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High order Sliding control with High order observer applied on a bioreactor

***Abstract**. The present paper provides a comparison between two different control strategies applied to a bioreactor system: A high-order sliding controller with a super-twist observer and a classic sliding control with a sliding observer. The performance of the two controllers is evaluated under different conditions, both with and without state perturbations, with a clear superiority for high order sliding control which eliminates chattering while maintaining performance. The results of this study are of great interest, as they offer insight into the efficacy of both control strategies in practical applications where the High Order Sliding Controller with the Super-Twist Observer provide an excellent response with an optimal control signal . Furthermore, the presented data demonstrates the response of the bioreactor system to the both controllers, providing valuable information for further research in this field.*

***Streszczenie.** Niniejsza praca badawcza zawiera porównanie dwóch różnych strategii sterowania zastosowanych w systemie bioreaktora: regulatora przesuwnego wysokiego rzędu z obserwatorem superskrętnym i klasycznego sterowania przesuwnego z obserwatorem przesuwnym. Wydajność tych dwóch kontrolerów jest oceniana w różnych warunkach, zarówno z zakłóceniami stanu, jak i bez, z wyraźną przewagą sterowania przesuwnego wysokiego rzędu, które eliminuje drgania przy zachowaniu wydajności. Wyniki tego badania są bardzo interesujące, ponieważ dają wgląd w skuteczność obu strategii kontroli w praktycznych zastosowaniach. Ponadto przedstawione dane obrazują reakcję układu bioreaktora na działanie obu regulatorów, dostarczając cennych informacji do dalszych badań w tym zakresie. (**Sterowanie przesuwne wysokiego rzędu z obserwatorem wysokiego rzędu zastosowanym w bioreaktorze**)*

Keywords: Sliding Control, Sliding Observer, High Sliding Control, *Super-Twist Observer* , Bioreactor
Słowa kluczowe: Kontrola przesuwania, Obserwator przesuwania, Kontrola wysokiego poślizgu, Super-Twist Observer, Bioreaktor

Introduction

Bioreactors are widely used in various industrial applications, especially in the biotechnology fields, to facilitate different biological reactions in a liquid medium, such as the stirred tank bioreactor (CTS). These systems are characterized by non-linear dynamics, which requires the use of advanced control strategies to achieve optimal performance.

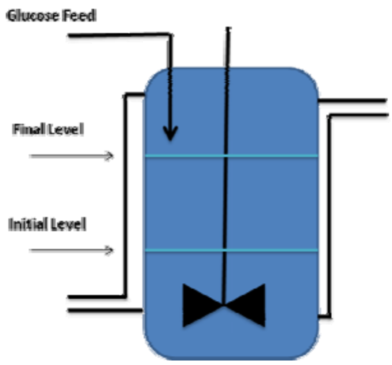
Previous research has investigated various control techniques for the bioreactor systems, including sliding mode control [1,2,3], adaptive sliding mode control [4], model predictive control [5], and fuzzy logic control [6]. These techniques have shown promise in achieving optimal performance in bioreactor systems, but the effectiveness can vary depending on the specific system and control objectives. For example, [7] showed that the sliding mode control with a linear observer can effectively regulate the temperature of a bioreactor system, while [8] demonstrated the effectiveness of the sliding mode control for glucose concentration control in a fed-batch bioreactor. In the contrast, [9] used a model predictive control to optimize the production of the bio-products in a fed-batch bioreactor. The work in this paper is in line with the sliding mode control for what presents the robustness combined with a sliding mode observer taking advantage of its convergence in finite time, valuable for the control of nonlinear systems; however this type of control presents a major handicap which limits its application in practice, namely the phenomenon of chattering. This represents the main motivation of our work, the best candidate to retain the robustness of sliding mode control by eliminating chattering, while maintaining performance is undoubtedly high order sliding control combined with supertwist observer.

In this research paper, we present a comparison between two different control strategies applied to a SISO bioreactor system: A classic sliding control with a sliding observer and a high-order sliding controller combined with a super-twist observer. Our results demonstrate that the High-Order Sliding Controller with the super-twist observer performs better than the classic Sliding Control with a classical sliding observer, providing an optimal control signal for the system. Moreover, the use of high-order sliding controllers reduces the chattering, which is often present in sliding controllers, without sacrificing performance. To provide a better understanding of the system dynamics, we present the non-linear model of the reactor.

