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The Substantiation of the Calculation Expression for the Extraction Force of Magnetic Separators

Abstract. *A review and analysis of methods for calculating the magnetic field force action on a ferromagnetic body are carried out. The integral equivalence of various formulas for the vector of the density of the force acting on a ferromagnetic body in a static magnetic field is shown. The possibility of applying the corresponding formula in engineering calculations of magnetic separators is confirmed theoretically and experimentally.*

Streszczenie. *Dokonano przeglądu i analizy metod obliczania intensywności oddziaływania pola magnetycznego na ciało ferromagnetyczne. Pokazano całkową równoważność różnych wzorów na wektor gęstości siły działającej na ciało ferromagnetyczne w statycznym polu magnetycznym.. Możliwość zastosowania odpowiedniego wzoru w obliczeniach inżynierskich separatorów magnetycznych została potwierdzona teoretycznie i eksperymentalnie.* **(*Uzasadnienie wzoru obliczeniowego siły ekstrakcji w separatorach magnetycznych*)**

Keywords: magnetic field global force, ferromagnetic body, electromagnetic separator.
Słowa kluczowe: siła pola magnetycznego, ciało ferromagnetyczne, separator elektromagnetyczny

Introduction

The use of the volume density $\overline{f}$ of the magnetic field ponderomotive force is one of the modern bases for describing the static magnetic field force action on ferromagnetic bodies. During the recent 30 years, quite a lot of papers have been devoted to the problem of determining both the volume force density and the global force acting from the electromagnetic field on ferromagnetic bodies and permanent magnets. This is explained by the fact that the correct calculation of these values is very important when considering not only the processes of magnetic separation, but primarily the processes of electromechanical energy conversion.

These papers can be roughly divided into two groups. The first one deals with the actual analytical concepts and comparative assessments of methods for calculating the mentioned forces [1–16]. The second one includes papers related to force calculations by various numerical methods, the results of which are intended either to confirm the correctness of the proposed calculation methods [17–18], or to compare two or more calculation methods in order to develop recommendations for their use [19–21].

The papers of the first group present methods relying on various theoretical principles.

First of all, it is Maxwell's Stress Tensor Method and Virtual Work Method (Principle). Calculation methods relying on Maxwell Stress Tensor can use surface or volume integration to calculate forces. Surface integration is certainly easier to use, but volume integration has its advantages, in particular, when solving three-dimensional problems [6, 20].

By the virtual work method (energetic principle) magnetic force can be calculated from expressions written using either a scalar or a vector potential. In both cases, it is possible to calculate both total (global) force and local force distribution [6, 9, 19].

Methods relying on equivalent source models use two types of models: equivalent magnetizing currents and equivalent magnetic charges. It should be noted that only the global (total) force is theoretically the same for both models [9].

In the second group of papers, the main attention is paid to the numerical calculations of the required forces. In most papers, the finite element method [17, 19–21] is used as the most developed. In part of the papers, the finite difference method is used. Some papers directly aim at comparing the methods for calculating global (total) force and local force and developing recommendations for their use, including the degree of discretization of the computational domain.

However, despite such a relatively large attention given to the problem of calculating the force action of an electromagnetic field, many issues remain debatable. This is noted in a number of papers [6–8]. In paper [6] the author writes: "From the point of view of engineers involved in calculations, forces in magnetized matter are not yet properly understood".

At present, when calculating the force action of magnetic separators of various designs on the extracted ferromagnetic bodies, the formula for the specific (per unit volume of the extracted body) extraction force is widely used [22]:

$$\overline{f_0} = \frac{1}{2}\mu_0\chi_b grad\left(H_0^2\right), \quad (1)$$

where μ_0 – vacuum magnetic permeability, $\mu_0 = 4\pi\times10^{-7}$ G/m; χ_b – magnetic susceptibility of the extracted ferromagnetic body, depending on its shape, size ratio and magnetic permeability of the substance; H_0 – magnetic field strength of the separator at the location of the unit volume of the extracted body in the absence of this body.

