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Dynamic Sliding Mode and Backstepping controllers for Trajectory Tracking of Mobile Robot Wheeled

Abstract. Trajectory tracking for a wheeled mobile robot is a complex problem encountered in robotics science. Many difficulties affect robot tracking control such as, nonlinear model, robot parameter uncertainties, external factors and disturbances. In this article, two controllers: dynamic sliding mode (DSMC) and dynamic backstepping (DBKC) are proposed for path following. The main objective of the developed algorithms is to control a non-holonomic wheeled mobile robot (WMR). The design of the two controllers (DSMC) and (DBKC) is based on Lyapunov's theorem, the kinematic controller is used to provide the desired values of the linear and angular velocities for the given trajectory. A stability analysis based on Lyapunov is presented to ensure the stability and convergence of WMR to the desired position, speed and velocity. The simulation results show the effectiveness of the robust controllers proposed in terms of precision and stability in different types of trajectories.

Streszczenie. Śledzenie trajektorii kołowego robota mobilnego to złożony problem napotykanym w robotyce. Na sterowanie śledzeniem robota wpływa wiele trudności, takich jak model nieliniowy, niepewność parametrów robota, czynniki zewnętrzne i zakłócenia. W tym artykule do śledzenia ścieżki zaproponowano dwa kontrolery: tryb dynamicznego przesuwania (DSMC) i dynamiczne cofanie (DBKC). Głównym celem opracowanych algorytmów jest sterowanie nieholonomicznym kołowym robotem mobilnym (WMR). Konstrukcja dwóch sterowników (DSMC) i (DBKC) opiera się na twierdzeniu Lapunowa, sterownik kinematyczny służy do zapewnienia pożądanych wartości prędkości liniowej i kątowej dla danej trajektorii. Przedstawiono analizę stabilności opartą na Lapunowie, aby zapewnić stabilność i zbieżność WMR do pożądanej pozycji, prędkości i prędkości. Wyniki symulacji pokazują skuteczność zaproponowanych solidnych sterowników pod względem precyzji i stabilności w różnych typach trajektorii. (Kontrolery trybu dynamicznego przesuwania i cofania do śledzenia trajektorii robota mobilnego na kołach)

Keywords: Nonlinear model, Kinematic and Dynamic control, Lyapunov method, Sliding mode and Backstepping control.

Słowa kluczowe: Model nieliniowy, sterowanie kinematyczne i dynamiczne, metoda Lapunowa, tryb ślizgowy i sterowanie cofaniem

Introduction

Mobile robots are increasingly present in our daily lives and in major fields of application such as industry, medicine, agriculture, security or home assistance. Land mobile robotics occupies an important historical place. In particular, wheeled mobile robots that use a particularly efficient rolling mode of locomotion are already used in the industrial field such as logistics.

Trajectory tracking and fixed configuration stabilization is the work we present. Our goal is also to find control laws to control the robot, such as the tracking control of wheeled mobile robots (WMR). The latter attracts the attention of many researchers in robotics science because of their difficulties in the design. Over the past decade, many studies have been performed on different aspects of control (WMR) such as kinematics, dynamics, and design of controllers that solve the desired path following problem [5].

In this article, we want to design a robust control that offers fast convergence and good robustness properties with respect to the variation of the robot parameters. One of the robust techniques is discontinuous control such as sliding mode control (SMC).

Sliding mode control (SMC) has many advantages, including its finite-time convergence to a stable manifold and its insensitivity to model perturbations and uncertainties. For this the in our work, the (DSMC) proposed, the dynamic model of WMR includes the inertia parameter including the total mass and the moment of inertia of WMR. Thus, the controller takes into consideration the processing of dynamic effects which make it more applicable in the real world. In the dynamic control of a WMR the inputs are generally the torques imposed on the driving wheels. Using the WMR dynamic model developed in polar coordinates, it has certain drawbacks such as chattering phenomena,

Also, the backstepping control is another nonlinear control technique considered as robust, the synthesis of such a control is done in a systematic way and based on Lyapunov's theory. Backstepping combines the choice of

the Lyapunov function with that of the control and adaptation laws. This allows it, in addition to the task for which the controller is designed (tracking and/or regulation), to guarantee the overall stability of the compensated system at all times. For this we have chosen the dynamic backstepping controller (DBTC) as a second controller for our robot [13].