This research paper is organized as follows: in section II, we discuss previous studies on control techniques for bioreactor systems. In section III, we describe the control law for the classic sliding Control with a classic sliding observer and the high-order sliding controller with a super-twist observer. We also present the performance of the observers for both controllers. In section IV, we present the results of the controllers, including the control signal and the performance of the system with and without perturbations. Finally, in section V, we conclude our findings and discuss the implications for future research

Plant model

Basically, a bioreactor is a tank in which several reactions occur simultaneously in a liquid medium. A standard schematic diagram of a completely mixed continuous stirred tank (CTS) bioreactor, with the dynamic model of the baker's yeast production process in a fed-batch reactor is shown in Fig.1.



Fig/ 1 Fed-batch Fermentor

This dynamic model relies on the existence of three limit physiological states of biomass:

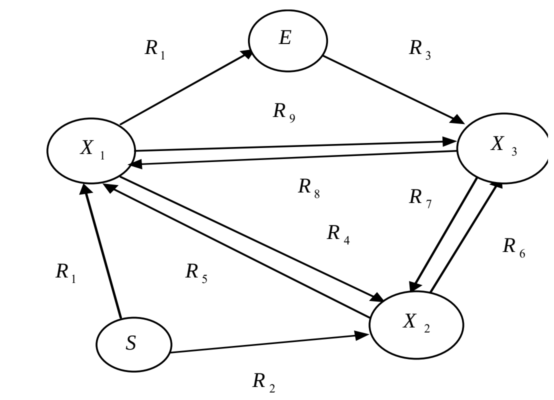


Fig. 2 Schema of reaction

E: Ethanol, S: Substrate (glucose), X: Biomass.

These physiological states correspond respectively to the glycemia fermentation, the respiration of the glucose and the respiration of the ethanol. The expressions of the kinetic model correspond to the aerobic growth, in the absence of other limitations than that of the substrate. The state vector x is of dimension 7. It is formed by concentrations (g/l) of Ethanol E, yeasts, glucose S, acetate A and the volume V of the reactor liquid.

The dynamic model of the reactor is represented by the following nonlinear system:

(1)
$$\begin{cases} \dot{E} = \frac{0.45}{0.14} R_1 - (1-\phi)\frac{R_3}{0.55} - \frac{E}{V} F_{in} \\ \dot{X}_1 = R_1 + R_5 + R_9 - R_4 - R_8 - \frac{X_1}{V} F_{in} \\ \dot{X}_2 = R_2 + R_4 + R_7 - R_5 - R_6 - \frac{X_2}{V} F_{in} \\ \dot{X}_3 = R_3 + R_8 + R_6 - R_9 - R_7 - \frac{X_3}{V} F_{in} \\ \dot{S} = -\frac{R_1}{0.14} - \frac{R}{0.5} - \frac{S}{V} F_{in} + \frac{S_{in}}{V} F_{in} \\ \dot{A} = \frac{0.01}{0.4} R_1 - \Phi \frac{R_3}{0.55} - \frac{A}{V} F_{in} \\ V = F_{in} \end{cases}$$

The input to be controlled is the glucose feed rate $u=F_{in}$
The output is the glucose concentration in the fermenter
y= S
R_1, *R_2* and *R_3* are biomass production rates defined by

$$(2)\quad \begin{cases} R_1 = \frac{0.6 x_2 x_5}{(0.5+x_5)\left(1+\frac{x_6}{0.4}\right)}; \; R_2 = \frac{0.29 x_3 x_5}{(0.04+x_5)} \\ R_3 = \frac{0.25 x_4}{\left(1+\frac{x_6}{0.2}\right)}\left(\frac{x_1}{(0.02+x_1)} + \phi \frac{x_6}{(0.02+x_6)}\right) \end{cases}$$

R_4, R_5, R_6, R_7, R_8 and R_9: the transition rates from one state to another one defined by:

$$(3)\quad \begin{cases} R_4 = 2x_2(1-\alpha); \; R_5 = 0.1 x_3 \alpha; \; R_6 = \frac{0.4 x_3 x_1}{0.5 + x_1} \\ R_7 = \frac{2 x_4 x_5}{(0.05+x_5)}; \; R_8 = \frac{0.2 x_2 x_1}{(0.2+x_1)}(1-\alpha); \; R_9 = \frac{0.5 x_4 x_5}{(0.01+x_5)} \end{cases}$$

The rates *R_1*,*R_2*,...,R_9 are expressed in g/(l.h), the dimensionless coefficients *α* and *Φ* are defined as follows:

$$(4)\quad \alpha = \frac{x_5^3}{0.1^3 + x_5^3}; \quad \begin{cases} \phi = 1 & if \; x_1 \le 10^{-6} g/l \\ \phi = 0 & if \; x_1 > 10^{-6} g/l \end{cases}$$

Control Theory

The super-twisting controller is used for systems of relative degree. In other words it can be used instead of a standard 1-sliding-mode controller in order to avoid the chattering. However for relative degree2 systems a 2-sliding controller, like a twisting one, is needed to stabilize system in finite time. In order to avoid the use of $\dot{\sigma}$ measurements, a differentiator (observer) is needed. The differentiator needed here has to feature robust exact differentiation with finite-time convergence in the absence of the measurement noise.