The practical use of formula (1) in each particular case of a particular design of a magnetic separator is based on the research of the distribution of the magnetic field strength of this separator, which makes it possible to determine the specific (reduced to μ_0 and χ_b) force:

$$\overline{f_r} = \frac{\overline{f_0}}{\mu_0\chi_b} = H_0 grad H_0, \quad (2)$$

called the reduced volume density of the magnetic field force [22, 23].

The derivation of formula (1), adopted in the theory of separator construction, is based on the energy method (energetic principle), which relies on the use of the energy formula for dia- and paramagnets in a static magnetic field [22]. However, since magnetic separators are mainly intended for extracting ferromagnetic, in this case such an energy approach needs to be clarified. In addition, for the system "magnetic field source – moving ferromagnetic", the problem of energy balance in the general case (without assumptions about the constancy of currents or source flux linkages) has not been solved [2, 3, 24], which creates certain difficulties in applying the energy method to the "separator - extracted body" system. Despite these theoretical uncertainties, formula (1) is successfully used in separator construction.

The substantiation of the applicability of this formula for practical calculations without involving the energy method is the purpose of this paper. In addition, our task was to obtain formulas for the direct calculation of the extraction force of magnetic separation devices, the derivation of which is based on taking into account the real properties of the magnetic field of these devices.

The analysis of the formulas for the magnetic field force action on magnets

In the absence of conduction currents from external current sources in a ferromagnetic, the above methods make it possible to obtain, in one way or another, the following formulas for the calculation of the specific (per unit volume) ponderomotive force of the magnetic field:

(3) $$\bar{f}_A = rot\bar{M} \times \bar{B}\,, \quad [2, 3],$$

(4) $$\bar{f}_P = -\mu_0 \bar{H} div\bar{M}\,, \quad [2, 3],$$

(5) $$\bar{f}_{M1} = -\frac{1}{2}\bar{H}^2 grad\mu\,, \quad [3],$$

(6) $$\bar{f}_{M2} = \mu_0 M grad H\,, \quad [18, 19],$$

(7) $$\bar{f}_{E\text{-}L} = \mu_0 (\bar{M} grad)\bar{H}\,, \quad [1],$$

(8) $$\bar{f}_{M.M} = M grad B\,, \quad [2]$$

where $\bar{M}, \bar{B}, \bar{H}$ – respectively the vectors of magnetization, induction and magnetic field strength in the considered unit volume of the ferromagnetic body (hereinafter, the module of vector values is denoted by the corresponding symbol without a dash above it); μ – magnetic permeability of the substance of a ferromagnetic body; $\bar{f}_A$ – the force according to the Ampère equivalent current model; $\bar{f}_P$ – the force according to Poisson's model of equivalent magnetic charges; $\bar{f}_{M1}$ – the force according to Maxwell's first formula; $\bar{f}_{M2}$ – the force according to Maxwell's second formula; $\bar{f}_{E\text{-}L}$ – the force according to the formula of Einstein and Laub; $\bar{f}_{M.M}$ – the force according to the model of equivalent magnetic moments.

A direct comparison of expressions (3)-(8) shows a significant difference between the above formulas for $\bar{f}$, both by module (it is sufficient to compare, for example, $\bar{f}_{M2}$ with $\bar{f}_{M.M}$), and by direction (e.g., vector $\bar{f}_{M2}$ is parallel to the vector of gradient $\bar{H}$, and vector $\bar{f}_{M1}$ – to the vector of gradient μ, vector $\bar{f}_P$ is parallel to vector $\bar{H}$, vector $\bar{f}_A$ is orthogonal to vector $\bar{B}$). Such a variety of formulas for $\bar{f}$ and their difference from one another poses the problem of choosing from the above list such a formula that allows getting a result that is adequate to real forces. It is this problem that is being solved during the discussion in the scientific literature. Moreover, the discussion on the force action of a static magnetic field on magnetized bodies is mainly aimed at considering nonlinearly magnetized (ferromagnetic) substances, since for dia- and paramagnets, the differences in the values f obtained by direct numerical calculations [2] according to formulas (3)-(8) are insignificant.