The main objective of this work is to develop control laws for the design of a robust controller for trajectory tracking of the robot subject to uncertainties. For this, we use two controllers one based on sliding mode and the other on backstepping. These two suggested controllers combine time-varying nonlinear feedback.

The outline of this article is as follows. In section 2, the mathematical representation of the complete unicycle type robot model with nonlinear model. The kinematics and the sliding mode (DSMC) and backstepping (DBTC) dynamic controllers are discussed, respectively, in sections 3 and 4. Simulations and interpretations are presented in section 5. Finally, section 6 concludes this article.

Mathematical model

In this section, we present the kinematic and dynamic model of a differential drive mobile robot (DDMR). The mobile robot used in this work is a unicycle type robot is based on two separately driven wheels placed on each side of the robot body. An additional idler (free) wheel is added in front of the robot to ensure its mechanical stability. Its center of rotation is located on the axis connecting the two driving wheels.

Fig. 1 presents the unicycle type mobile robot in the fixed and mobile frames.

A: is the midpoint of the wheel axle.

2R: represents the diameter of the robot wheel;

2L: represents the width of the robot;

$\dot{\phi}_R$ and $\dot{\phi}_L$: respectively represent the speed of rotation of the right wheel and of the left wheel;

e: is the orientation angle of the robot;

d: is the distance between point A and point C.

Two different coordinate systems are used: the fixed coordinate system of any $\{x_i, y_j\}$ and the movable coordinate system of the robot $\{x_r, y_r\}$. Let $q^i = [x^i, y^i, \theta^i]^T$ be a point of the frame $\{x_i, y_j\}$ and $q^r = [x^r, y^r, \theta^r]^T$ a point of the frame $\{x_r, y_r\}$. The points q^i and q^r are linked by the orthogonal matrix $R(\theta)$.

$$(1) \quad q^i = R(\theta)q^r \text{ with } R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

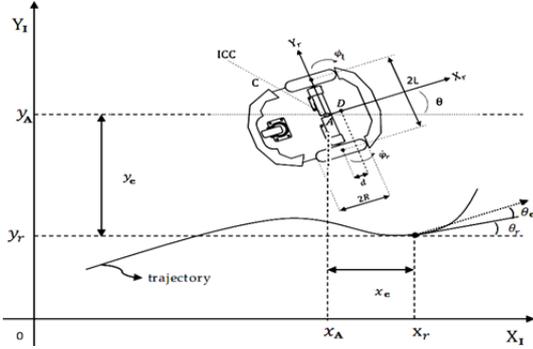


Fig. 1. Mobile robot and coordinate system

Kinematic model

The linear speed of our robot following DDMR is the average of the linear speeds of the two wheels [1].

$$(2) \quad \begin{cases} v = \frac{(v_R + v_L)}{2} = \frac{R(\dot{\phi}_R + \dot{\phi}_L)}{2} \\ \omega = \frac{(v_R - v_L)}{2L} = \frac{R(\dot{\phi}_R - \dot{\phi}_L)}{2L} \end{cases}$$

Thus the linear speed of each wheel of the robot is given by:

$$(3) \quad \begin{cases} v_L = R\dot{\phi}_L \\ v_R = R\dot{\phi}_R \end{cases}$$

In the mobile frame the coordinates of point A are:

$$(4) \quad \begin{pmatrix} x_A^r \\ y_A^r \\ \theta_A^r \end{pmatrix} = \begin{pmatrix} \frac{R}{2} & \frac{R}{2} \\ 0 & 0 \\ \frac{R}{2L} & -\frac{R}{2L} \end{pmatrix} \begin{pmatrix} \dot{\phi}_R \\ \dot{\phi}_L \end{pmatrix}$$

