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Distributed Velocity Control in Cooperative Multi-Agent Moving Source Seeking

Abstract. *This paper uses distributed velocity control to address the agent-based moving source-seeking problem. The source is represented by a scalar field. In cooperative multi-agent seeking, agents communicate with one another using a communication topology to exchange and collect information regarding their positions and the value of the scalar field at any given time. This information is then compiled to create a single overall prediction that is used to locate the moving source that moves in a linear or sinusoidal manner. The velocity at which the agents track the moving source is determined using distributed velocity control that examines two agent velocity types: the gradient velocity and the desired velocity. Formation control is also used to maintain the desired formation of the agents. Finally, a computer simulation is conducted to examine how different agents search for a moving source. In both cases, the agents can find the source at a different seeking time. The results show that the agents under a given desired velocity may perform better than those under gradient velocity for the desired velocity* $v_d > 1.85\ m/s$ *and* $v_d > 1.9\ m/s$ *for linear and sinusoidal moving sources, respectively.*

Streszczenie. *W tym artykule zastosowano rozproszoną kontrolę prędkości, aby rozwiązać problem wyszukiwania źródła ruchu w oparciu o agenta. Źródło jest reprezentowane przez pole skalarne. W kooperacyjnym poszukiwaniu wielu agentów agenci komunikują się ze sobą za pomocą topologii komunikacji w celu wymiany i gromadzenia informacji dotyczących ich pozycji i wartości pola skalarnego w dowolnym momencie. Informacje te są następnie kompilowane w celu stworzenia jednej ogólnej prognozy, która jest wykorzystywana do zlokalizowania ruchomego źródła, które porusza się w sposób liniowy lub sinusoidalny. Prędkość, z jaką agenci śledzą poruszające się źródło, jest określana za pomocą rozproszonej kontroli prędkości, która bada dwa typy prędkości agenta: prędkość gradientu i prędkość pożądaną. Kontrola tworzenia jest również stosowana do utrzymania pożądanego tworzenia środków. Na koniec przeprowadzana jest symulacja komputerowa w celu zbadania, w jaki sposób różni agenci wyszukują poruszające się źródło. W obu przypadkach agenci mogą znaleźć źródło w innym czasie wyszukiwania. Wyniki pokazują, że czynniki przy danej pożądanej prędkości mogą działać lepiej niż te przy gradientowej prędkości dla pożądanej prędkości* v_d*>1,85 m/s i* v_d*>1,9 m/s odpowiednio dla źródeł ruchu liniowego i sinusoidalnego. (**Rozproszona kontrola prędkości w kooperacyjnym, wieloagentowym, ruchomym źródle wyszukiwania**)*

Keywords: Distributed velocity control, Cooperative multi-agent moving source seeking, Gradient velocity.
Słowa kluczowe: Rozproszona kontrola prędkości, kooperacyjne, wieloagentowe wyszukiwanie źródła ruchu, prędkość gradientu.

Introduction

Motivation

Source seeking problem can be defined as the problem of finding a maximum value of some potential induced by the source that describes, for example, a temperature level, the hazardous concentration of substances, or vapour. Some problems with cooperative source-seeking have been discussed. One way to find a common source of information is by working together. However, this may take some effort. Prior research has studied various methods of cooperation in information-seeking tasks, and some of these ideas have been applied in real-life situations involving plants, such as unmanned aerial vehicles (UAVs) and mobile robots [1].

Source seeking algorithm is designed to drive single or multi-agents to a source represented by the scalar field signal that all the agents can measure. The scalar field is defined as a function that produces a value at every point in space. As the gradient of the scalar field signal is measured, a gradient climbing algorithm can be developed [2]. In this case, gradient estimation can be used from the distributed measurement of the scalar field of the agents.

The problem in the single-agent case is that all measurements are performed by a single agent [3], [4] as the position changes in every instance. In this research, the estimation is conducted based on a minimum of two measurements taken from two different positions. If the sensor is sensitive enough, the agent must move further from the source to get a better signal. However, it might take a while for the signal to get stronger, which may cause a delay for the agent. If it cannot detect a signal, the agent may remain stationary.

In the source seeking problem, the agents cooperate to measure values of the scalar field from distinct positions of agents simultaneously [5]–[7]. This allows the seeking agents to locate the right source more easily, since they have known the measured scalar field of each agent to determine the correct value of the scalar field.

Related results

The issue of source-seeking has been discussed in the field of control systems, where researchers have explored several different algorithms that agents can use to locate sources of information. Moreover, there are different ways to deal with the problem of sourcing products, e.g., either focus on only one or multiple sources.

