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Field oriented control technique applied to the PMSM without mechanical sensor, based on the cubature KALMAN filter observer

***Abstract**. This paper presents a sensorless field oriented control (FOC) for permanent magnet synchronous motor (PMSM) drives by using a Cubature KALMAN Filter Observer (CKFO). This paper mainly focuses on designing the (CKFO) algorithm in order to estimate the speed and rotor position. For the design of the observer, we use the nonlinear model of the PMSM represented in the frame α, β (For the design of the observer, the nonlinear model of the PMSM represented in the reference α, β is used). This observer uses the cubature transform to solve the nonlinearity problem, unlike extended KALMAN filter (EKF). The simulation results of the sensorless FOC show accurate tracking of the reference speed and smooth transition dynamics. With a good state estimation by the CKF observer, which is robust against noise (internal and measurement) and parametric insertions (With a good state estimate by the CKF observer, which presents robustness against noises (internal and measurement) and also against parametric insertitudes).*

***Streszczenie.** W artykule przedstawiono bezczujnikowe sterowanie zorientowane na pole (FOC) dla napędów silników synchronicznych z magnesami trwałymi (PMSM) przy użyciu kubaturowego obserwatora filtrującego KALMAN (CKFO). W artykule skupiono się głównie na projektowaniu algorytmu (CKFO) służącego do szacowania prędkości obrotowej i położenia wirnika. Do projektowania obserwatora wykorzystujemy nieliniowy model PMSM przedstawiony w układzie α, β (Do projektowania obserwatora wykorzystujemy nieliniowy model PMSM przedstawiony w odnośniku α, β). Obserwator ten wykorzystuje transformację kubaturową do rozwiązania problemu nieliniowości, w przeciwieństwie do rozszerzonego filtru KALMANA (EKF). Wyniki symulacji bezczujnikowego FOC pokazują dokładne śledzenie prędkości odniesienia i płynną dynamikę przejścia. Z dobrą estymacją stanu przez obserwatora CKF, która jest odporna na szumy (wewnętrzne i pomiarowe) i wstawkami parametrycznymi (Z dobrą estymacją stanu przez obserwatora CKF, która wykazuje odporność na szumy (wewnętrzne i pomiarowe), a także na wstawki parametryczne) . (**Technika sterowania zorientowanego polowo zastosowana w PMSM bez czujnika mechanicznego, oparta na kubaturowym obserwatorze z filtrem KALMANA**)*

Keywords: Cubature KALMAN filter Observer (CKFO), field oriented control (FOC), Sensorless, PMSM, Position estimation.
Słowa kluczowe: Kubaturowy filtr KALMAN Obserwator (CKFO), sterowanie zorientowane na pole (FOC), Bezczujnikowy, PMSM, Oszacowanie położenia.

Introduction

Today, permanent magnet synchronous motors are recommended in industrial technology. This is due to the fact that they are reliable, the is less bulky than DC motors. Thus, their construction is simpler since it does not belong to a mechanical commutator. Therefore, this increases their lifetime and avoids permanent maintenance [1, 2].

The PMSM is known for its robustness, which allows to create speed and torque controls with precision and very interesting dynamic performances (robotics actuators, servomotors, variable speed drives, etc.). However, its control is more difficult than that of a DC machine because the system is non-linear and it is very difficult to obtain decoupling between the armature current and the field current [1,3].

The FOC strategy applied to the PMSM allows in order to decouple dynamic behavior. Therefore, the use of robust control algorithms to maintain an acceptable level of decoupling and performance is necessary [4]. The Kalman filter is a stocastic state observer based on a number of assumptions, including internal and measurement noise, which are assumed to be centered and white. Their basic principle is to minimize the estimation error [5-7].

Nowadays, there is a growing interest in the development of estimation strategies for rotor position and speed for PMSM drivers. Many of these methods for estimating non-measurable states and disturbances in partially known systems are based on the KALMAN Filter and its extensions [8].

In [9], EKF is used to estimate the position and speed of PMSM, and in [10], load torque is also estimated in addition to position and speed. In [11], a reduced-order linear Kalman filter is employed as a phase-locked loop (PLL) to extract the frequency and phase of the stator flux vector; however, the dynamic response to stepping load has not been investigated. In [12], various optimization methods for Kalman filter extensions are presented. In [13], two parallel EKFs are used to estimate the back-emf, and then the position and speed are estimated. In [14], mechanical equations of PMSM are utilized to estimate position and angular velocity at low speeds, thus reducing the computational burden.