Using equation (1), we can write

$$(5) \quad \begin{pmatrix} \dot{x}_A^i \\ \dot{y}_A^i \\ \dot{\theta}_A^i \end{pmatrix} = \begin{pmatrix} \frac{R}{2} \cos\theta & \frac{R}{2} \cos\theta \\ \frac{R}{2} \sin\theta & \frac{R}{2} \sin\theta \\ \frac{R}{2L} & -\frac{R}{2L} \end{pmatrix} \begin{pmatrix} \dot{\phi}_R \\ \dot{\phi}_L \end{pmatrix}$$

By combining equations (2) and (5), we can also rewrite DDMR velocities in terms of linear and angular velocities in the kinematic model of the unicycle mobile robot as follows:

$$(6) \quad \dot{q}_A^i = \begin{pmatrix} \dot{x}_A^i \\ \dot{y}_A^i \\ \dot{\theta}_A^i \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$

Dynamic model

Using the Lagrange Dynamics approach, the dynamic model of the DDMR is [1]:

$$(7) \quad M(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q) + \tau_d = B(q)\tau - A^T(q) \cdot \lambda$$

Where :

- $M(q)$: an $n \times n$ symmetric positive definite inertial matrix,
 - $V(q, \dot{q})$: the matrix of centrifugal forces and Coriolis forces;
 - $G(q)$: the gravitational vector,
 - τ_d : the vector of bounded unknown disturbances including unstructured unmodeled dynamics,
 - $B(q)$: the input transformation matrix;
 - τ : the input vector,
 - $A^T(q)$: the matrix associated with the kinematic constraints,
 - λ : the Lagrange multipliers vector.
- q, \dot{q} and \ddot{q} : denote position, velocity and acceleration vectors respectively.

The potential energy being zero because, assuming that the mobile robot moves in the horizontal plane and that there is no Disturbances. Then, the dynamic model of the wheeled mobile robot can be put in the form:

$$M(q) = \begin{pmatrix} m & 0 & -m d \sin\theta & 0 & 0 \\ 0 & m & m d \cos\theta & 0 & 0 \\ -m d \sin\theta & m d \cos\theta & I & 0 & 0 \\ 0 & 0 & 0 & I_w & 0 \\ 0 & 0 & 0 & 0 & I_w \end{pmatrix}$$

$$V(q, \dot{q}) = \begin{pmatrix} 0 & m d \dot{\theta} \cos\theta & 0 & 0 & 0 \\ 0 & -m d \dot{\theta} \sin\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad B(q) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^T(q) \cdot \lambda = \begin{bmatrix} -\sin\theta & \cos\theta & \cos\theta \\ \cos\theta & \sin\theta & \sin\theta \\ 0 & L & -L \\ 0 & -R & 0 \\ 0 & 0 & -R \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \end{bmatrix}$$

It is possible to obtain the expression of the dynamic model of the robot by eliminating the term $A^T(q) \cdot \lambda$ which corresponds to the stress forces related to the kinematic stresses.

Defining $\eta = \begin{pmatrix} \dot{\phi}_R \\ \dot{\phi}_L \end{pmatrix}$ to be the vector of auxiliary velocities, we can write

$$(8) \quad \begin{pmatrix} \dot{x}_A \\ \dot{y}_A \\ \dot{\theta} \\ \dot{\phi}_R \\ \dot{\phi}_L \end{pmatrix} = \begin{pmatrix} (R/2)\cos\theta & (R/2)\cos\theta \\ (R/2)\sin\theta & (R/2)\sin\theta \\ R/2L & -R/2L \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{\phi}_R \\ \dot{\phi}_L \end{pmatrix} \Rightarrow \dot{q} = S(q)\eta$$