First, regarding the problem of sourcing concepts for one agent, some researchers have focused on using exploratory missions to take measurements when agents change their positions regularly over time. Furthermore, for isolated non-holonomic agents, angular velocity settings have been proposed to find the maximum of the scalar plane as mentioned earlier in [3]. Source seeking under periodic back-and-forth movement of the unicycle is used in [8], where the forward speed can be changed to move the agent toward the source. An extension of [3] to search for 3-D sources is proposed in [4]. A sliding mode navigation strategy has been proposed in [1] to move the agent to the maximum of the scalar field. An approach to finding stochastic sources for non-holonomic mobile agents is presented in [9]. The issue discussed in research [10] is the problem of source localization of a diffusion process for an agent with multiple sensors which is based on the calculation of gradients and high-order derivations, such as the Hessian matrix of the Poisson integral. For the case of noisy environment with heterogeneous background signals, Rolf et al. have developed a general algorithm for adaptive source seeking in [11].

Secondly, several algorithms for improving mission performance have been developed for multi-agents. The sourcing problem can be solved by breaking it down into two parts: formation maintenance and leader-follower as shown in [7]. Formation maintenance refers to the action of ensuring that the system is organized and running smoothly. At the same time, the leader-follower part involves providing the leader with some guidance and support, so they can carry out their tasks successfully. The coordination framework uses virtual vehicles and artificial potential functions in this case. Kalman cooperative filter is designed to estimate gradients at the center of formations in [12]. The gradient ascent method to direct the formation of agents towards the maximum of the scalar plane is carried out as mentioned earlier in [13]. A leader which can estimate the gradient is required. The problem of finding a source for a group of agents by distributing them uniformly in a fixed circular formation has been discussed in [14]. To estimate the gradient in the center of mass, the agent rotates around the center, and the information in the central agent is known to all agents. Localization of distributed sources without position information is presented in [10]. The researchers consider a time-ring topology to describe the communication network sensors and the gradient is estimated based on the distributed implementation of the Poisson integral formula. Furthermore, distributed source seeking in a 3-dimensional environment without explicit estimation of the gradient of the field is discussed in [15]. They have developed a strategy to perform a source seeking behavior for a swarm of an arbitrary number of agents and analyzed input-to-state stability.

Several studies have discussed the arrangement of formations. One example is the problem of controlling the formation of identical N agents under an unknown topology, which has been studied in [16]. The study derived the robust stability conditions for the formation, equivalent to those for the single agent. The framework of research in [16] is further developed in [17], which considers the design of a distributed Linear Quadratic Regulator (LQR) in agent formation. A feedback strategy based on the method of decomposition and inequality of linear matrices is proposed in [18]. In research [19], using the framework graph theory results, Popov and Werner turn the problem of formation stability into an important regulatory problem for a single agent. This way, the management of performance requirements becomes more accessible, and stability for fixed and varied topologies with communication delays is guaranteed [20], [21]. A control protocol for robust distance-based formation control in which agents are subjected to disturbances is studied in [22]. Here, connectivity maintenance and collision avoidance among neighbouring agents are also handled by the appropriate design of certain performance bounds. A related work by Sikorski in [23] discusses development of agent-oriented programming for multi-robot system. There, the robots coordinate to find a path and avoid obstacles.

This paper focuses on distributed velocity control for cooperative multi-agent moving source seeking. N identical agents can communicate in two directions. The gradient of the scalar field helps to guide the movement of the agents, which then directs the formation towards the moving source. It is assumed that all agents are identical, and each of them has sensors to measure the intensity of the scalar field and its position relative to other agents. Each agent calculates the gradient of its measurements and those of its neighbours. These scenarios consist of two kinds of velocities. First, the velocity is derived from gradient estimation which has been discussed in our previous result [24], and second, it is derived from the desired velocity, which is a constant velocity that is applied to the agents at certain time interval, as discussed later.

Main contributions

This paper presents a novel distributed velocity control for cooperative multi-agent moving source seeking. The main findings of this research are summarized as follows. Gradient consensus and distributed velocity control are first provided for the cooperative multi-agent moving source seeking. We continue our previous results in [21] which utilizes the distributed velocity control to handle the problem of static source seeking. Here, we focus on addressing the moving source seeking problem, which has not been discussed in [21]. The constant desired velocity for approaching a moving source is proposed as an alternative to the gradient velocity approach that requires the agents to move at a higher peak velocity; this constant desired velocity can be thought as a cruising phase of the moving agents under certain formation. It is shown that cooperative multi-agent moving source seeking is achieved with both the gradient velocity and desired velocity, with different seeking time. The cooperative multi-agent moving source-seeking system involves a moving scalar field with linear, sinusoidal, expansion, and contraction movements, with its scalar field being a single maxima scalar function.

Outline

This paper discusses how to calculate distances and gradients between two points in the context of a multi-agent system. In Background Section, some basic information, such as the coordinate points of the agents, is provided. This information is then utilized in Gradient Estimation and Gradient Consensus Section to calculate the distances and gradients between the points. In addition, the gradients can be used to make decisions regarding the points. Furthermore, an analysis of distributed velocity control and simulation results is given in Distributed Velocity Control for Moving Source Seeking Section and also discussed in Simulation Results Section, respectively. Finally, conclusions from the simulation results are given.