In this paper, a new state observer based on the Cubature Kalman filter (CFKO) is used to observe the rotor position and speed. The observer model is set up in the rotor flux-oriented synchronous coordinate, so it can be used easily in salient or non-salient pole motors because the stator inductances in synchronous coordinate are always constant. By using numerical iteration, the Cubature Kalman filter solves nonlinear equations directly.

Mathematical PMSM model

The PMSM has been described in a stationary two-axes reference frame (α,β) in order to obtain the system equations in the most appropriate form. This choice yields a model in which the derivatives of the motor currents are linearly related to both currents and applied voltages [4- 6]. By assuming:

$$x(t) = [i_\alpha \; i_\beta \; \Omega \; \theta_e]^T \, , u(t) = [v_\alpha \; v_\beta \,]^T$$

And $y(t) = [i_\alpha \; i_\beta \,]^T$

$$(1) \quad \begin{cases} f\big(x(t), u(t)\big) = \begin{bmatrix} \frac{-R_s}{L_s} . i_\alpha - \frac{p.\Psi_r}{L_s} . \sin(\theta_e) . \Omega + \frac{1}{L_s} v_\alpha \\ \frac{-R_s}{L_s} . i_\beta - \frac{p.\Psi_r}{L_s} . \cos(\theta_e) . \Omega + \frac{1}{L_s} v_\beta \\ \lambda . \sin(\theta_e) . i_\alpha - \lambda . \cos(\theta_e) . i_\beta \\ -\frac{f}{J} . \Omega - \frac{1}{J} . \mathrm{T}_l \\ p.\Omega \end{bmatrix} \\ h\big(x(t)\big) = [i_\alpha \; i_\beta \,]^T \end{cases}$$

With: $\lambda = \frac{-3.p.\Psi_r}{2.J}$ and $L_s = L_q = L_d$

Observability study of PMSM model

To estimate the state of a process, it must be observable. In the case where the process is nonlinear, write the following state equation:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = h(x(t)) \end{cases} \quad (2)$$

Where $x \epsilon R^n, u \epsilon R^m$ and $y \epsilon R^p$ are the states, input and output vectors, respectively.

$f: R^n x R^m \to R^n$ and $h: R^n \to R^P$ are smooth maps.

A system under the form of (2) is said to be locally observable at a point x_0 if all state x can be instantaneously distinguished by a judicious choice of input *u* on a neighborhood *U* of x_0. The system is locally observable if there exist integers $k_1 \geq k_2 \geq . k_p$ with $\sum_{i=1}^{p} k_i = n$ and a neighborhood U of x_0 such that [6,15,16]:

$$\dim\{L_f^j, / \ i = 1, \dots, p; J = 0, \dots, k_i\}(x) = n$$

For all $x \ \epsilon \ U$ Recall that the Lie derivative of h along the vector field f is defined as $L_f = \frac{\partial h}{\partial x} f$

Either the space '*O*' called observation space defined by the smallest vector space containing $h_1, \dots, h_p$ and all their successive Lie derivatives:

$$O = span\{L_f^j h(x); i = 1, \dots, p; j = 0,1,2, \dots\} \quad (3)$$

With: $L_f^j h(L_f^{j-1} h), j \geq 2$ and $L_f^0 h = h$

Where: $L_f h(x) = \sum_{i=1}^{n} f_i(x) . \frac{\partial h(x)}{\partial (x_1)}$

For the model of the PMSM described by equation (01), the observability matrix (*OB*) which is generated by $L_f^0 dh_1$, $L_f^0 dh_2$, $L_f^1 dh_1$ and $L_f^1 dh_2$ given by:

$$OB = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{R_s}{L_s} & 0 & \frac{p.\Psi_r}{L_s}.\sin(\theta_e) & \frac{p.\Psi_r}{L_s}.\Omega.\cos(\theta_e) \\ 0 & -\frac{R_s}{L_s} & -\frac{p.\Psi_r}{L_s}.\cos(\theta_e) & \frac{p.\Psi_r}{L_s}.\Omega.\sin(\theta_e) \end{bmatrix} \quad (4)$$

With: $\Delta(OB) = \frac{p^2.\Psi_r^2.\Omega.(sin^2(\theta_e) + cos^2(\theta_e))}{L_s^2}$

Is full rank if and only if $\Omega \neq 0$. Hence, one can conclude that system (1) is locally observable for all Ω values except at the point $\Omega = 0$. Indeed, if $\Omega = 0$ in the system (1), electrical variables are decoupled mechanical variables and we can't reconstruct the mechanical variables from only the electrical measurements.