And

$$(9) \quad \ddot{q} = \dot{S}(q)\eta + S(q)\dot{\eta}$$

$S(q)$ is a full-rank matrix that satisfies the following condition:

$$S^T(q)A^T(q) = 0$$

Substituting (9) with equivalent terms in (7), it becomes:

$$(10) \quad M(q)[\dot{S}(q)\eta + S(q)\dot{\eta}] + V(q, \dot{q})S(q)\eta = B(q)u - A^T(q) \cdot \lambda$$

By multiplying equation (10) by $S^T(q)$, we have:

$$(11) \quad S^T(q)M(q)\dot{S}(q)\eta + S^T(q)M(q)S(q)\dot{\eta} + S^T(q)V(q, \dot{q})S(q)\eta = S^T(q)B(q)u - S^T(q)A^T(q) \cdot \lambda$$

The new dynamic equation becomes:

$$(12) \quad \bar{M}(q)\dot{\eta} + \bar{V}(q, \dot{q})\eta = \bar{B}(q)u$$

Where:

$$(13) \quad \begin{cases} \bar{M}(q) = S^T(q)M(q)S(q) \\ \bar{V}(q, \dot{q}) = S^T(q)M(q)\dot{S}(q) + S^T(q)V(q, \dot{q})S(q) \\ \bar{B}(q) = S^T(q)B(q) \end{cases}$$

$$\bar{M}(q) = \begin{bmatrix} I_w + \frac{R^2}{4I^2}(mL^2 + 1) & \frac{R^2}{4I^2}(mL^2 - 1) \\ \frac{R^2}{4I^2}(mL^2 - 1) & I_w + \frac{R^2}{4I^2}(mL^2 + 1) \end{bmatrix}$$

$$\bar{V}(q, \dot{q}) = \begin{bmatrix} 0 & \frac{R^2}{2L^2}(m_c d \dot{\theta}) \\ -\frac{R^2}{2L^2}(m_c d \dot{\theta}) & 0 \end{bmatrix}, \quad \bar{B}(q) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} m = m_c + 2m_w \\ I = I_c + m_c d^2 + 2m_w L^2 + 2I_m \end{cases}$$

By inserting (5) in (12) we get:

$$(14) \quad \begin{bmatrix} m + \frac{2I_w}{R^2} & 0 \\ 0 & I + \frac{2L^2}{R^2}I_w \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} 0 & -m_c d \omega \\ m_c d \omega & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{R} & 0 \\ 0 & \frac{L}{R} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

With: $\mu = \begin{cases} \mu_1 = \tau_R + \tau_L \\ \mu_2 = \tau_R - \tau_L \end{cases}$.

Where: (τ_R, τ_L) is the input torque expressed in Newton meters (N.m).

Equation (14) is the nonlinear form of the dynamic model of the mobile robot Submitted to our study.

Controller design

In this part of the work, we will first present the kinematic controller which was proposed in [2], and secondly we will present two dynamic controllers. The first uses the sliding mode control technique and the second the backstepping control technique.

Kinematic controller

Two postures of the robot are defined, namely:

-the reference posture $p_r = (x_r, y_r, \theta_r)^T$ considered as the goal posture.

-the current posture $p_A = (c, y_A, \theta_A)^T$ considered as the real posture of the robot.

Thus the posture error in the fixed frame is:

$$(15) \quad e_p = \begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} (P_r - P_A)$$

The kinematic controller proposed by [2] makes it possible to estimate the velocities (linear and angular velocities) of the nonholonomic mobile robot, which make the system asymptotically stable is:

$$(16) \quad \begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} v_r \cos\theta_e + k_e x_e \\ \omega_r + v_r (k_y y_e + k_\theta \sin\theta_e) \end{pmatrix}$$

Where k_x , k_y and k_θ are positive constants.

Dynamic controller

To design the dynamic tracking controller, we use two control methods, the first in sliding mode and the second in backstepping. We design DSMC and DBTC for our robot where disturbances are zero, system parameters are known and the robot model is nonlinear.