Let the input signal *f(t)* be a function defined on $[0,\infty)$ consisting of a bounded Lebesgue-measurable noise with unknown features and an unknown base signal $f_0(t)$ with the first derivative having a known global Lipschitz constant L > 0.The problem is to find real-time robust estimations of $f_0(t)$ and $\dot{f}_0(t)$ which are exact in the absence of measurement noise. Consider the auxiliary system $\dot{z}_0 = v$ where *v* is a control input.

Let $\sigma_0 = z_0 - f_0(t)$ and let the task be to keep $\sigma_0 = 0$ in a 2-Sliding mode. In that case $\sigma_0 = \dot{\sigma}_0 = 0$, which means that $z_0 = f_0(t)$ and $\dot{f}_0 = v$ .The system can be rewritten as:

$$\dot{\sigma}_0 = -\dot{f}_0(t) + v, \quad \left|\ddot{f}_0\right| \le L$$

The global closed loop of this scheme is restricted to stability of output error because the relative order of the system is 1, the resulting sliding motion then evolves on a reduced order manifold of dimension (n - 1) which is related to dynamic of zeros manifold, since the system is minimum phase then this manifold is stable.

The function $\dot{f}_0$ can be not smooth, but its derivative $\ddot{f}_0$ exists almost everywhere due to the Lipschitz property of $\dot{f}_0$. A modification of the super-twisting controller:

$$(5)\quad \begin{cases} v = -\lambda_1 |\sigma_0|^{1/2} sign(\sigma_0) + z_1 \\ \dot{z}_1 = -\lambda_2 sign(\sigma_0) \end{cases}$$

Is applied here, the medication is needed.

For neither $\dot{f}_0(t)$ nor *v* is bounded. The resulting form of the differentiator is:

$$\dot{z}_0 = v = -\lambda_1 |z_0 - f(t)|^{1/2} sign(z_0 - f(t)) + z_1$$
$$\dot{z}_1 = -\lambda_0 sign(z_0 - f(t))$$

Where, both *v* and *z_1* can be taken as the differentiator outputs.

Super-Twisting Observer

One of the popular second-order sliding mode algorithms which offer a finite reaching time and which can be used for sliding mode based observation is the super-twisting algorithm considered previously. The proposed super-twisting observer has the form:

$$\text{(6)} \quad \begin{cases} \dot{\hat{z}}_1 = \hat{z}_1 + \sigma_1 \\ \dot{\hat{z}}_2 = f\left(t, x_1, \hat{z}_2, u\right) + \sigma_2 \end{cases}$$

Where $\hat{z}_1$ *and* $\hat{z}_2$ are the state estimates, while the correction variables σ_1 *and* σ_2 are output error injections of the form.

$$\sigma_1 = \lambda \left| x_1 - \hat{z}_1 \right|^{1/2} sign\left(x_1 - \hat{z}_1\right)$$

$$\sigma_2 = \alpha sign\left(x_1 - \hat{z}_1\right)$$

Taking $\tilde{z}_1 = x_1 - \hat{z}_1$ *and* $\tilde{z}_2 = x_2 - \hat{z}_2$ us obtains the error equations:

$$\dot{\tilde{z}}_1 = \tilde{z}_2 - \lambda \left|\tilde{z}_1\right|^{1/2} sign\left(\tilde{z}_1\right)$$

$$\dot{\tilde{z}}_2 = F\left(t, x_1, x_2, \hat{z}_2\right) - \alpha sign\left(\tilde{z}_1\right)$$

Where

$$F\left(t, x_1, x_2, \hat{z}_2\right) = f\left(t, x_1, x_2, u\right) - f\left(t, x_1, \hat{z}_2, u\right) + \xi\left(t, x_1, x_2, y\right)$$

Suppose that the system states are bounded, then the existence of a constant f^+ is ensured, such that the inequality

$$\left|F\left(t, x_1, x_2, \hat{z}_2\right)\right| < f^+$$

Holds for any possible *t*, x_1, x_2 and $\left|\hat{z}_2\right| \le 2\sup\left|x_2\right|$.

