizy numerycznej i analitycznej systemu bezprzewodowego przesyłu mocy (WPT). System składa się z powierzchni nadawczej i odbiorczej, z których każda składa się z płaskich cewek spiralnych. Oba proponowane podejścia redukują rozmiar i złożoność modeli. Rozważono również dwa układy cewek: periodyczny i ze zmiennym kierunkiem nawinięcia uzwojenia. Różnica między proponowanymi układami polega na innym sposobie nawijania cewek. W układzie periodycznym wszystkie cewki są nawijane w tym samym kierunku. Porównano wpływ rodzaju cewek, kierunku nawinięcia uzwojenia, liczby zwojów i odległości między cewkami na sprawność systemu WPT. Analiza objęła szeroki zakres częstotliwości od 100 kHz do 1000 kHz. Do analizy wykorzystano metodę elementów skończonych (MES). Porównując oba układy, modele ze zmiennym uzwojeniem cewek wykazują wyższe wartości sprawności. Proponowany układ umożliwia jednoczesne ładowanie wielu czujników (znajdujących się np. w ścianach). (**Analityczna i numeryczna ocena sprawności systemu WPT małej mocy**).*

Keywords: wireless power transfer (WPT), planar coils, magnetic fields, FEM.
Słowa kluczowe: bezprzewodowa transmisja energii (WPT), cewki planarne, pole magnetyczne, FEM.

Introduction

One way to power mobile devices is to charge them using wireless power transfer (WPT) [1, 2, 3]. Thanks to the concept of inductive power transfer (IPT), it is possible, among others, to wirelessly charge modern technology devices (phones, smartphones, laptops). WPT is becoming more and more widely used. Wireless charging is also suitable for lighting hard-to-reach places or intelligent buildings with sensors in the walls. WPT is considered an alternative method of charging wireless devices.

Typical charging of devices with cables and wires turned out to be very lossy, difficult to install and subject to frequent breakdowns [4]. The number of portable devices is still growing, but wired chargers mean that the devices are not fully portable. For this reason, alternative solutions related to the energy transmission and distribution were sought. Such solution is the wireless power transfer (WPT) technology described in [1-3]. WPT system can effectively transmit power from a source to a device using the rule of electromagnetic induction. The advantage of this system is the mobility of the devices and non-radiation. This system uses i.a. the pair of coils [5]. It is also possible to use intermediate coils [6] and even the coil system [7-8].

For low frequencies, mainly the coil system in the form of domino resonators [9] and linear resonator systems [10] is used, where energy transfer in the space between the transmitter and the receiver is supported by the use of several resonators. The purpose of this solution is to increase the efficiency of the system. Disadvantage of this approach is it requires more place than any other transmission system. There are many studies and experiments on electric vehicles to reduce fuel and energy consumption [11,12]. Also in the medical field, WPT is studied and even used when charging or powering medical devices (eg pacemakers, endoscopy) [13,14]. WPT system is also considered in inteligent buildings containing the sensors placed inside walls [15]. A detailed analysis was performed for a series configuration of resonators [16,17], while parallel-series topology of planar coils [18], acting as group of energy transmitters and receivers, are still not fully developed.

In this article, the wireless power transfer system with periodically arranged planar coils was presented. The author propose and study numerical and analytical model, which can be used to analyse power transfer conditions in presented WPT systems. Both solutions reduce size and complexity of the numerical and analytical models. The proposed single cell analysis with periodic boundary conditions does not require the three-dimensional model with many coils. The advantage of this solution is a large reduction in the degrees of freedom. Two types of the coils: the circular and the square were analysed. Two systems were also considered: with periodic and antiperiodic boundary conditions. The difference between these systems concerned the winding direction of the turns in adjacent coils. Calculations of exemplary WPT systems were performed over frequency range from 0.1 MHz to 1 MHz. By using both proposed method, the author analysed the influence of: type of the coils, the winding direction and geometric parameters (number of turns and distance between the coils) on power transfer efficiency.

Geometry and description of the analysed models of WPT system

The presented WPT system is composed of pairs of transmitting-receiving coils which are called WPT cells (Fig. 1). Two main approaches to the WPT are considered. One is connected with the circular coils (Fig. 1a) and the other with the square coils (Fig. 1b). The outside dimensions of each transmitter/receiver cell are $d \times d$, the same radius (r) and number of turns wounded around a dielectric carcass (n).

The transmitting surface is powered so that each transmitter is connected in parallel to a sinusoidal voltage source with the effective value (U). The coils creating the receiving surface are connected directly to the load. Each WPT cell is assigned a separate load ($\underline{Z}$). The analysis concerned two waraints of the coils. The first one was made of circular coils (Fig. 1a) and the second one was made of square coils (Fig. 1b).