Dynamic sliding mode controller

Considering that our system is nonlinear and disturbance-free, equation (14) becomes:

$$(17) \quad \dot{x} = g \cdot u(t) + fx$$

The output equation

$$(18) \quad \begin{cases} \dot{v}(t) = \frac{m_c d}{m_0} \omega^2 + \frac{1}{m_0 R} u_1(t) \\ \dot{\omega}(t) = \frac{-m_c d}{I_0} v \cdot \omega + \frac{L}{I_0 R} u_2(t) \end{cases}$$

We define the error and the variation of the error as follows:

$$(19) \quad e = v_c - v(t) \quad \text{and} \quad \dot{e} = \dot{v}_c - \dot{v}(t)$$

First step we choose the sliding surface:

$$(20) \quad S(t) = \begin{bmatrix} S_1(t) \\ S_2(t) \end{bmatrix} = e_c(t) + \beta \int e_c(t) dt; \quad \beta > 0$$

Consider the candidate Lyapunov function and its derivative:

$$(21) \quad V = \frac{1}{2} s^T s \quad \text{and} \quad \dot{V} = s^T \dot{s}$$

For $\dot{V} < 0$ (stability condition) we choose the equation (20).

$$(22) \quad \dot{s} = -k \cdot \text{sign}(s) \quad k > 0$$

So:

$$(23) \quad \begin{cases} \dot{S}_1(t) = \dot{v}_c(t) - \dot{v}(t) + \beta e_v(t) = 0 \\ \dot{S}_2(t) = \dot{\omega}_c(t) - \dot{\omega}(t) + \beta e_\omega(t) = 0 \end{cases}$$

We replace equation (18) and (22) in equation (23):

$$(24) \quad \begin{cases} \dot{v}_c - \left[\frac{m_c d}{m_0} \omega^2 + \frac{1}{m_0 R} u_1(t) \right] + \beta e_v(t) = k_1 \text{sgn}(S_1) \\ \dot{\omega}_c - \left[\frac{-m_c d}{I_0} v \cdot \omega + \frac{L}{I_0 R} u_2(t) \right] + \beta e_\omega(t) = k_2 \text{sgn}(S_2) \end{cases}$$

So:

$$(25) \quad \begin{cases} \frac{m_c d}{m_0} \omega^2 + \frac{1}{m_0 R} u_1(t) = \dot{v}_c + \beta e_v(t) + k_1 \text{sgn}(S_1) \\ \frac{-m_c d}{I_0} v \cdot \omega + \frac{L}{I_0 R} u_2(t) = \dot{\omega}_c + \beta e_\omega(t) + k_2 \text{sgn}(S_2) \end{cases}$$

From the equation preceding .we can deduce the control law:

$$(26) \quad \begin{cases} u_1(t) = m_0 R \left[\dot{v}_c + \beta e_v(t) + k_1 \text{sgn}(S_1) - \frac{m_c d}{m_0} \omega^2 \right] \\ u_2(t) = \frac{I_0 R}{L} \left[\dot{\omega}_c + \beta e_\omega(t) + k_2 \text{sgn}(S_2) + \frac{m_c d}{I_0} v \cdot \omega \right] \end{cases}$$

Where: $m_0 R = \gamma$, $\frac{I_0 R}{L} = \alpha$

Dynamic backstepping controller

The backstepping control design procedure has been used to develop stabilizing controllers for time-invariant factories that are linear or belong to a class of nonlinear systems. Stability analysis in the context of the use of backstepping is based on Lyapunov methods which constitute a very powerful tool for testing and finding sufficient conditions for the stability of dynamical systems. In our case we design a controller (DBTC) that stabilizes our mobile robot for path following. [4]