Cubature Kalman filter observer design

We can have a stochastic state observer algorithm based on the CKF filter applied to the PMSM model, in order to achieve vector control without a speed sensor.

This algorithm is applied to the following nonlinear stochastic model [17,18]:

$$\begin{cases} \frac{dx}{dt} = f(x(t), u(t), t) + w(t) \\ y(t) = h(x(t)) + v(t) \end{cases} \quad (5)$$

Where $x(t)$ and $y(t)$ are the state and the measurement vectors, respectively. *w(t)* and *v(t)* are zero-mean white Gaussian noises with covariance *Q(t)* and *R(t)*, respectively.

The overall sensorless FOC control block diagram involving the proposed Cubature Kalman Filter Observer is presented in Fig. 1.

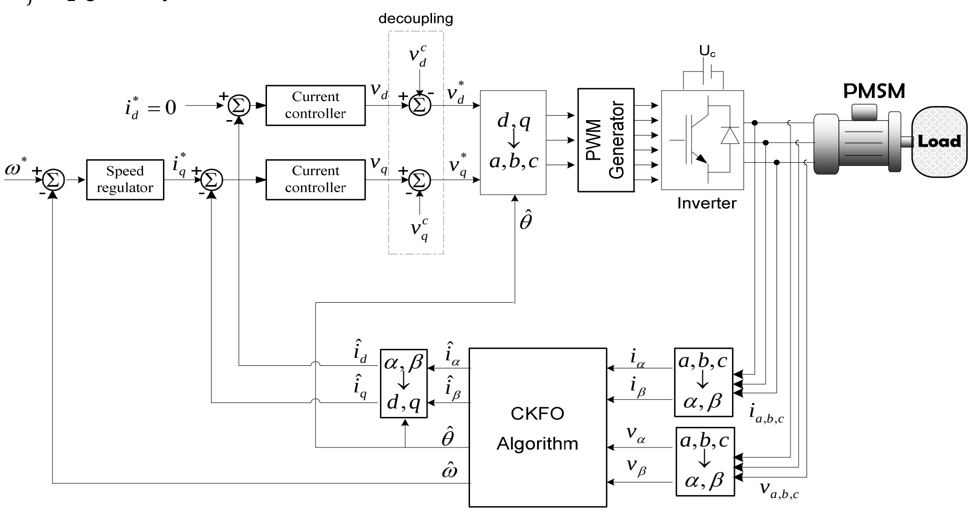


Fig.1. Control block diagram with the proposed rotor position and speed (CKF) observer

The observer (CKF) uses the cubature transform to solve the nonlinearity problem. There are two steps, one of prediction and the other of correction or update. The prediction equations of the KALMAN cubature filter and the modifications to the correction step are [19-22]:

A. *Prediction step*

This step contains eight operations:

1. *Calculate sigma points*

By applying the cubature transformation equations to find the sigma points, we have:

$$\chi_{i,k|k} = [\hat{x}_{i,k|k-1} + (\sqrt{P_{k-1}})_i \ \ \hat{x}_{i,k|k-1} - (\sqrt{P_{k-1}})_i] \quad (6)$$

With $\chi_{k|k-1} = [\hat{\imath}_\alpha \ \ \hat{\imath}_\beta \ \ \hat{\Omega} \ \ \hat{\theta}_e]$ estimated state vector and P_{k-1} is the error covariance matrix, which was determined in the previous iteration.

2. *Transformation of sigma points.*

Using the state model represented by equation (01) we will transformed the sigma points

$$\chi_{i,k+1|k} = f[\chi_{i,k|k}, u(k), k] \quad (7)$$

where $u(k) = [v_{\alpha,} \ \ v_{\beta,}]^T$ represents the input or control vector.

3. *The mean of the predicted state :*

$$\hat{x}_{k+1|k} = \sum_{i=0}^{2n} W_i . \chi_{i,k+1|k} \quad (8)$$

Where $W_i = \frac{1}{2n}$

4. *The covariance of the predicted state :*

$$P_{k+1|k} = \sum_{i=0}^{2n} W_i \, [\chi_{i,k+1|k}^x - \hat{x}_{k+1|k}^-][\chi_{i,k+1|k}^x - \hat{x}_{k+1|k}^-]^T + Q \quad (9)$$

5. *Calculated of the predicted* observation*:*

The transformation of sigma points with observation model is represented by the following equation:

(10) $$\gamma_{i,k+1|k} = h[\chi_{i,k+1|k}, u(k), k]$$

6. *Prediction of observation.*

(11) $$\hat{y}_{k+1|k} = \sum_{i=0}^{2n} W_i \cdot \gamma_{i,k+1|k}$$

where $\hat{y}(k) = [\hat{\imath}_{\alpha,}\ \ \hat{\imath}_{\beta,}]^T$ is the estimated output vector.