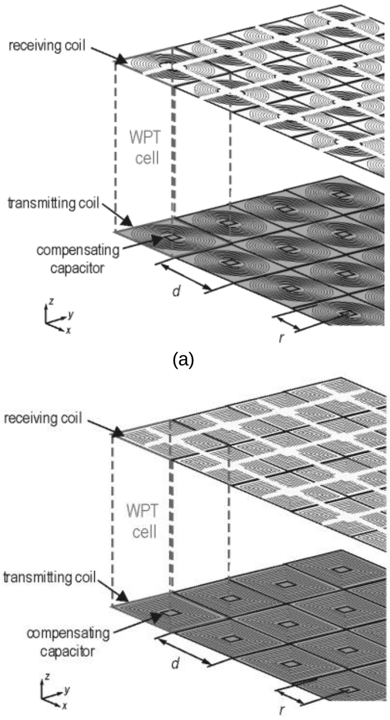


Fig. 1. Proposed WPT system consisting of planar: (a) circular coils, (b) square coils

Additionally, the influence of the winding direction on the parameters of the system was considered. The system, which has the transmitting / receiving surface composed of coils wound in the same direction, is called the periodic system (Fig. 2a). On the other hand, the WPT system with alternately wound turns was called aperiodic (Fig. 2b).

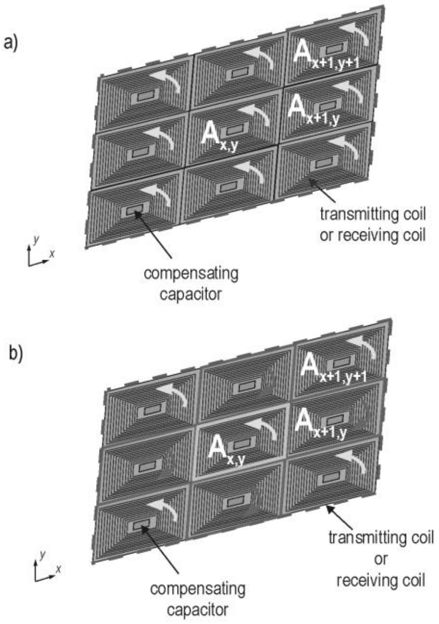


Fig. 2. The analysed transmitting/receiving surface composed of: (a) square coils with the same winding direction, (b) square coils with the opposite winding direction

The influence of the change in the number of turns and the distance between the transmitting and receiving surface (*h*) on the efficiency was analysed (Table 1). Additionally, the influence of the winding direction on the parameters of the system was considered. The presented models, which are a solution for the WPT system, were solved by the numerical and analytical methods. Of course, the results of both methods were compared.

Table 1. Variants of geometric parameters accepted for analysis

r (mm)	*n* (-)	*h* (mm)	
		h = 0.5 *r*	*h* = *r*
5	10	2.5	5.0
	30	2.5	5.0

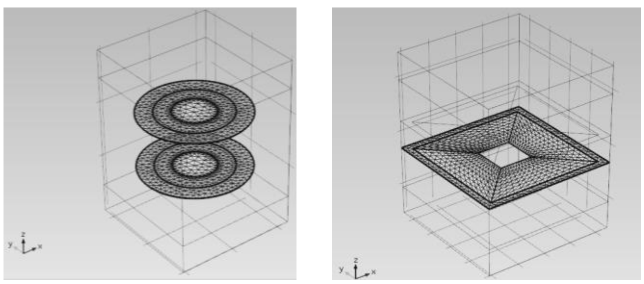
Solution by numerical method

The analysis of the WPT system can be performed using the numerical methods (e.g. FEM, FDTD, FDFD) [19-23], analytical method [13] or experimentally. Each numerical method requires preparing exact three-dimensional model and assigning different boundary conditions. The Finite Element Method (FEM) was used to analyze the models of the WPT system [31]. This analysis requires taking into account e.g.: the geometry and distribution of the coil turns, the number of WPT cells and elements of the electric circuit connected to each coil. The turns of the coils were made of thin wire (*w*) with insulation layer (*i*). The compensating capacitor is modelled as the element with a concentrated capacity (*C*). The parameters for the WPT model, coil windings and the frequency domain are shown in Table 2.

Table 2. Parameters of the wire used to the analysis

Parameter	Symbol	Value
Diameter of the wire	*w*	150 μm
Thickness of the wire insulation	*i*	1 μm
Conductivity of the wire	*σ*	$5.6{\cdot}10^{7}$ S/m
Voltage source	*U*	1 V
Load impedance	*Z*	50 Ω

In the proposed numerical approach, the entire WPT system is simplified to the single cell $A_{x,y}$ containing a pair of transmitting and receiving coils. Analyzed cell $A_{x,y}$ is filled with air (Fig. 3). On the top and the bottom of the model, the perfectly matched layer (PML) was put to imitate infinite dielectric background. In the periodic models, an infinite array of the coils was modeled utilizing the periodic boundary conditions (PBC) [21]. Whereas, in aperiodic models an antiperiodic boundary conditions are applied, in order to project infinite set of the WPT cells.