The first error variable is defined as:

$$(27) \quad e = v_c - v$$

Its derivative is written:

$$(28) \quad \dot{e} = \dot{v}_c - \dot{v}$$

Or both equations (27), (28) are associated to the following Lyapunov function:

$$(29) \quad V = \frac{1}{2} e^2$$

Its derivative is written:

$$(30) \quad \dot{V} = e \dot{e} \quad \text{With} \quad \dot{V} \leq 0$$

Derivative also wrote:

$$(31) \quad \dot{V} = -K e^2 \quad \text{With} \quad K > 0$$

From equation (42) and (44) we obtain:

$$(32) \quad e \dot{e} = -K e^2$$

So:

$$(33) \quad \dot{e} = -K e$$

From equation (28) and (33) we obtain:

$$(34) \quad \dot{v}_c - \dot{v} = -K e$$

So:

$$(35) \quad \dot{v} = \dot{v}_c + K e$$

We replace the equation (18) in (35), we obtain:

$$(36) \quad \begin{cases} \dot{v}(t) = \frac{m_c d}{m_0} \omega^2 + \frac{1}{m_0 R} u_1(t) = \dot{v}_c + K_1 e_v(t) \\ \dot{\omega}(t) = \frac{-m_c d}{I_0} v \cdot \omega + \frac{L}{I_0 R} u_2(t) = \dot{\omega}_c + K_2 e_\omega(t) \end{cases}$$

From the equation preceding .we can deduce the control law:

$$(37) \quad \begin{cases} u_1(t) = m_0 R \left[\dot{v}_c + K_1 e_v(t) - \frac{m_c d}{m_0} \omega^2 \right] \\ u_2(t) = \frac{I_0 R}{L} \left[\dot{\omega}_c + K_2 e_\omega(t) + \frac{m_c d}{I_0} v \cdot \omega \right] \end{cases}$$

Where: $m_0 R = \gamma$, $\frac{I_0 R}{L} = \alpha$

Simulation result

Demonstrate the efficiency and robustness of the proposed controllers; Simulations were carried out in the presence of uncertainties for the nonlinear model of the DDMR. Figure 2 presents the architecture of the controller (kinematics and dynamics) proposed in our simulation under the environment MATLAB-SIMULINK.

The parameters in the simulation are listed as follows $m_c = 17$ kg; $m_w = 0.5$ kg; $r = 0.095$ m; $L = 0.24$ m; $I_c = 0.537$ kg.m² $I_m = 0.0011$ kg.m²; $I_w = 0.0023$ kg.m²; $m = m_c + 2 * m_w = 18$ kg; $I = I_c + m_c * d^2 + 2 * m_w * L^2 + 2 * I_m = 0.6393$ kg.m² $K_x = 0.5$ s⁻¹; $K_y = 1$ m⁻¹; $K_\theta = 0.2$ m⁻¹; $\beta = 50$; $k_1 = k_2 = 0.45$; $K_1 = K_2 = 30$.

We choose two trajectories to check the reaction and robustness of our proposed controllers. The trajectories and the initial condition are defined as:

Straight Line Trajectory

$$\text{Trajectory } \begin{cases} x_r = t \\ y_r = 3, \\ \theta_r = 0 \end{cases}, P_0 = \begin{cases} x_0 = 0 \\ y_0 = 0 \\ \theta_0 = 0 \end{cases}$$