Background

Notation

In this paper, the interaction of agents is modeled using an undirected graph denoted by $G = (V, E)$, where $V = \{1,2,\dots,N\}$ is the set of vertices that each represent the agents, and $E \subseteq V \times V$ is the set of edges that each represent the communication link between two adjacent agents. Edge $(i,j) \in E$ indicates that agent i sends information to agent j. The set of neighbours of agent i is denoted as $N_i = \{i \in V: a_{ij} \neq 0\}$.

The adjacency matrix is denoted by $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ with $a_{ij} = 1$ if agent i and agent $j \in V$ communicate with each other and $a_{ij} = 0$ otherwise. Note that A is symmetric.

Laplacian matrix is denoted by $L = \Delta - A$, where $\Delta = diag(A \cdot \mathbf{1})$ is a diagonal matrix with the degree of agents as the diagonal elements, i.e., $\Delta_{ii} = d_i = \Sigma_j a_{ij}$, and $\mathbf{1} = [1\ 1\ 1\ \dots\ 1]^\top \in \mathbb{R}^N$ is the vector with 1 as its element. Right eigenvector L is the eigenvector from the eigenvalue $\lambda_1 = 0$ and $L \cdot \mathbf{1} = 0$. The second smallest eigenvalue λ_2 determines the convergence rate of the algorithm. The notation $|N_i|$ represents the number of neighbours in the neighbour set N_i, and $\otimes$ represents the Kronecker product.

Problem description

Given a scalar field $\psi = \psi(r)$ which is a mapping of $\psi\colon \mathbb{R}^P \to \mathbb{R}^+$, where $p = 1, 2,$ or 3 and $r \in \mathbb{R}^P$ that defines the position coordinate of the agent in space. The agents' sensing capability is illustrated by $p = 2, r = [x\ y]^\top \in \mathbb{R}^2$, as mentioned later. The source is found indicated by the maximum value of ψ. Source seeking problem is to find the value of r such that the scalar field ψ is maximum. This can be formulated by the following optimization equation

$$r^* = arg\,max\,\psi(r).$$

The moving source scalar function is required for simulation and cooperative analysis of multi-agent mobile source seeking. As shown in Fig. 1, the static source scalar function is used first at the initial time, followed by the moving source scalar function. The static source scalar function is formulated as follows:

$$\psi_s(x,y) = A_1 e^{-\frac{(x-x_1)^2}{\sigma_{x_1}^2}-\frac{(y-y_1)^2}{\sigma_{y_1}^2}} + A_2 e^{-\frac{(x-x_2)^2}{\sigma_{x_2}^2}-\frac{(y-y_2)^2}{\sigma_{y_2}^2}}. \quad (1)$$

Then, the peak of the scalar function is at $(x,y) = (x_1,y_1)$ and (x_2,y_2) and can be moved linearly or sinusoidally with a certain speed. It can also be done to scale the scalar function so that it becomes expansion or contraction from before. Because there are two peak points and constant scalar velocities v_x, v_y occurs on the x-axis and y-axis, hence it can be defined as the linear movement of the point of the peak by (2) and (3),

$$x_x(t) = v_x t[1\ 1]^\top \quad (2)$$

on the x-axis, and

$$x_y(t) = v_y t[1\ 1]^\top \quad (3)$$

on the y-axis, where $x_x = [x_1\, x_2]^\top$ is the position on the x-axis, and $x_y = [y_1\, y_2]^\top$ is the position on the y-axis. The term $[1\ 1]^\top$ is used to denote the column vector with length 2 and both entries 1. While the formulation for the sinusoidal movement of the peak point of the scalar function made two kinds, namely sinusoidal movement on either the x-axis or the y-axis only, in this case, no combination of the two is used.

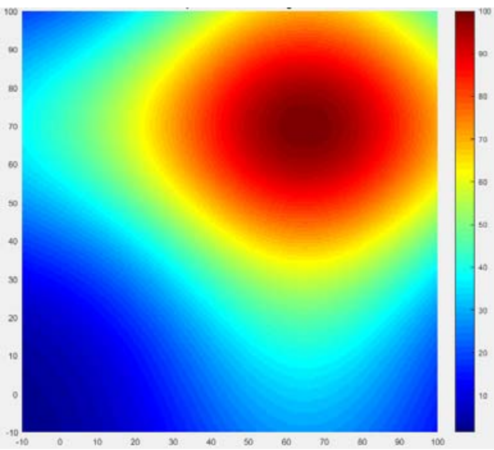

Fig. 1. Example of scalar field $\boldsymbol{\psi}$

Sinusoidal movement is represented as

$$x_x(t) = (v_x t + Asin(\omega t))[1\ 1]^\top \quad (4)$$

on the x-axis, and

$$x_y(t) = v_y t[1\ 1]^\top \quad (5)$$

on the y-axis, where A is the amplitude of the sinuses and ω is the angular velocity.