It should be noted that at present there is no consensus on the calculation of the real distribution of internal stresses in a magnetized ferromagnetic. Its finding is a complex physical problem of the force action of a magnetic field on the microcarriers of magnetism in a magnetized substance [27]. However, if we proceed from the need in most calculations to determine the total force of the magnetic field acting on the entire ferromagnetic body, the question of the applicability of a particular formula can be solved on the basis of establishing the equivalence of the considered formulas when integrating over the entire volume of the magnetized body to each other and to the real total force.

Such an integral approach, in particular, was used in [2] to establish the equivalence of two different formulas for the magnetic field force acting on permanent magnets.

The use of this integral approach in the case of designing magnetic separation devices is associated with the need for theoretical confirmation of the practical applicability of formula (1) for calculating the total force acting on a ferromagnetic body in a static magnetic field, in the absence of knowledge of the exact distribution of the specific force (internal stresses) over the volume of the researched body.

The integral equivalence of various formulas for the density of the force acting on a ferromagnetic body in a static magnetic field

The electromagnetic (ponderomotive) force acting on some isotropic ferromagnetic body with volume V_b, placed in a static magnetic field can be defined as

(9) $$\bar{F} = \int_{V_b} \bar{f} dV\,.$$

Obviously, the complex form of differential-vector expressions for $\bar{f}$ by formulas (3)-(8) does not allow applying the well-known theorems on volume integration to the volume integral (9). However, it is possible to consider surface equivalent magnetization currents and surface equivalent magnetic charges as the limits of the corresponding volume currents and charges near the surface of the body, and the corresponding vectors of the magnetic field, discontinuous on the surface of the body, as generalized functions [28]. Force volume densities $\bar{f}_i$ can also be regarded as generalized functions, which make it possible to avoid, when determining the corresponding total forces, the writing of surface integrals on the discontinuity surfaces of the field vectors and obtain relations for solving the problem of integral equivalence of the considered formulas for the forces.

In addition, the proof of the possibility of applying formula (1) to calculate the total magnetic field force on the entire ferromagnetic body requires the use of some additional physical concepts. They are based on the decomposition of the real strength and induction of the magnetic field in the form of the sum of the strength and induction of the magnetic field of the source (in the considered case – a separator) and the same magnetic field vectors from a magnetized body [2]:

(10) $$\bar{H} = \bar{H}_0 + \bar{H}_M\,, \qquad \bar{B} = \bar{B}_0 + \bar{B}_M\,.$$

Here: $\bar{H}$ and $\bar{B}$, $\bar{H}_0$ and $\bar{B}_0$, $\bar{H}_M$ and $\bar{B}_M$ – real vectors of intensity and induction of the magnetic field, respectively, real (in the presence of a magnetized body), from the source of the magnetic field and from the magnetized substance of the body introduced into the magnetic field of the source.

Using the above said, let us determine the volume density of the magnetic field force according to the two indicated models of the magnetized state. For the model of equivalent magnetizing currents, the volume force density $\overline{f}_A$ can be defined as the force acting on a current of known volume density $rot\overline{M}$ in some magnetic field with known induction $\overline{B}$, which, taking into account (10), gives

(11) $$\overline{f}_A = rot\overline{M} \times \overline{B} = rot\overline{M} \times \overline{B}_0 + rot\overline{M} \times \overline{B}_M .$$

For the model of equivalent magnetic charges, the volume force density $\overline{f}_P$ can be defined as the force acting on a magnetic charge distributed with a known density $\mu_0 div\overline{M}$ in some magnetic field with a known strength $\overline{H}$, which, taking into account (10) gives

(12) $$\overline{f}_P = -\mu_0 \overline{H} div\overline{M} = -\mu_0 \overline{H}_0 div\overline{M} - \mu_0 \overline{H}_M div\overline{M} .$$

Of course, neither force density $\overline{f}_A$, nor force density $\overline{f}_P$ is adequate to the real density of the force of magnetic field in magnetized substance [2, 24, 29]. However, it is quite possible to use force volume densities $\overline{f}_A$ and $\overline{f}_P$ if it is necessary to determine the total magnetic field force acting on the whole volume of the magnetized body. It is demonstrated as follows.