According to principle of first order differentiator, the parameters of observer α and λ could be selected As $\alpha = a_1 f^+$ *and* $\lambda = a_2\left(f^+\right)^{1/2}$, where $a_1 = 1{,}1;\quad a_2 = 1{,}5;$

Convergence of the observer states $\left(\hat{z}_1, \hat{z}_2\right)$ to the system state variables (x_1,x_2)occurs in finite time, [theory ref]

Second Order Sliding Mode Control

The second order sliding control (*r* = 2) (called 2-sliding) allows to eliminate or to reduce the chattering phenomenon. Its main purpose is to generate a second order sliding mode on a selected sliding surface *S*(*t, x*). This can be done by imposing:

$$S\left(t, x\right) = \dot{S}\left(t, x\right) = 0$$

This is achieved by considering a second order system whose states are the sliding function *S*(*t*,*x*) and its derivative $\dot{S}(t,x)$. To have a second order sliding mode, i.e. $S(t,x) = 0$ *and* $\dot{S}(t,x) = 0$, a sliding mode control is applied to this new second order system. Therefore, we obtain the finite time convergence of *S*(*t*,*x*) and of its derivative $\dot{S}(t,x)$. to zero.

Consider a dynamic system described by equation expressed by:

$$\dot{x} = f\left(t, x, u\right)$$

The derivative of *S*(*t, x*) is written as:

$$\frac{d}{dt}S(t,x) = \frac{\partial}{\partial t}S(t,x) + \frac{\partial}{\partial x}S(t,x)\frac{\partial x}{\partial t}$$

Then:

$$\dot{S}(t,x) = \frac{\partial}{\partial t}S(t,x) + \frac{\partial}{\partial x}S(t,x)\, f(t,x,u)$$

The second derivative of *S*(*t, x*) is written as:

$$\frac{d}{dt}\dot{S}(t,x) = \frac{\partial}{\partial t}\dot{S}(t,x,u) + \frac{\partial}{\partial x}\dot{S}(t,x)\frac{\partial x}{\partial t} + \frac{\partial}{\partial u}\dot{S}(t,x)\frac{\partial u}{\partial t}$$

This gives:

$$\frac{d}{dt}\dot{S}(t,x) = \frac{\partial}{\partial t}\dot{S}(t,x,u) + \frac{\partial}{\partial x}\dot{S}(t,x)\, f(t,x,u) + \frac{\partial}{\partial u}\dot{S}(t,x)\dot{u}(t)$$

Suppose that:

$$\text{(7)} \quad \begin{cases} \theta(t,x) = \frac{\partial}{\partial t}\dot{S}(t,x,u) + \frac{\partial}{\partial x}\dot{S}(t,x)\, f(t,x,u) \\ \varsigma(t,x) = \frac{\partial}{\partial u}\dot{S}(t,x) \end{cases}$$

Consider now the new system in which state variables are the sliding function *S*(*t, x*) and its derivative $\dot{S}(t,x)$. which are noted respectively by $y_1(t, x)$ and $y_2(t, x)$:

$$\text{(8)} \quad \begin{cases} y_1(t,x) = S(t,x) \\ y_2(t,x) = \dot{S}(t,x) \end{cases}$$

Using eqs. (7) and (8), we obtain:

$$\text{\(9)} \quad \begin{cases} \dot{y}_1(t,x) = y_2(t,x) \\ \dot{y}_2(t,x) = \theta(t,x) + \varsigma(t,x)\dot{u}(t) \end{cases}$$

The system described by (9) is a second order one. We propose for this new system a new sliding function:

$$\sigma(t,x) = y_2(t,x) + \alpha\, y_1(t,x) = \dot{S}(t,x) + \alpha\, S(t,x) \quad (10)$$

The system whose input is $\dot{u}(t)$ and whose output is $\sigma(t,x)$ is of relative degree 'one', so a sliding mode can exist on $\sigma(t,x)$ [10]. So the input $\dot{u}(t)$ can be taken as:

$$\dot{u}(t) = -M\, sign\left(\sigma(t,x)\right).$$

$$\dot{u}(t) = \dot{u}_{eq}(t) - k\, sign\left(\sigma(t,x)\right).$$

Application to bioreactor system

The dynamic model of the reactor is represented by the following nonlinear system, in the state space, affine with respect to the control input:

With $x = \left[E \quad X_1 \quad X_2 \quad X_3 \quad S \quad A \quad V\right]^T$;

$$U = F_{in} \text{ and } y = S = x_5$$

$$\begin{cases} \dot{x}_1 = \frac{0.45}{0.14}R_1 - (1-\phi)\frac{R_3}{0.55} - \frac{x_1}{x_7}U \\ \dot{x}_2 = R_1 + R_5 + R_9 - R_4 - R_8 - \frac{x_2}{x_7}U \\ \dot{x}_3 = R_2 + R_4 + R_7 - R_5 - R_6 - \frac{x_3}{x_7}U \end{cases}$$

$$\begin{cases} \dot{x}_4 = R_3 + R_8 + R_6 - R_9 - R_7 - \frac{x_4}{x_7}U \\ \dot{x}_5 = -\frac{R_1}{0.14} - \frac{R_2}{0.5} - \frac{x_5}{x_7}U + \frac{S_{in}}{x_7}U \\ \dot{x}_6 = \frac{0.01}{0.14}R_1 - \phi\frac{R_3}{0.55} - \frac{x_6}{x_7}U \\ \dot{x}_7 = U \end{cases} \quad (11)$$