7. Innovation covariance (residual) calculation.

(12) $$P_{yy,k+1|k} = \sum_{i=0}^{2n} W_i \left[\gamma_{i,k+1|k} - \hat{y}_{k+1|k}\right][\gamma_{i,k+1|k} - \hat{y}_{k+1|k}]^T + R$$

8. Autocorrelation matrix calculation.

(13) $$P_{xy,k+1|k} = \sum_{i=0}^{2n^a} W_i \left[\chi_{i,k+1|k} - \hat{x}_{k+1|k}\right][\gamma_{i,k+1|k} - \hat{y}_{k+1|k}]^T$$

B. *Correction step:*

This step contains three operations:

1. *KALMAN gain calculation*

(14) $$K_k = P_{xy} P_{yy}^T$$

2. *Update the state estimate with the measurement.*

(15) $$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K(y_k - \hat{y}_{k+1|k})$$

Where $y(k) = [i_\alpha\ i_\beta]^T$ is the measured variables.

3. *Update of the error covariance. Update of the covariance matrix of state.*

(16) $$P_{k+1|k+1} = P_{k+1|k} - K_k P_{yy} K_k^T$$

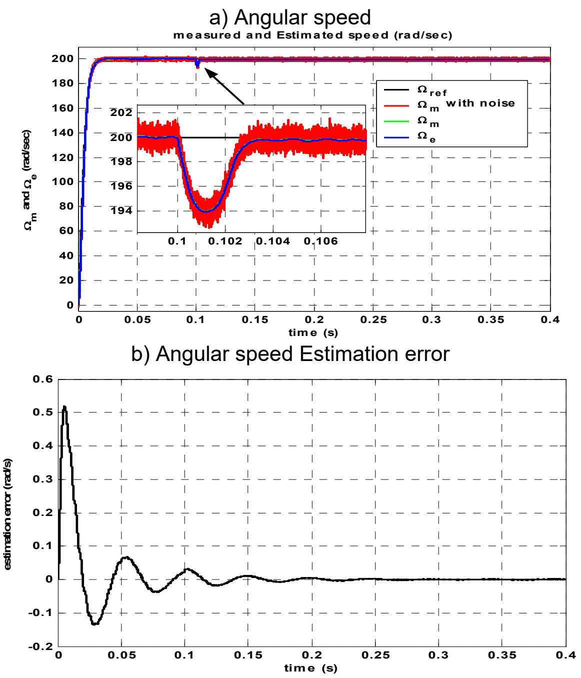
Simulation results

The simulation results using the MATLAB-SIMULINK software package and the motor parameters listed in Table1. Internal and measurement zero-mean white Gaussian noises are injected.

Table 1. Parameters of the PMSM

Symbol	Parameters	Value
L_s [mH]	Stator inductance	2,2
R_s [Ω]	Stator resistance	0,8
J [kg.m^2]	Rotor inertia	0,74*10^{-3}
f [N.m.sec/rad]	Frictional constant	2,6*10^{-3}
ψ_r [Wb]	Rotor magnetic flux	0.133
p	Number of pole pairs	4

Figure 2 shows the simulation results of CKF state estimation performance and the sensorless FOC control trajectory tracking performance for PMSM drive.