(a) (b)

Fig. 3. The numerical model of the single WPT cell with: (a) circular coils, (b) square coils

The problem of energy transport in the analysed model is solved using magnetic vector potential

(1) $$\mathbf{A} = [\mathbf{A}_x \ \mathbf{A}_y \ \mathbf{A}_z]$$

and description of magnetic phenomena in the frequency domain using the Helmholtz equation

(2) $$\nabla \times \left(\mu_0^{-1} \nabla \times \mathbf{A}\right) - \mathrm{j}\omega\sigma\mathbf{A} = \mathbf{J}_{ext}$$

where: $\mathbf{J}_{ext}$ – external current density vector [A/m^2]; ω – pulsation [rad/s]; σ – conductivity [S/m].

Periodicity conditions on four side surfaces are given in the form of magnetic isolation where **n** is a surface normal vector $\mathbf{n}=[\mathbf{1}_x \ \mathbf{1}_y \ \mathbf{1}_z]$

(3) $$\mathbf{n} \times \mathbf{A} = 0 \, .$$

Solution by the analytical method

As optional for the numerical solution, an analytical model with analytical equations for calculating lumped parameters was proposed (Fig. 4). In this method, the WPT wide area set analysis was reduced to the single WPT cell.

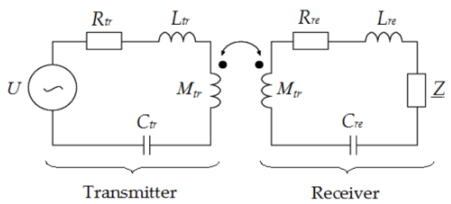


Fig. 4. Circuit model of the cell in the WPT system

The length of all windings in the circular coil (l_{cir}) is presented by equation (3), whereas for the square coil (l_{squ}) by equation (4):

(4) $$l_{cir} = 2\pi n\left[r - 0.5(n-1)(w+i)\right],$$

(5) $$l_{squ} = 4n\left[2r - n(w+i)\right].$$

The greater problem is to determine the values of the lumped parameters taking into account the influence of the adjacent cells on the equivalent inductances of the transmitting coil (L_{tr}), receiving coil (L_{re}) and their mutual inductance (M_{tr}). The analytical solution was presented in [7,8]. The mutual inductance (M_{tr}) between the transmitter and the receiver is presented by:

(6) $$M_{tr} = \left| \frac{\underline{U}_{r,\infty}}{2\pi f \, \underline{I}_{t,\infty}} \right|$$

where: $\underline{U}_{r,\infty}=|\underline{U}_{r,\infty}|e^{j\theta}$ – voltage induced in the receiving coil, $|\underline{U}_{r,\infty}|$ – RMS value of the induced voltage, θ – phase angle between the source voltage and induced voltage.

The compensating capacity at a specific frequency is determined by the formula:

(7) $$C(f) = \frac{1}{4\pi^2 f^2 L_c} = \frac{1}{4\pi^2 f^2 \left(L_{self} - M_{pe}\right)}.$$

In order to confirm the correctness of the assumptions for the analytical and numerical model, an analysis was carried out for models with a coil with a radius of r = 5 mm. Variability of the number of turns $n \in \{10;\ 30\}$ and the distance between the transmitting and receiving surfaces $h \in \{2.5;\ 5\}$ mm are taken into account.

The results of the WPT system analysis

Figures 5 and 6 compare the efficiency results of the WPT system for a circular coil for the number of turns n equal to 10 and 30, for the model with periodic and antiperiodic boundary conditions. The efficiency of the WPT system increases over the entire range of the analyzed frequencies. Regardless of the number of turns, higher efficiency is obtained for the system with antiperiodic boundary conditions. At f = 1 MHz and at h = 2.5 mm, this difference is even 28% and at h = 5 mm it exceeds 19%.