First of all, we note that in expression (11) on the right, the second term is the volume density of the force of action on the equivalent magnetization currents from the magnetic field of these currents, which for a specific magnetized body represent a certain closed system located in the finite volume of the body under consideration. In this case, the total force is equal to zero, since it is the sum of the internal forces of a closed physical system [30]:

(13) $$\int_{V_b} rot\overline{M} \times \overline{B}_M \times dV = 0 .$$

Similarly, the second term on the right in formula (12) is also the sum of internal forces in a closed physical system [30]:

(14) $$\int_{V_b} -\mu_0 \overline{H}_M div\overline{M} \times dV = 0 .$$

Thus, in the case of modeling a magnetized substance of a certain body of finite volume V_b by equivalent magnetization currents, the total force of the magnetic field on this body can be determined as

(15) $$\overline{F}_A = \int_{V_b} \overline{f}_A dV = \int_{V_b} rot\overline{M} \times \overline{B}_0 \times dV .$$

If the magnetized substance of the same body is modeled by equivalent magnetic charges, the total force of the magnetic field on the body under consideration can be determined as

(16) $$\overline{F}_P = \int_{V_b} \overline{f}_P dV = -\int_{V_b} \mu_0 \overline{H}_M div\overline{M} \times dV .$$

According to expressions (15) and (16) the total force of the magnetic field acting on a certain body in it is determined by both models of magnetization as the force of the external magnetic field (with induction $\overline{B}_0$ and strength $\overline{H}_0$) respectively for equivalent magnetizing currents or for equivalent magnetic charges. Moreover, based on the third law of mechanics on action and reaction, in the case under consideration it is easy to obtain that

(17) $$\overline{F}_A = \overline{F}_P = \overline{F}_R ,$$

where $\overline{F}_R$ – the real total force acting on a magnetized body in the magnetic field of the source.

(18) $$\overline{F}_R = \int_{V_b} \overline{f}_R dV .$$

Here $\overline{f}_R$ – the volume density of the real force of the magnetic field acting on the magnetized substance of the body, considered as a generalized function.

The last relation can also be written in integral form, if we take into account expressions (15), (16) and (18):

(19) $$\int_{V_b} \overline{f}_A dV = \int_{V_b} \overline{f}_P dV = \int_{V_b} \overline{f}_R dV .$$

Thus, according to formula (19), the total force of the magnetic field of some source, acting on a magnetized body placed in this field, can be determined via volume integration (by full body volume) or by volume density $\overline{f}_A$ of force, corresponding to the model of substance magnetization in the form of equivalent magnetization currents, or volume density $\overline{f}_P$ of force, corresponding to the model of substance magnetization in the form of equivalent magnetic charges. In other words, both considered models of a magnetized substance give a result adequate to reality when calculating the total force of the magnetic field acting on a magnetized body.

The similarity of the calculated values of the electromagnetic force found using other models (formulas (5) - (8)), for the case under consideration, when the ferromagnetic body is located in a non-magnetic medium, is shown in a number of papers, including ones with the participation of the author [23, 24, 31], and is not considered further here.

A calculation formula for determining the specific extraction force of magnetic separation devices

We use the obtained conclusion and the models of the magnetized substance considered above, as well as Brown's theorem [29] to substantiate the applicability of formula (1).

According to Brown's theorem, the total force of the magnetic field acting on some arbitrary volume filled with magnetic dipoles distributed continuously with a volume density $\overline{M}_d$ is equal to the force acting on a system of magnetic charges (poles) distributed continuously in the same volume with density $-\mu_0 div\overline{M}_d$. Its application to the case of a magnetized body placed in an external magnetic field allows us to write the following integral equality:

(20) $$\int_{V_b} \left(\overline{M} grad\right)\overline{B}_0 \times dV = -\int_{V_b} \mu_0 \overline{H}_0 div\overline{M} \cdot dV ,$$

which corresponds to the distribution of magnetic dipoles over the volume of the body V_b with a density equal to the magnetization of the substance of the considered body ($\overline{M}_d = \overline{M}$).