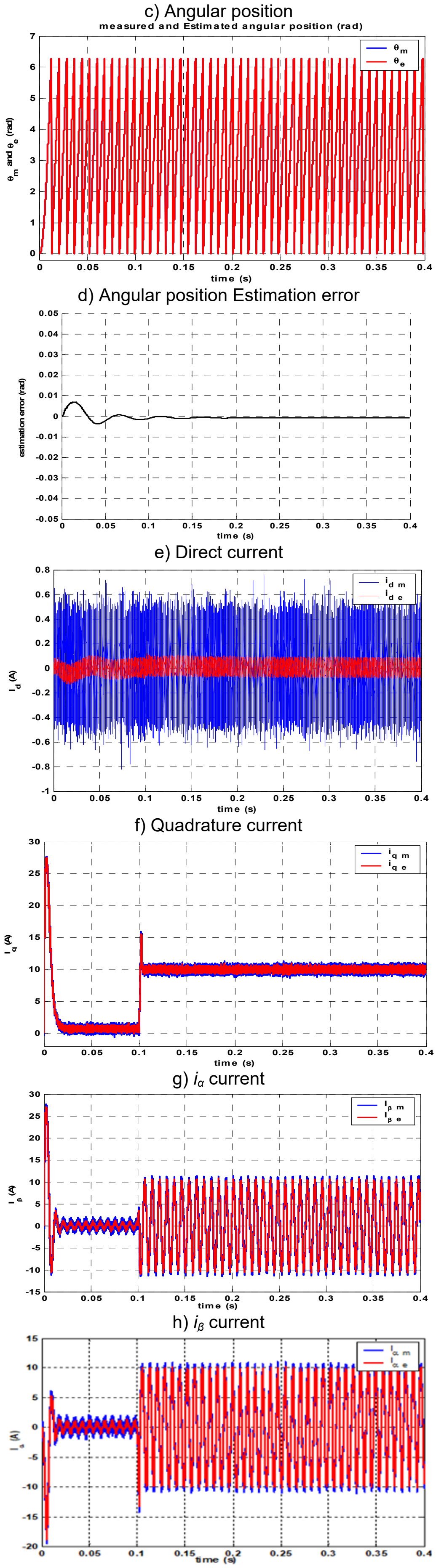
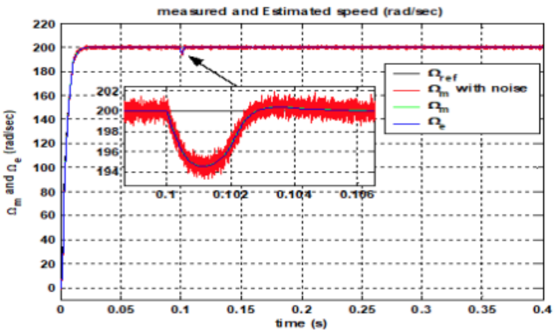


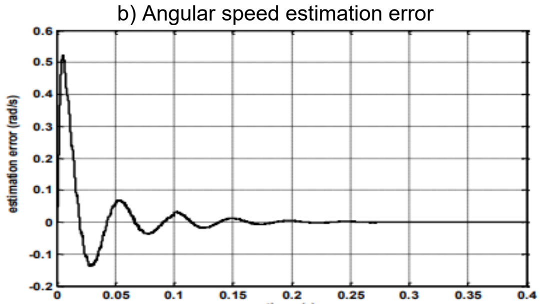
Fig. 2. Control with noisy measurements

It can be stated that CKFO is capable of tracking the speed state satisfactorily under noisy machine operation. it is observed that achieved the objective of the FOC technic, where the direct current i_d is hovering around zero (Fig. 2 (e) and (h)), Besides, In Fig. 2 (g) and (h), the measured and estimated stator currents in the α, β reference are demonstrated.