Using the scaling function formula, the movement to the scalar function becomes expansion or contraction compared to before, as

$$\boldsymbol{\sigma}(t+t_s) = \alpha(\boldsymbol{\sigma}(t)) \quad (6)$$

where $\boldsymbol{\sigma} = \left[\sigma_{x_1}\,\sigma_{y_1}\,\sigma_{x_2}\,\sigma_{y_2}\right]^\top$. The time-varying parameter of the movement of the scalar function α in (6) can be categorized as expansion or contraction if the parameter value $\alpha > 1$ or $\alpha < 1$, respectively. In this paper, both expansion and contraction cases are considered in the same experiment, i.e., the value of α may switch over time.

In this paper, the following assumptions are made:

- Assumption 1: the scalar field of source $\psi(r)$ has only a single maximum and no local maximum.
- Assumption 2: the moving scalar field of the source has a straight line, sinusoidal, expansion, and contraction movements.
- Assumption 3: the cooperative multi-agent has a communication topology represented by an undirected and connected graph.

Gradient estimaation and gradient consensus

This section discusses the gradient estimation of each agent and the gradient consensus of the formation of cooperative multi-agent in source seeking.

Gradient estimation

In the multi-agent-based gradient estimation, each agent approximates its own gradient, and the connected agents receive the estimations from their neighbours to compute the global gradient. The proposed method to compute the gradient estimation is based on the least squares [24], [25].

Each agent i measures the intensity of the scalar field ψ from its position. This is denoted by $\psi_i = \psi(r_i)$, where $i = 1,2,3,\dots N$. Given the position of agent j, r_j, that is close to agent i, r_i, the estimation can be calculated by using the first-order Taylor series. The estimation of ψ_j at r_i is given by:

$$\psi(r_j) \approx \psi(r_i) + (r_j - r_i)^\top\, \hat{\boldsymbol{g}}(r_i), \quad (7)$$

where $\hat{\boldsymbol{g}}(r_i)$ is the estimated gradient calculated by agent i. For $p = 2$, the estimated gradient becomes $\hat{\boldsymbol{g}}(r_i) = \left[\hat{\boldsymbol{g}}_{\boldsymbol{x}}(r_i)\hat{\boldsymbol{g}}_{\boldsymbol{y}}(r_i)\right]^\top$. If agent i has $|N_i|$ neighbours, then (7) becomes

$$\begin{bmatrix} \psi(r_1) - \psi(r_i) \\ \psi(r_2) - \psi(r_i) \\ \vdots \\ \psi\left(r_{|N_i|}\right) - \psi(r_i) \end{bmatrix} = \begin{bmatrix} (r_1 - r_i)^\top \\ (r_2 - r_i)^\top \\ \vdots \\ \left(r_{|N_i|} - r_i\right)^\top \end{bmatrix} \hat{\boldsymbol{g}}(r_i)$$

$$b_i = A_i \hat{g}_i \quad (8)$$

where $b_i \in \mathbb{R}^{|N_i| \times 1}$, $A_i \in \mathbb{R}^{||N_i|| \times p}$, and $\hat{\boldsymbol{g}}_{\boldsymbol{i}} \in \mathbb{R}^{p\times 1}$. This problem can be solved by using the least-squares method

$$\hat{g}_i = \left(A_i^\top A_i\right)^{-1} A_i^\top b_i. \quad (9)$$

Gradient of the consensus filter

The consensus algorithm proposed by Olfati-Saber and Shamma in [26] is the average consensus filter for spatially distributed sensor networks. In this algorithm, each sensor receives inputs from its neighbours, estimates them, and then averages them. The average consensus was applied to gradient consensus as follows

$$\dot{g}_i = \beta \sum_{j\epsilon N_i} a_{ij} e_{g_{ij}}(t) + \beta(1+d_i)\big(\hat{g}_i(t) - g_i(t)\big), \tag{10}$$

where

$$e_{g_{ij}}(t) = \big(\hat{g}_i(t) - g_i(t)\big) - \left(\hat{g}_j(t) - g_j(t)\right)$$

and $\beta \geq 1$ is a control parameter for tracking the performance of the gradient as the agent moves. By using the definition of the Laplacian graph, the following equation is obtained:

$$\begin{aligned} \dot{g} &= -\beta\big(I_N \otimes I_p + \Delta \otimes I_p + L \otimes I_p\big) g \\ &\quad + \beta\big(I_N \otimes I_p + \Delta \otimes I_p + L \otimes I_p\big)\hat{g}, \end{aligned} \tag{11}$$

where $I_N \in \mathbb{R}^{N \times N}$ and $I_p \in \mathbb{R}^{p \times p}$ are identity matrices. Thus (11) can be rewritten as:

$$\dot{g} = \beta(-Bg + B\hat{g}), \tag{12}$$

where $B = I_N \otimes I_p + \Delta \otimes I_p + L \otimes I_p$, $B \in \mathbb{R}^{Np \times Np}$.