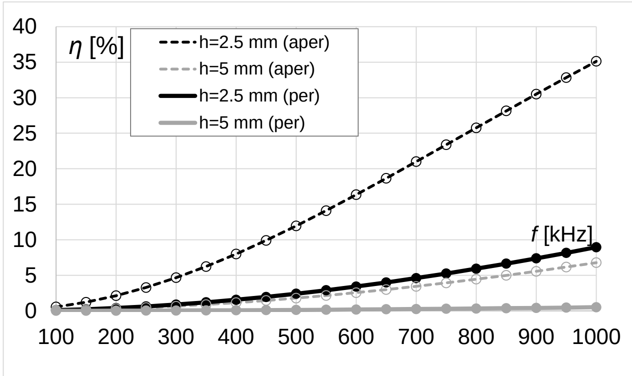


Fig. 5. Results of the efficiency for circular coils with r = 5 mm and n=10 at two distances (h = 2.5 mm and h = 5 mm)

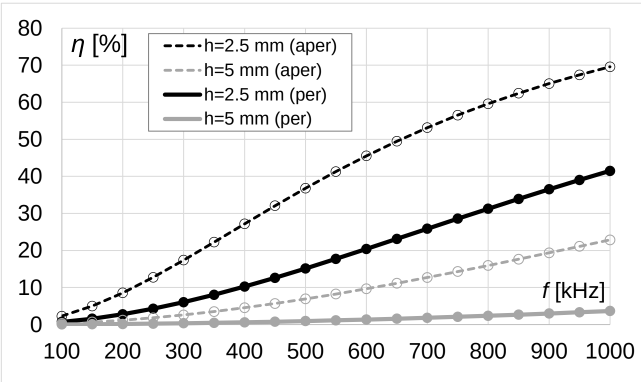


Fig. 6. Results of the efficiency for circular coils with r = 5 mm and n=30 at two distances (h = 2.5 mm and h = 5 mm)

Figures 7 and 8 compare the efficiency results of the WPT system for a square coil for the number of turns n equal to 10 and 30, for the model with periodic and antiperiodic boundary conditions. The results were calculated at two distances h (2.5 mm and 5 mm).

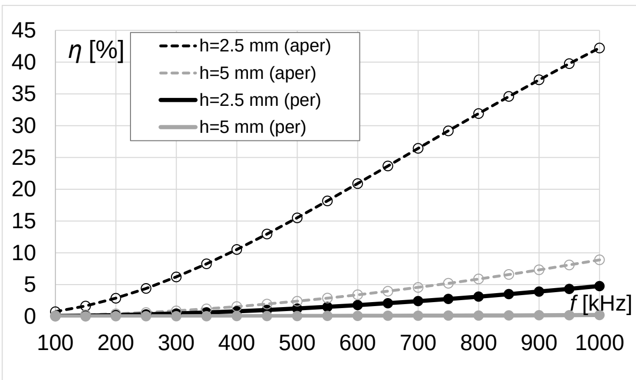


Fig. 7. Results of the efficiency for square coils with r = 5 mm and n=10 at two distances (h = 2.5 mm and h = 5 mm)

The efficiency of the WPT system increases over the entire range of the analyzed frequencies. Regardless of the number of turns, higher efficiency is obtained for the system with antiperiodic boundary conditions. At f = 1 MHz and at h = 2.5 mm, this difference is almost 40% and at h = 5 mm is almost 27%.

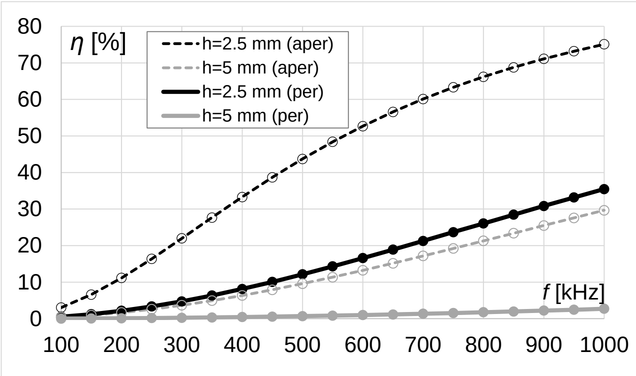


Fig. 8. Results of the efficiency for square coils with r = 5 mm and n=30 at two distances (h = 2.5 mm and h = 5 mm)

Comparing the results for systems with periodic boundary conditions, higher efficiency values were obtained for the circular coil than for the square one. The difference is the greatest for n = 30 and amounts to 6%. However, comparing the results for systems with antiperiodic boundary conditions, higher efficiency values were obtained for the square coil. This difference exceeds 7% for n = 10.

Conclusions

The article presents the results of the proposed numerical and analytical analysis of Wireless Power Transfer System (WPT). The system consists of a transmitting surface and a receiving surface, where each of them is composed of planar spiral coils. Both proposed approaches reduce the size and complexity of the models. Two types of the coils (circular and square) were included in the analysis. Two coil systems were considered: periodic and aperiodic. In the periodic system, higher efficiency was obtained with circular coils (even 6% at f = 1 MHz). In the aperiodic system, higher efficiency was obtained with square coils (even 7% at f = 1 MHz). When comparing both systems (periodic and aperiodic), both for the circular and the square coils, the aperiodic systems show higher efficiency values (even 40% at f = 1 MHz). The author plans to consider coils with other geometries in the future.

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