Relative Degree of the System: In the present case, the Lie derivatives of the output $y=S=h(x)=x_5$ are:

$$L_f^0 h(x) = L_f^0 x_5 = x_5$$

$$L_f^1 h(x) = L_f^1 x_5 = \frac{\partial x_5}{\partial x} f = -\frac{R_1}{0.14} - \frac{R_2}{0.5}$$

$$L_g L_f^0 h(x) = L_g L_f^0 x_5 = L_g x_5 = \frac{\partial x_5}{\partial x} g = \frac{S_{in} - x_5}{x_7} \neq 0 \quad as \quad S_{in} \neq x_5$$

The relative degree of system is thus r =1.

Second Order sliding mode control for bioreactor

Synthesis of the control law:

Let $S(t,x) = x_5 - y_r$

Then $\dot{S}(t,x) = \dot{x}_5 - \dot{y}_r = -\frac{R_1}{0.14} - \frac{R_2}{0.5} + \frac{S_{in} - x_5}{x_7}U - \dot{y}_r$

The new system defined by the sliding surface and his derivative is given by: $\begin{cases} \dot{y}_1(t,x) = \dot{S}(t,x) \\ \dot{y}_2(t,x) = \theta(t,x) + \varsigma(t,x)\dot{u}(t) \end{cases}$

With $\begin{cases} y_1(t,x) = S(t,x) \\ y_2(t,x) = \dot{S}(t,x) \end{cases}$

We propose for this new system a new sliding function:

$$(12) \begin{cases} \sigma(t,x) = y_2(t,x) + \alpha y_1(t,x) = \dot{S}(t,x) + \alpha S(t,x) \\ \sigma(t,x) = -\frac{R_1}{0.14} - \frac{R_2}{0.5} + \frac{S_{in} - x_5}{x_7}U - \dot{y}_r + \alpha(x_5 - y_r) \end{cases}$$

Super-Twisting Observer for bioreactor

The Super-Twisting Observer for bioreactor is given by:
With:

$$(13) \begin{cases} \dot{\hat{z}}_1 = \frac{0.45}{0.14}R_1 - (1-\phi)\frac{R_3}{0.55} - \frac{\hat{z}_1}{\hat{z}_7}U + \gamma_1 \\ \dot{\hat{z}}_2 = R_1 + R_5 + R_9 - R_4 - R_8 - \frac{\hat{z}_2}{\hat{z}_7}U + \gamma_2 \\ \dot{\hat{z}}_3 = R_2 + R_4 + R_7 - R_5 - R_6 - \frac{\hat{z}_3}{\hat{z}_7}U + \gamma_3 \\ \dot{\hat{z}}_4 = R_3 + R_8 + R_6 - R_9 - R_7 - \frac{\hat{z}_4}{\hat{z}_7}U + \gamma_4 \\ \dot{\hat{z}}_5 = -\frac{R_1}{0.14} - \frac{R_2}{0.5} - \frac{x_5}{\hat{z}_7}U + \frac{S_{in}}{\hat{z}_7}U + \gamma_5 \\ \dot{\hat{z}}_6 = \frac{0.01}{0.14}R_1 - \phi\frac{R_3}{0.55} - \frac{\hat{z}_6}{\hat{z}_7}U + \gamma_6 \\ \dot{\hat{z}}_7 = U + \gamma_7 \end{cases}$$

$\gamma_1 = \eta sign(x_5 - \hat{z}_1)$; $\gamma_2 = \eta sign(x_5 - \hat{z}_2)$; $\gamma_3 = \eta sign(x_5 - \hat{z}_3)$;
$\gamma_4 = \eta sign(x_5 - \hat{z}_4)$; $\gamma_6 = \eta sign(x_5 - \hat{z}_6)$; $\gamma_7 = \eta sign(x_5 - \hat{z}_7)$;
$\gamma_5 = \lambda |x_5 - \hat{z}_5|^{1/2} sign(x_5 - \hat{z}_5)$

Let

Assumption 1: The system model is minimum phase.