Distributed velocity control for moving source seeking

This section describes the dynamic equation of an individual agent i. Specifically, a two-dimensional single integrator model, which is described as

$$\dot{x}_i = I_2 u_i, \tag{13}$$

where u_i is a control signal specified later, and $\dot{x}_i, u_i \in \mathbb{R}^2$. A two-dimensional model is considered because the agents can detect the scalar field of the source under this model. The state equation of all agents is written as

$$\dot{x} = (I_N \otimes I_2) u, \tag{14}$$

with $\dot{x}, u \in \mathbb{R}^{14}$ since there are 7 agents moving in three dimensions.

This paper modified a distributed control for cooperative multi-agent proposed in [25]. This work divides the distributed control into two parts, called formation control and velocity tracking. The formation control part can be formulated as $k_F\left(\sum_{j\in\mathcal{N}_i} a_{ij}\left(r_{F_i} - r_{F_j}\right) - \left(r_i - r_j\right) - \theta\left(v_i - v_j\right)\right)$, whereas the velocity tracking is $k_V(\dot{g}_i - \gamma v_i)$.

In the setting, the velocity can be taken from either the gradient of the cooperative multi-agents or from a manually-controlled desired velocity. By applying the superposition law, the basic rules that are distributed across all agents are obtained as follows

$$\begin{aligned} u_i &= k_F\left(\textstyle\sum_{j\in\mathcal{N}_i} a_{ij}\left(r_{F_i} - r_{F_j}\right) - \left(r_i - r_j\right) - \right. \\ &\quad \left. \theta\left(v_i - v_j\right)\right) + k_V\big(\dot{g}_i - \gamma v_i\big), \end{aligned} \tag{15}$$

where the coefficients k_F, θ, k_V, γ are positive. The coefficients k_F and k_V denote the weight of movement based on formation control and velocity tracking, respectively.

In equation (15), the term $k_V(\dot{g}_i - \gamma v_i)$ is calculated in two different ways: first by using gradient consensus in $\dot{g}_i$ called *gradient velocity* and second by combining gradient velocity with a constant velocity called *desired velocity*. The gradient consensus approach has been used in our previous result [24] for addressing the static source case. On the other hand, in the desired velocity case, $\dot{g}_i$ is replaced with a constant v_d for some specific time interval. As we can see later, the use of desired velocity may reduce the higher peak velocity of agents' movements without sacrificing their performance. One can think of this desired velocity as a cruising phase of a vehicle with cruise control.

We next analyze the interaction between the velocity and moving source. One problem that may arise for pursuing a moving source is that the agents may not be able to move close enough toward the source. To that end, we assume that the source does not move with too fast of a speed, i.e., we assume that there exist times where the velocity satisfy $(\dot{g}_i - \gamma v_i) > v_x$ and $(\dot{g}_i - \gamma v_i) > v_y$.

Furthermore, we notice the agents need to respond quickly due to the changing scalar field over time. Because of this, later we see that using only gradient for determining agents' velocity may not always be optimal.

Simulation resultsS

The parameter values used in the simulation are shown in Table 1. The parameter of the movement of the scalar function in (6) consists of expansion $(\alpha > 1)$ with $\alpha = 1.002$ and contraction $(\alpha < 1)$ with $\alpha = 0.998$. In this paper, the same peak point of the source at $(x_1, y_1) = (x_2, y_2)$ is used in order to produce a single optima scalar field. It is worth noting that considering different values of x and y is also possible in the case of multiple optima.

The gradient consensus uses $\beta = 1$. For the distributed velocity control of agents, $k_F = 0.5$ and $\theta = 1$ are considered for formation control, while $k_V = 0.5$ for velocity control are designed.

The agents are assumed to have a formation, as shown in Fig. 2, to characterize better the time at which the agents reach the source; all seven agents are said to reach the source if agent #1 positioned in the center reaches the source.

Simulation of the source seeking

The simulation results depicted from Fig. 3 to Fig. 8 show the position of the cooperative multi-agents. The parameters used in linear moving source in (2) and (3) are $v_x = 0.02 m/s$, $v_y = -0.005 m/s$. Fig. 3 shows the case of both gradient and desired velocity; the gradient velocity case (abbreviated as LG for Linear source seeking with Gradient velocity) uses the value of $\dot{g}_i$ as in (9) which later affects the control signal u_i defined in (14). On the other hand, in the desired velocity case (abbreviated as LD for Linear source seeking with Desired velocity), $\dot{g}_1$ is replaced with v_d for some specific time interval.