As can be seen from equality (20), the volumetric force density given by the formula can be introduced into the consideration

(21) $$\overline{f}_m = \left(\overline{M} grad\right)\overline{B}_0 ,$$

which, according to expressions (16) and (20), is integrally equivalent to the force density $\overline{f}_P$ which, in turn, is integrally equivalent to the real volume density of the magnetic field force on the magnetized substance of the body under consideration.

Note that, as the volume force densities $\overline{f}_A$ and $\overline{f}_P$ considered above, the volume force density $\overline{f}_m$ is not equal to the real volume density of the magnetic field force on the magnetized substance, since when obtaining it, the internal magnetic field caused by the magnetization of the substance in the body is not taken into account [29].

The integral equivalency of force densities $\bar{f}_m$ and $\bar{f}_R$ can also be determined using a model of equivalent magnetization current to describe the magnetized state of the substance. To do this, it suffices to represent any differentially small volume inside a magnetized body in the form of a small current loop located on the surface of this differential volume, as, for example, in [32]. As a result of this representation, the continuous system of volumetrically distributed equivalent magnetization currents is replaced by the corresponding continuous system of current loops in the volume of the body, the magnetic moment of each of which is equal to the magnetic moment of the corresponding differential volume ($\bar{M}\,dV$). Moreover, the force of the magnetic field acting on any small current loop inside the volume of the body, which has a magnetic moment equal to the magnetic moment of the volume dV, can be determined by formula [32]

(22) $$d\bar{F}_m = \left(\bar{m}grad\right)\bar{B}_0 = \left(\bar{M}grad\right)\bar{B}_0 dV ,$$

where $\bar{m}$ – magnetic moment of volume dV; $\bar{m}=\bar{M}dV$.

It follows from (22) that the specific force of the magnetic field in the considered model of the magnetization of the substance is also determined by formula (21). However, with such an approach, force density $\bar{f}_m$ is integrally equivalent to force density $\bar{f}_A$, which, in turn, is integrally equivalent to force density $\bar{f}_R$.

Thus, the total force acting on a magnetized body in a certain magnetic field can be determined from the volume density of the magnetic field force given by formula (21).

Upon further consideration, we will take into account that magnetic separators are mainly designed to extract soft magnetic ferromagnetics with isotropic magnetic properties. For such bodies, their magnetization in an external field can be represented as [22, 33]

(23) $$\bar{M} = \chi_b \bar{H}_0 .$$

Then substituting formula (23) in formula (21) and taking into account that $\bar{B}_0 = \mu_0 \bar{H}_0$, after simple transformations, one can get a formula that completely coincides with formula (1). Therefore, if the substance of the body extracted by the magnetic separator is a magnetoisotropic volume-singly connected material (for example, a magnetically soft ferromagnetic) and the magnetization of this body is proportional to the strength of the external magnetic field (in the case under consideration, the strength of the magnetic field of the separator), the specific magnetic field extraction force is described by formula (1). It is integrally equivalent under the above conditions to the real specific force of the magnetic field, which confirms the possibility of practical application of formula (1) in engineering calculations of magnetic separation devices.

Of course, in calculating magnetic separation devices, other formulas can be used to calculate magnetic extraction forces. However, based on the experience of separator construction [22], for the considered type of electric devices, it is recommended to use formula (1). Its use seems to be more convenient for the following reasons. Firstly, with small dimensions of the extracted body (compared with the volumes of the magnetic field in the working interpole gaps of the separators), this body can be characterized by parameter χ_b constant for the entire volume of the body (depending only on the material, shape and size ratio of the body). Secondly, formula (1) contains only one characteristic of the magnetic field – intensity H_0, to determine which there are currently various calculation methods and methods of direct measurement.