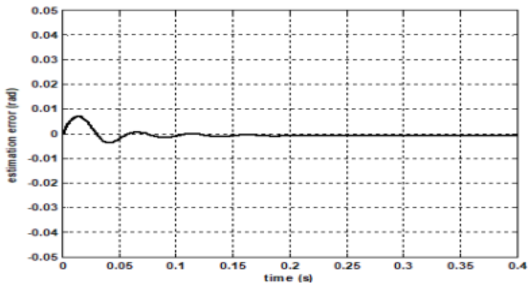
a) Angular speed



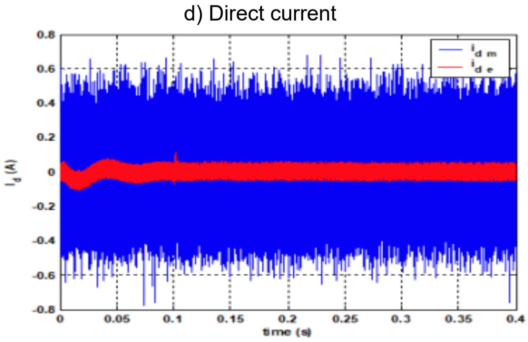
b) Angular speed estimation error



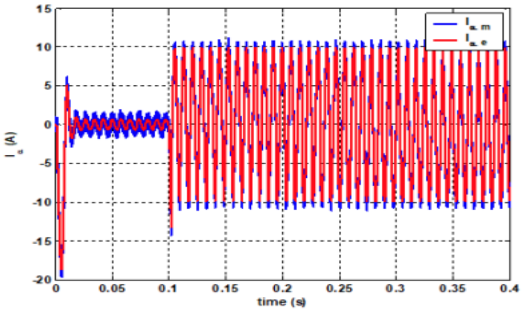
c) Angular position estimation error



d) Direct current



e) i_α current



f) i_β current

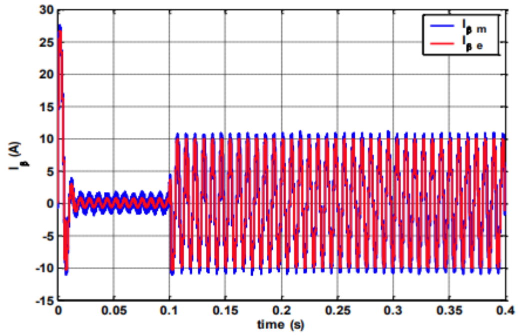


Fig. 3. Control with estimated states.

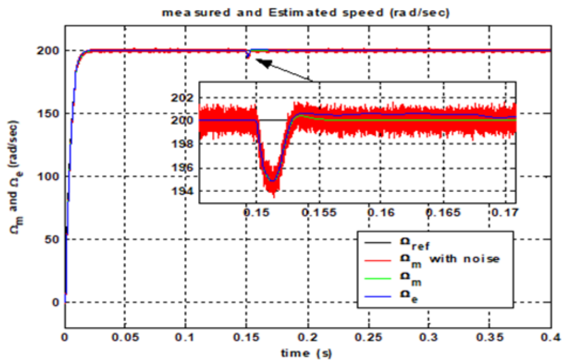
The control is carried out by noisy measurements in Fig. 2 but in Fig. 3 the feedback is realised by estimated states, it can be deduced that the achieved objective of FOC, the CKFO gives good estimates of the speed with the rejection of noises, and the control response is improved. In another scenario, the simulation results are given when the reference speed is 200 rad/s and the motor is loaded with a step load of 7.4 Nm (rated torque) at t=0.1s.

All figures of simulation shows the FOC strategy performance under the condition of the load torque variations.

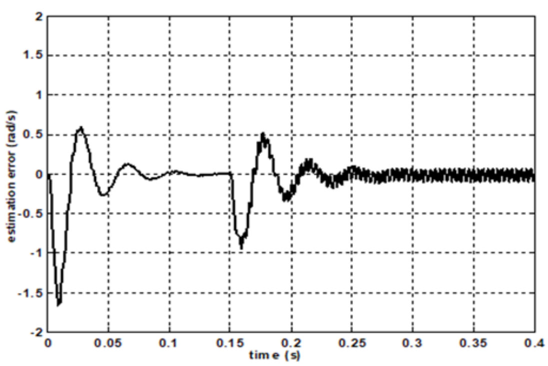
Finally, we examine the robustness of the proposed approach in the presence of the model uncertainty.

The major advantage of CKF observers is that they can be made considerably more robust to parametric uncertainties. For this purpose, we consider a mismatch between the real stator resistance and the moment of inertia and their values in the model. A difference up to 20% between the real parameters and their values in the model is considered. Simulation results are depicted in Figs. 4 and 5, the performance does not degrade significantly because the observer can well estimate the uncertainty.