Assumption 2: $\max|\hat{h}_5| \leq \Gamma_M \quad with \quad \Gamma_M > 0.$

Theorem:

The closed loop with bioreactor model given by the equations (11), second order sliding mode control given by $u(t) = \int -M\, sign(\sigma(t,x))dt$ with the choice of $M = c_0\left(\frac{\hat{z}_7}{S_{in} - x_5}\right)$ with $c_0 \geq 1$ where $\sigma(t,x)$ is given by (12) and supertwist observer given by (13), are globally asymptotically stable if the assumption 1 and 2 are satisfied and the following condition:

$$c_0\alpha > \Gamma_M$$

Proof:

The global closed loop of this scheme is restricted to the stability of the output error, because the relative order of the system is 1, the resulting sliding motion then evolves on a reduced order manifold of dimension (n- 1) which is related to the dynamic of zeros manifold, since the system is minimum phase [11], then this manifold is stable and assumption1 are satisfied.

The assumption 2 is satisfied by the stabilization of observer by second order sliding mode control in the closed loop, so the dynamic of observer are bounded.

1. *Stability of output error :*

(1) Let $e_y = \hat{z}_5 - y_r \quad and \quad \tilde{x}_5 = x_5 - \hat{z}_5$

And define as Lyapunov function:

(2) $V = \frac{1}{2}e_y^2$

From Super Twist Observer we have:

(3)

$$\dot{\hat{z}}_5 = -\frac{R_1(x_5, \hat{z}_2, \hat{z}_6)}{0.14} - \frac{R_2(x_5, \hat{z}_3)}{0.5} + \left(\frac{S_{in} - x_5}{\hat{z}_7}\right)u(t) + \lambda|x_5 - \hat{z}_5|^{1/2} sign(x_5 - \hat{z}_5)$$

With (1) and (3) the derivative of V is given by:

(4)

$$\dot{V} = e_y\dot{e}_y = e_y\left(\dot{\hat{z}}_5 - \dot{y}_r\right) = e_y\left(\hat{h}_5 + \left(\frac{S_{in} - x_5}{\hat{z}_7}\right)u(t) + \lambda|\tilde{x}_5|^{1/2} sign(\tilde{x}_5) - \dot{y}_r\right)$$

Where $\hat{h}_5 = -\frac{R_1(x_5, \hat{z}_2, \hat{z}_6)}{0.14} - \frac{R_2(x_5, \hat{z}_3)}{0.5}$

Since

(5) $u(t) = \int -M\, sign(\sigma(t,x))dt$

And

(6) $\sigma(t,x) = \hat{h}_5 + \left(\frac{S_{in} - x_5}{\hat{z}_7}\right)u(t) - \dot{y}_r + \alpha(x_5 - y_r)$

Let $x_5 - y_r = \underbrace{x_5 - \hat{z}_5}_{\tilde{x}_5} + \underbrace{\hat{z}_5 - y_r}_{e_y} = \tilde{x}_5 + e_y$

Then

(7) $\sigma(t,x)=\hat{h}_5+\left(\frac{S_{in}-x_5}{\hat{z}_7}\right)u(t)-\dot{y}_r+\alpha\tilde{x}_5+\alpha e_y$

Substitute (5) and (7) in (4) give:

(8)

$$\dot{V}=e_y\left(\begin{array}{l}\hat{h}_5-\left(\frac{S_{in}-x_5}{\hat{z}_7}\right)M\int sign\left(\hat{h}_5+\left(\frac{S_{in}-x_5}{\hat{z}_7}\right)u(t)-\dot{y}_r+\alpha\tilde{x}_5+\alpha e_y\right)dt+\\ \lambda\left|\tilde{x}_5\right|^{1/2}sign\left(\tilde{x}_5\right)-\dot{y}_r\end{array}\right)$$

Choosing

$$M=c_0\left(\frac{\hat{z}_7}{S_{in}-x_5}\right)\ with\ c_0\geq 1\Rightarrow M\geq\frac{\hat{z}_7}{S_{in}-x_5}$$, and

We have $\int sign(\sigma(t,x))dt<\int\sigma(t,x)dt$ if

$\sigma(t,x)>1\ and\ -1<\sigma(t,x)<0$

If the state x of system is far from slide surface, then $\sigma(t,x)>1$ is verified,

And if the state x thanks to control is in a neighborhood of slide surface then for a system with a relative order equal to 1 with respect to $\sigma(t,x)$ there exist positive constant S_0,such that in a neighborhood $\left|\sigma(t,x)\right|<S_0$ with S_0= 1 then $-1<\sigma(t,x)<0$ is verified.