In this simulation, the source is assumed to stop moving at $t = 50\,s$. Thus, on the desired velocity case, v_d is used on agent #1 from the time where initial formation has been established to $t = 50\,s$, with other agents following agent #1's movement based on linear moving the formation control. After that, agent #1 use $\dot{g}_1$ again until it reaches the source.

The velocity of the agents under a linear moving source is shown in Fig. 3 with $v_d = \{1.6, 1.85, 2\} m/s$ for the desired velocity case. From Fig. 3 and Fig. 4, it is observed that agents under gradient velocity are able to reach the source that moves linearly, with agent #1 exactly hitting the source at $t = 83.5\,s$. The figure shows that the agents successfully keep their formation for most of the source-seeking process. Similarly, Fig. 3 to Fig. 7 illustrates that agents under desired velocity $v_d = 2\,m/s$ can also reach the linearly moving source; in this case under desired velocity, the agents reach the source faster at $\mathrm{t} = 72.7\,s$.

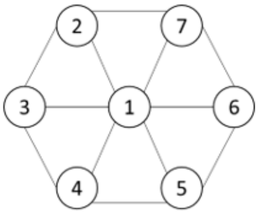

Fig. 2. The formation used in the simulation

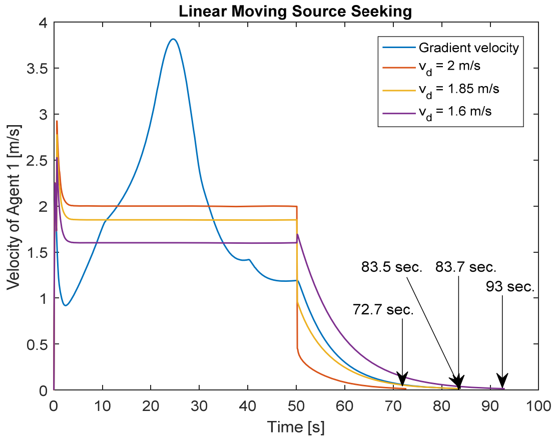

Fig. 3. Gradient and desired velocity used in linear moving source seeking with various v_d

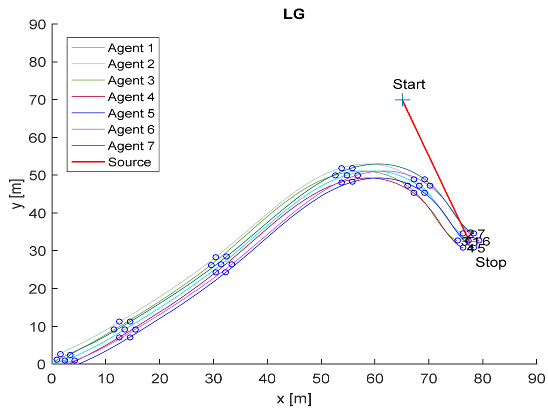

Fig. 4. Agents and source movements under LG

See animated for Fig.4-7, and Fig. 10-13:
http://bit.ly/LinearAndSinusoidalMovingSourceSeeking