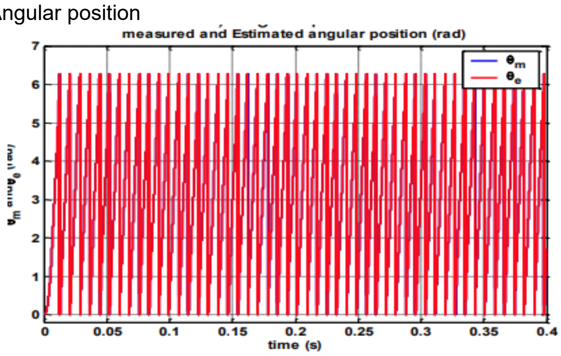
a) Angular speed



b) Angular speed estimation error



c) Angular position



d) Angular potition Estimation error

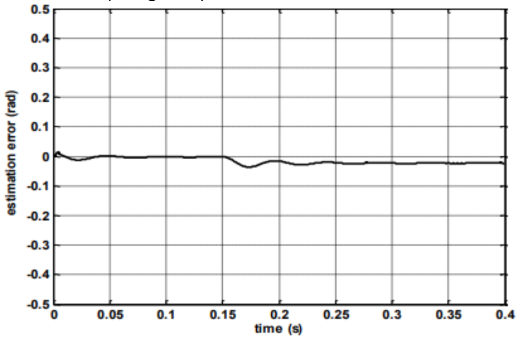
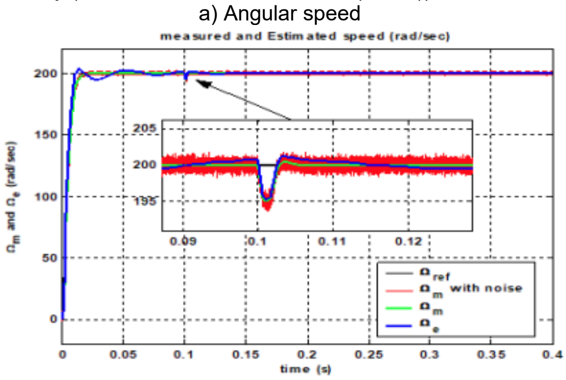


Fig. 4. Control with estimated states in presence of parameters uncertainty (variation of the resistance R_s (+20%))

a) Angular speed



b) Angular speed Estimation error

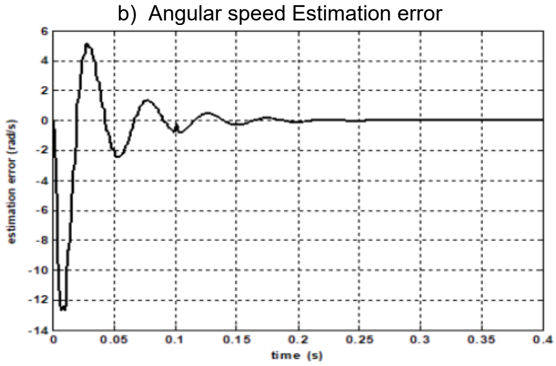
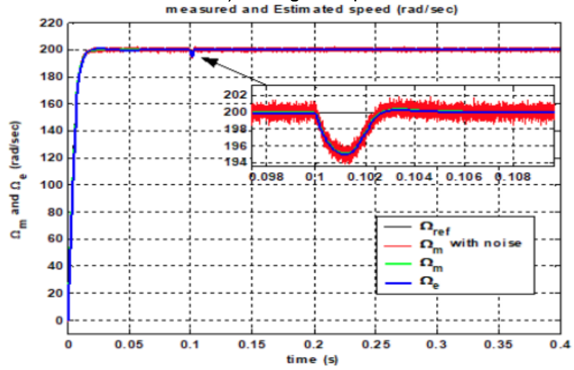


Fig. 5. Control with estimated states in presence of parameters uncertainty (variation of the moment of inertia J (+10%))

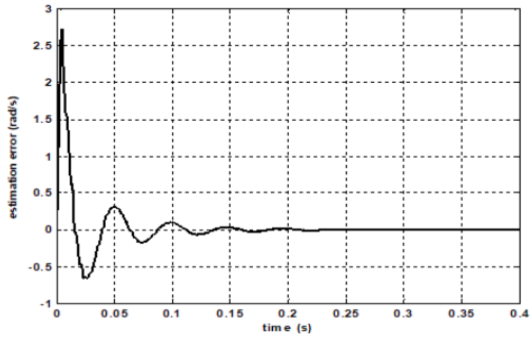
Figure 6 shows measured and estimated state variables (speed and position) with a non zero initial angular position ($\theta_0 = \pi/12$ rad), but the estimation errors quickly tend to zero.

The results show the effectiveness and fast response without overshoot at tracking a reference speed under parameter and load torque variations throughout the system. Also in trajectory tracking (fig.7), the CKFO gives estimates almost identical to the measurements, even in cases of reduced or zero speed