Apply this result to (8) gives:

$$\dot{V}<e_y\left(\hat{h}_5-c_0\int\left(\hat{h}_5+\left(\frac{S_{in}-x_5}{\hat{z}_7}\right)u(t)-\dot{y}_r+\alpha\tilde{x}_5+\alpha e_y\right)dt+\lambda\left|\tilde{x}_5\right|^{1/2}sign\left(\tilde{x}_5\right)-\dot{y}_r\right)$$

$$\dot{V}<e_y\left(\hat{h}_5-c_0\int\left(\hat{h}_5+\left(\frac{S_{in}-x_5}{\hat{z}_7}\right)u(t)-\dot{y}_r+\alpha\tilde{x}_5+\alpha e_y\right)dt+\lambda\left|\tilde{x}_5\right|^{1/2}sign\left(\tilde{x}_5\right)-\dot{y}_r\right)$$

$$\dot{V}<e_y\left(\hat{h}_5-c_0\int\left(\hat{h}_5+\left(\frac{S_{in}-x_5}{\hat{z}_7}\right)u(t)\right)dt+c_0\int\dot{y}_r dt-c_0\alpha\int\tilde{x}_5 dt-c_0\alpha\int e_y dt+\lambda\left|\tilde{x}_5\right|\left|\tilde{x}_5\right|^{-1/2}-\dot{y}_r\right)$$

We now that the sliding mode occurs in $\tilde{x}_5=0$, then from equation of observer we have:

$$\dot{\hat{z}}_5=\hat{h}_5+\left(\frac{S_{in}-x_5}{\hat{z}_7}\right)u(t)+\lambda\left|\tilde{x}_5\right|^{1/2}sign\left(\tilde{x}_5\right)=\hat{h}_5+\left(\frac{S_{in}-x_5}{\hat{z}_7}\right)u(t)$$

And so,

$$\dot{V}<e_y\left(\hat{h}_5-c_0\int\dot{\hat{z}}_5\,dt+c_0\int\dot{y}_r dt-c_0\alpha\int e_y dt-\dot{y}_r\right)$$

$$\dot{V}<e_y\left(\hat{h}_5-c_0\hat{z}_5+c_0 y_r-c_0\alpha\int e_y dt-\dot{y}_r\right)$$

$$\dot{V}<e_y\hat{h}_5-c_0 e_y\hat{z}_5+c_0 e_y y_r-c_0\alpha e_y\int e_y dt-e_y\dot{y}_r$$

$$\dot{V}<e_y\hat{h}_5-c_0 e_y\left(\hat{z}_5-y_r\right)-c_0\alpha e_y\int e_y dt-e_y\dot{y}_r$$

$$\dot{V}<-c_0 e_y^{\,2}-c_0\alpha e_y\int e_y dt+e_y\hat{h}_5-e_y\dot{y}_r$$

We have the regulation scheme, so $\dot{y}_r=0$ then

$$\dot{V}<-c_0 e_y^{\,2}-c_0\alpha e_y\int e_y dt+\hat{h}_5 e_y$$

With assumption $\max\left|\hat{h}_5\right|\leq\Gamma_M\quad with\quad\Gamma_M>0$

$$\dot{V}<-c_0 e_y^{\,2}-c_0\alpha e_y\int e_y dt<0\quad if\ c_0\alpha>\Gamma_M$$

Then $e_y\to 0,$ in virtue of (6) and $\tilde{x}_5=0.$

$x_5-y_r=\tilde{x}_5+e_y=0\Rightarrow x_5\to y_r$ Globally asymptotically.

Stability of global closed loop dynamic.

Since the bioreactor are minimum phase and supertwist observer is finite time convergent error, the separation principle in nonlinear system allow us then to conclude to global stability of closed loop dynamic.

Simulation results

This section presents simulation results for the bioreactor regulation using the high-order sliding mode controller with a super-twist observer and the classic sliding mode controller with a sliding observer under state under state perturbations. The responses of the bioreactor system for both controllers are compared, assessing their performance in maintaining desired setpoints and minimizing deviations. The control signal is included to validate the proposed control scheme's optimality. Additionally, a comparison of estimated states from both observers reveals insights into the accuracy and reliability of state estimation.

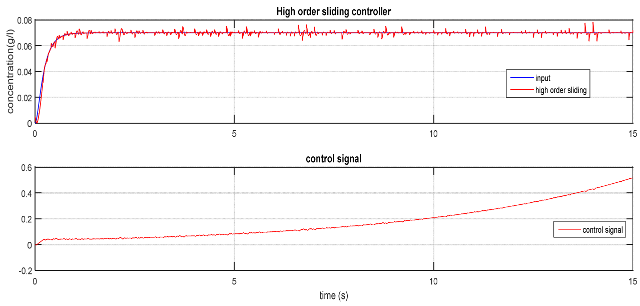


Figure 3 response of the bioreactor controlled by the high order sliding mode controller and supertwist observer with state perturbation