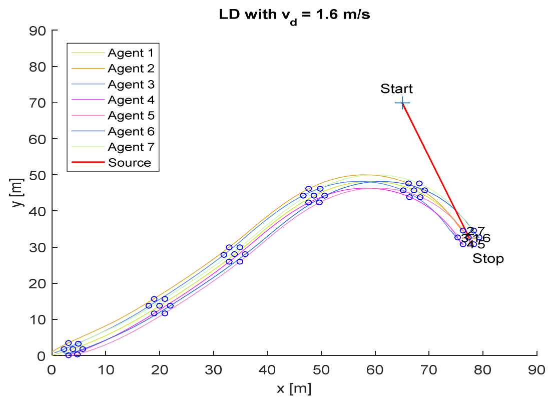

Fig. 5. Agents and source movements under LD with $v_d = 1.6\,m/s$

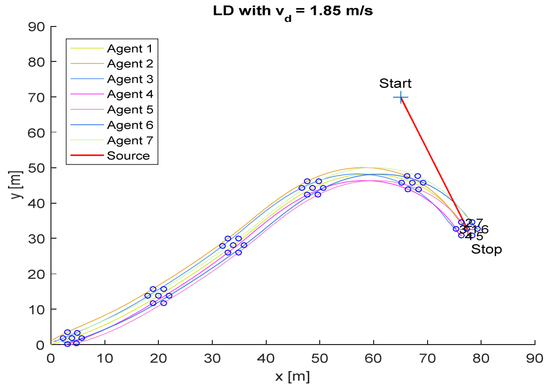

Fig. 6. Agents and source movements under LD with $v_d = 1.85\,m/s$

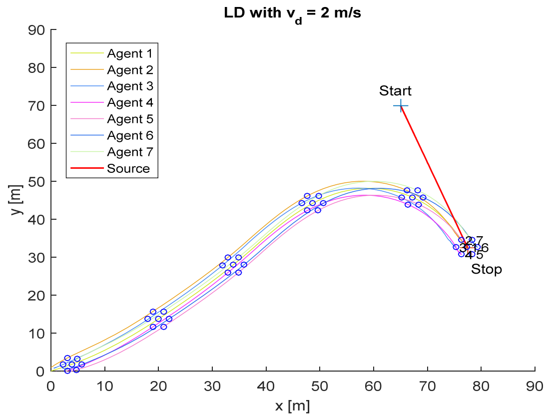

Fig. 7. Agents and source movements under LD with $v_d = 2\,m/s$

Table 1. Static parameters used for the moving source

Parameter	Values
$A_1 = A_2$	$50\,m$
$x_1 = x_2$	$65\,m$
$y_1 = y_2$	$70\,m$
σ_{x_1}	30
σ_{y_1}	75
σ_{x_2}	90
σ_{y_2}	25

It is illustrated in Fig. 8 that the distances between agent #1 and the source over time. The plot shows that under LD, the distance between agent #1 and source changes almost linearly since the source also moves in a linear fashion. On the other hand, under LG's case the agent initially moves slowly but later keeps up with the pace, depending how close it is to the source.

Table 2 contains the summary of the seeking time for linear moving source with both gradient velocity (LG) and manually-determined desired velocity (LD) with several values of v_d. It can be seen here that the higher v_d is, the faster the agents reach the linearly moving source. Under this simulation setting, the cases LG and LD have similar seeking time if $v_d \approx 1.85\,m/s$.

Simulation of the sinusoidal moving source seeking

Similar to the linear moving source above, the cases of gradient velocity (SG for Sinusoidal source seeking with Gradient velocity) and desired velocity (SD for Sinusoidal source seeking with Desired velocity) are considered in sinusoidal moving source. It is also considered that the source stops moving after 50 s. The parameters used here in sinusoidal moving source in (4) and (5) are $v_x = -0.03\,m/s$, $v_y = 0.001\,m/s$, $A = 0.2\,m$, $\omega = 2\,rad/s$.

It is observed from Fig. 9 to that agents under both gradient velocity and fixed desired velocity are also able to reach the source that moves in a sinusoidal way, with agent #1 exactly hitting the source at $t = 95.9\,s$ under gradient velocity and at $t = 84.4\,s$ under desired velocity $v_d = 2.2\,m/s$. In addition, it can be seen from Fig. 10 and Fig. 11 that for sinusoidal cases, the agents maintain their formation for most of the source-seeking process.

It is showed in Fig. 10 that with sinusoidal movement of the source, there are intervals where agent #1 moves slightly further from the source; however, it eventually reaches the source. Similar to the linearly-moving source discussed before, the agent under gradient velocity initially moves slower but later keeps up with the pace.

Comparison between several values of v_d for sinusoidal case can be seen in Table 3which shows that the seeking time also becomes shorter as v_d increases; this is almost similar for SG and SD cases for $v_d = 1.9\,m/s$.

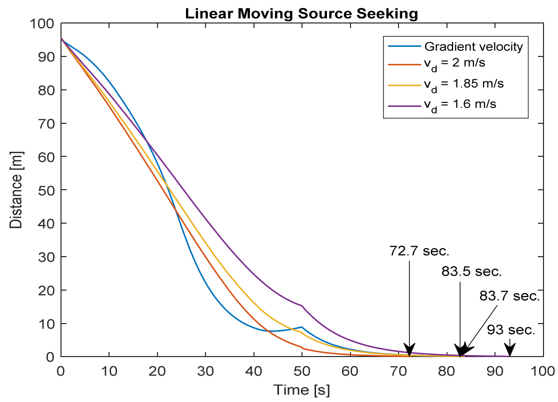

Fig. 8. Distance between agent #1 and source of linear moving source seeking with LG and LD

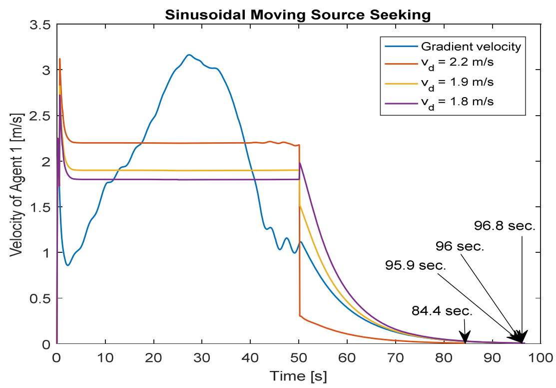

Fig. 9. Gradient and desired velocity used in sinusoidal moving source seeking with various v_d