Calculation expressions for the extraction force of magnetic separating devices

Exact volume integration of differential-vector operations on magnetic field vectors, which form the basis of the formulas for the specific force of the magnetic field, is very difficult. Therefore, in practical engineering calculations of magnetic separation devices, when it is required to determine the total extraction force, the necessary volumetric integration of the corresponding formula for the specific force is carried out, taking into account the specific conditions that occur in the real case and which help to simplify the process of this integration. Simplification of integration for magnetic separation devices is achieved by taking into account the relative smallness of the dimensions of the extracted body compared with the working interpole gaps of magnetic separators. As a result, for some extractable body of volume V_b, we can neglect the change of function $grad\left(H_0\right)^2$, taking its value within this volume constant, which, when determining the total extraction force acting on the body under consideration, by integrating formula (1) over volume V_b, allows us to write the following transformations:

(24) $$\bar{F} = \int_{V_b} \frac{1}{2}\mu_0\chi_b grad\left(H_0\right)^2 dV = \frac{1}{2}\mu_0\chi_b grad\left(H_0\right)^2 \times \int_{V_b} dV = \frac{1}{2}\mu_0\chi_b grad\left(H_0\right)^2 V_b .$$

Thus, expanding the gradient operation in formula (24), for the extraction force of magnetic separators, we finally obtain the following expression, widely used in practice [22]

(25) $$\bar{F} = \mu_0\chi_b\left(H_0 gradH_0\right)V_b ,$$

from which it can be seen that the direction of the extraction force coincides with the direction of the magnetic field gradient vector. Moreover, if all the quantities included in expression (25) are taken in SI units, then the module of the extraction force $\left|\bar{F}\right|$ is expressed in newtons.

In the practical use of formula (25), it is recommended [17] to calculate the product $\left(H_0 gradH_0\right)$ for the point corresponding to the center of gravity $\left(H_0 = H_{0c}\right)$ of the extracted particle. However, this makes it possible to obtain an exact result only for the case when condition $\left(H_0 gradH_0\right) = const$ is met. Of course, in magnetic fields of real separator designs, product $\left(H_0 gradH_0\right)$ is not constant, but the error in the practical use of formula (25) can be reduced by multiplying force $\bar{F}$ determined by formula (25) by some correction factor depending on the ratio of the average size of the extracted body to the average interpole distance of the magnetic system of the separator [22]. Moreover, according to the data of experimental research [22], this correction factor has a value greater than one, which means an underestimation of the value of the extraction force, determined by formula (25), in comparison with the actual extraction force of the magnetic separator.

The described approach was applied to an axisymmetric two-pole electromagnet and a spherical body made of soft magnetic material with a high magnetic permeability. As a result, the following expression is obtained

(26) $$F = 1.5\mu_0 V grad H_{0c}^2$$

which can be easily tested by an experiment.

The latter was carried out on a physical model of the Sh140-100 electromagnetic pulley (scale 1:5) using a steel ball with a diameter of 17.5 mm, weighing 22 g, located symmetrically on the vertical axis of the interpole gap at a distance of 15 mm from the surface of the poles. As can be seen from the Table 1, where the corresponding results are given, the calculated values agree relatively well with the experimental data.

Table 1. Results of experimental verification

Magnetizing current [A]	Force value [H]		Error [%]
	Experiment	Calculation	
1	0.275	0.2653	-3.4
1.5	0.510	0.567	+11
2	0.834	0.949	+13.8
3	1.46	1.63	+9.4
4	2.01	2.14	+6.4
5	2.75	2.45	-10.8

Conclusions

The analysis of the general methods for determining the volume density and the resulting force of the interaction of a ferromagnetic body with an electromagnetic field have shown that many provisions (representations) regarding real processes are debatable.

The equality of the total forces obtained for various models of the representation of a magnetized ferromagnetic body has been shown. A calculation formula has been obtained for determining the specific extraction force of magnetic separating devices.

Experimental verification has confirmed the validity of using the received and known calculation expressions for preliminary design calculations of magnetic separators.

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