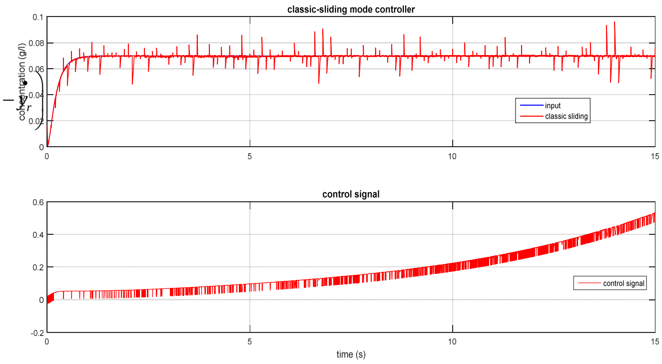


Figure 4 response of the bioreactor for the classic sliding mode controller and sliding observer with state perturbation

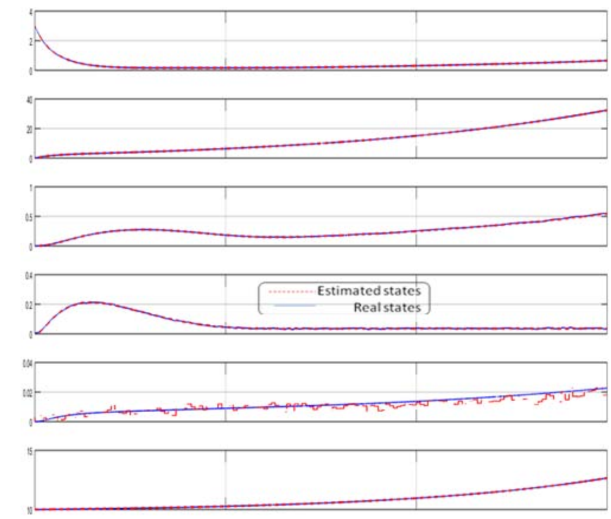


Figure 5 States of the bioreactor for the classic sliding mode controller and sliding observer with state perturbation

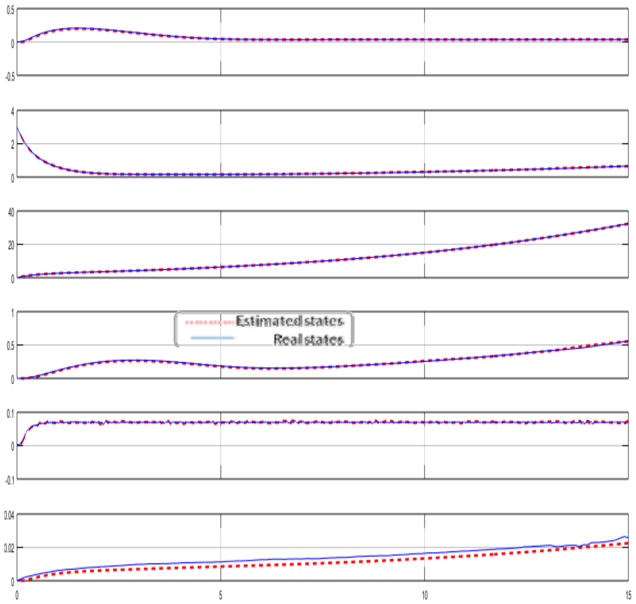


Fig.6 States of the bioreactor controlled by the high order sliding mode controller and supertwist observer with state perturbation

Table 1 variance of the control signal and the MSE

	Classic Sliding	High order sliding
Variance of the control signal	Var= 0.0177	Var=0.0167
M-S of error	mse=3.8658 e-05	mse= 8.0748e-06

MSE: mean suared error

For the high order sliding mode response , we have a smooth control signal with the absence of chattering, which is always present for the sliding control, this phenomena is eliminated for this controller with guaranteeing a good performances(time response,steady state error...), in contrast for the classic sliding mode control where the chattering is always present, and the performances are not satisfying.

Despite the presence of disturbances on the states, the system operates effectively and in good performances, this confirms the robustness of this controller.

For the state observer we have an excellent estimation for both observers : high order sliding mode and classic order sliding mode, but for the classic one we have bad estimation for some states where all the estimated states of high order sliding observer were identifiable as the orginal states.

Conclusion

The bioreactor being a system of relative degree 1, allowed us to be satisfied with the second order sliding control as well as the supertwist observer, the results of simulations showed that their application to the control of the system of the bioreactor give excellent results compared to to those of the classical sliding mode control with the classical sliding mode observer, the elimination of chattering is visible even in the presence of disturbances on the state and the performances are satisfactory.

The bioreactor being a minimum phase system, the other states considered as internal dynamics are bounded as shown by the simulation results for the two control schemes.

The results obtained show that the high order sliding control with the supertwist observer offers a promising solution for the control of bioreactor systems.

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