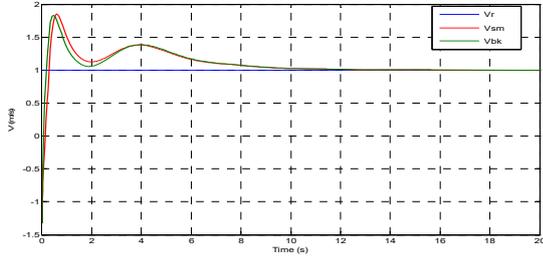


Fig. 2. Robot velocities

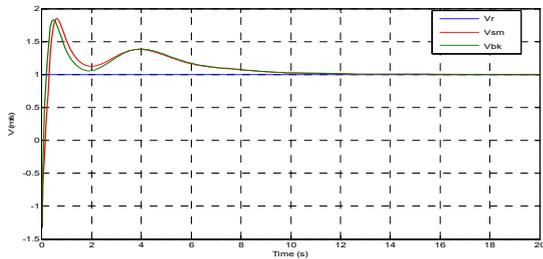


Fig. 3. Robot angular velocities

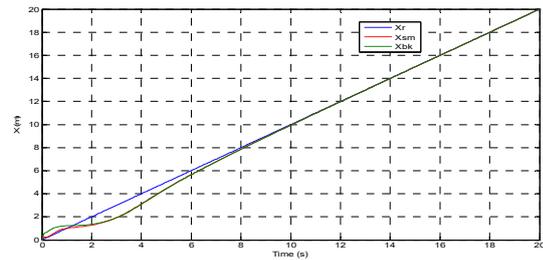


Fig. 4. X Evolution

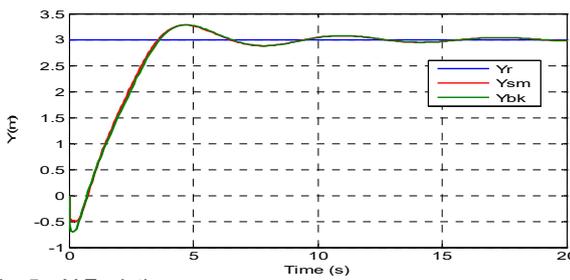


Fig. 5. Y Evolution

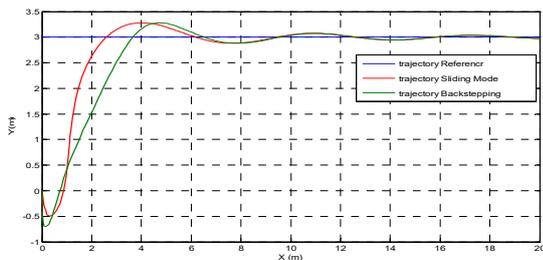


Fig. 6. Trajectory R and Tacking

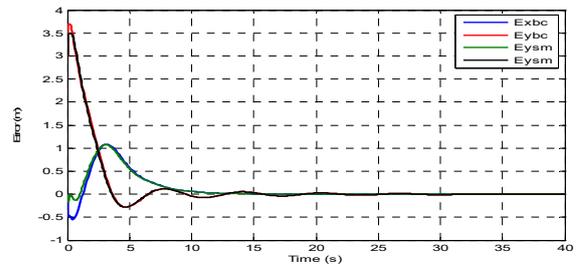


Fig. 7. Tacking errors

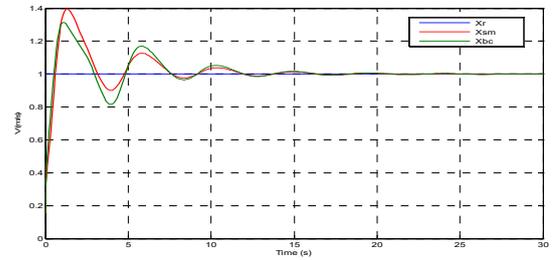


Fig. 8. Robot velocities

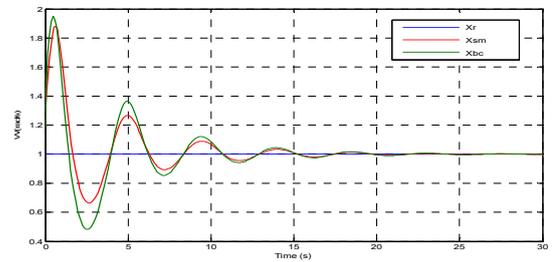


Fig. 9. Robot angular velocities

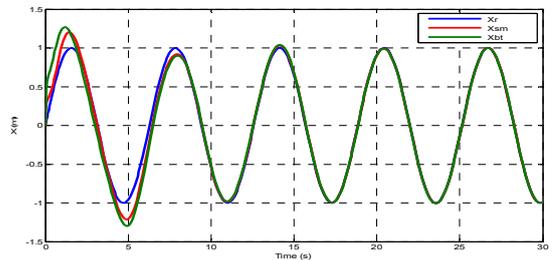


Fig. 10. X Evolution

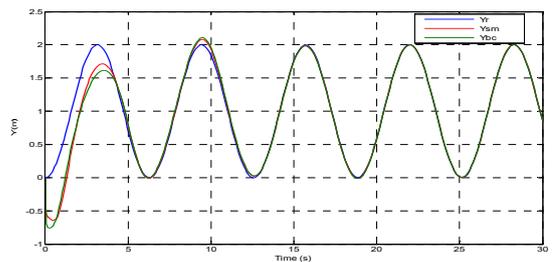


Fig. 11. Y Evolution

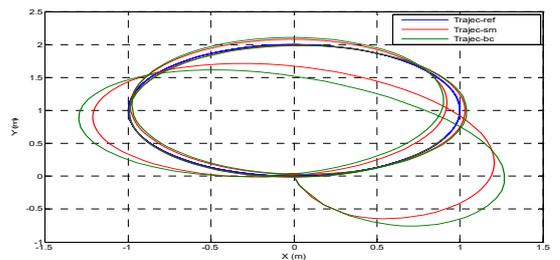


Fig. 12. Trajectory R and Tacking

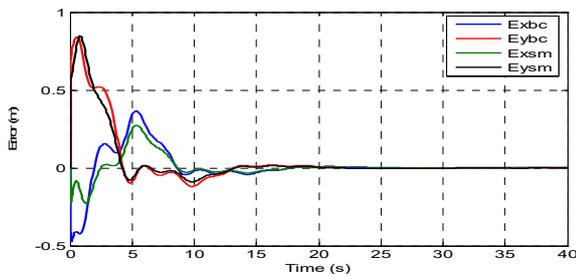


Fig. 13. Tacking errors

The simulations are carried out by following a straight trajectory in which the linear velocity and angular center of gravity of the WMR is present in figure 2 and 3. Figures 4 and 5 show the position of our robot in the lair (x, y). In figure 6 which represents the tracking of the WMR trajectory using the two controllers used (DSMC) and (DBKC). According to Figure 7 there is a large initial tracking error due to the starting point of the robot from point P_0 where (x=0) and (y=0).

One can easily see that by using the (DSMC) and (DBKC) proposed, the errors of posture and velocity and angular velocity converge towards zero and the trajectory tracking is perfectly achieved

Circle Trajectory