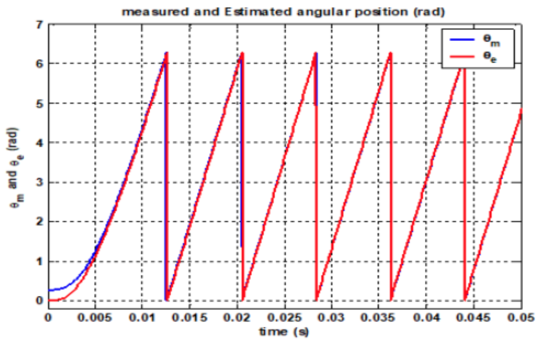
a) Angular speed



b) Angular speed estimation error



c) Angular position



d) Angular potition estimation error

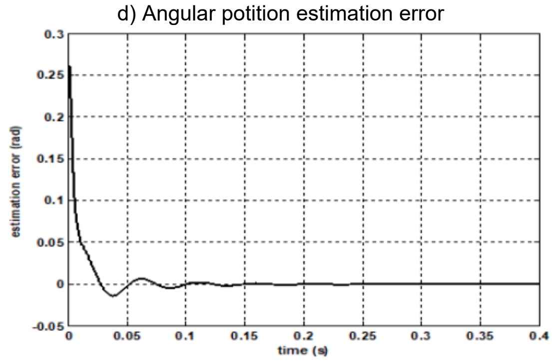
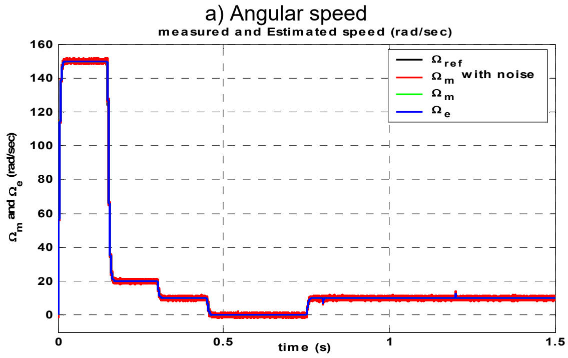
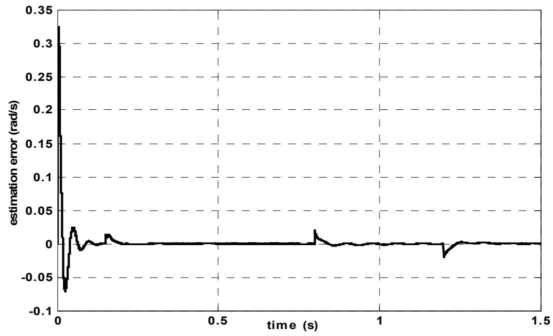


Fig. 6. Control with estimated states (nonzero initial angular position ($\theta_0 = \pi/12$ rad)).

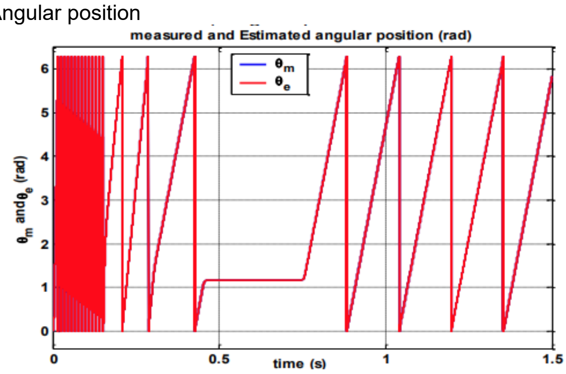
a) Angular speed



b) Angular speed estimation error



c) Angular position



d) Angular potition estimation error

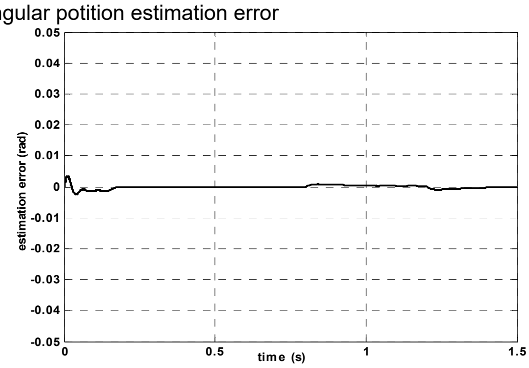


Fig. 7. Dynamic responses of the PMSM trajectory tracking

Conclusion

This paper proposed an CKFO-based PMSM sensorless control method. For controlling the speed of PMSM, the estimated speed and rotor position are fed back to calculate the reference currents. The drive performance is demonstrated at different working conditions. The simulation results using MATLAB/Simulink are included, which shows good estimate of the state vector by the CKFO also its robustness against noises, and non zero initial position. The performance of proposed observer is found satisfactory in presence of parametric uncertainty during operation of the motor. The findings enumerated above confirms the suitability of CKFO based vector control of PMSM drive. One of the advantages of this observer is that it does not require linearization of the model like other observers, based on the above; we suggest making an analytical comparison with other observers.

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