$$\text{Trajectory} \begin{cases} x_r(t) = \sin(t) \\ y_r(t) = 2 \cos(t), P_0 = \begin{cases} x_0 = 0 \\ y_0 = 0 \\ \theta_0 = 0 \end{cases} \\ \theta_r(t) = t \end{cases}$$

The simulation results for this trajectory show the posture stabilization of the WMR for both controllers (DSMC) and (DBKC), the robot velocities are plotted in Figures 9-10 and follow their desired reference asymptotically. We can see on figures 11-12, the evolution of the position of our robot. On figure 13 we can observe that the two controllers (DSMC) and (DBKC) proposed successfully stabilize the WMR at its origin from initial posture (0, 0, 0°), and we also notice that the robot arrives at the end second turn to the reference trajectory and catches up to the desired coordinates.

Conclusion

In this work a review of the kinematics and dynamics of a wheeled mobile robot with differential drive. We have developed two control laws, one low on the sliding mode technique and the other on the backstepping technique for the tracking control of WMR. The non-linearity of the dynamic model robot, the uncertainties greatly affect the trajectory tracking control. The two control techniques used are based on Lyapunov's theory, which shows the effectiveness of two control laws for trajectory tracking and maintaining the stability of the closed-loop dynamics of the mobile robot. The simulation results showed that the two proposed controllers have almost the same results in response time with a slight difference at startup. They also offer excellent trajectory tracking attenuation as well as robustness against parameter uncertainties.

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