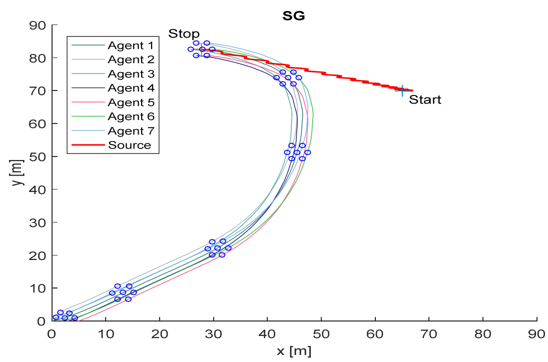

Fig. 10. Agents and source movements under SG

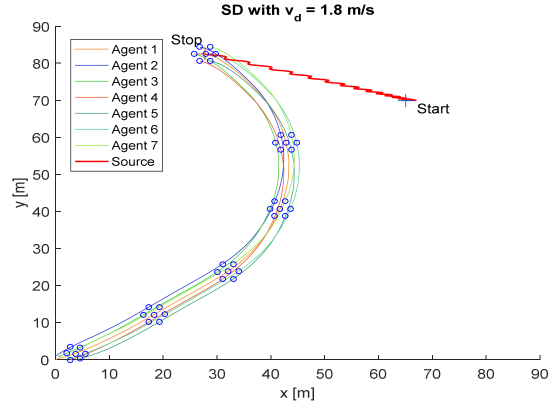

Fig. 11. Agents and source movements under SD with $v_d = 1.8m/s$

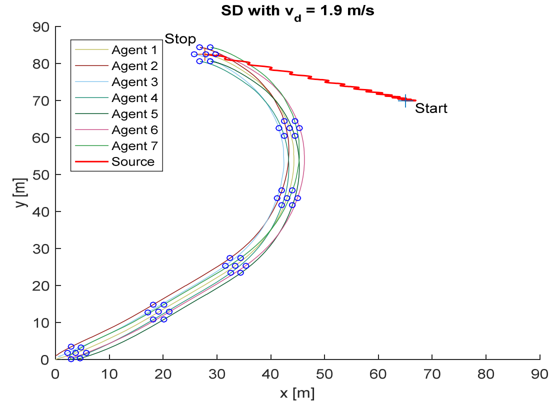

Fig. 12. Agents and source movements under SD with $v_d = 1.9m/s$

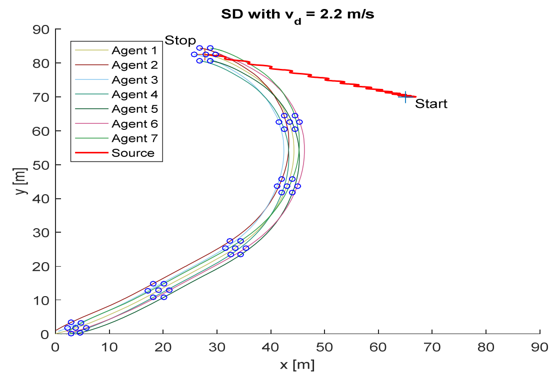

Fig. 13. Agents and source movements under SD with $v_d = 2.2m/s$

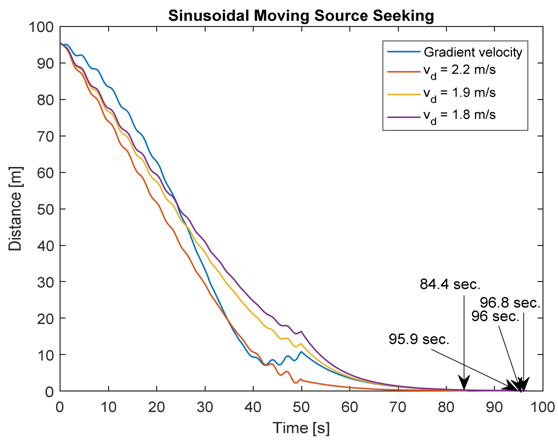

Fig. 14. Distance between agent #1 and source of sinusoidal moving source seeking with SG and SD

Table 2. Comparison of gradient velocity and desired velocity under linear moving source

No	Experiment	v_d [m/s]	Seeking Time [s]
1	LG in Fig. 4	-	83.5
2	LD in Fig. 5	1.6	93
3	LD in Fig. 6	1.85	83.7
4	LD in Fig. 7	2	72.7

Table 3. Comparison of gradient velocity and desired velocity under sinusoidal moving source

No	Experiment	v_d [m/s]	Seeking Time [s]
1	SG in Fig. 10	-	95.9
2	SD in Fig. 11	1.8	96.8
3	SD in Fig. 12	1.9	96
4	SD in Fig. 13	2.2	84.4

Conclusion

This paper considers a distributed velocity control of a multi-agent system which has to trace a moving source while maintaining a certain formation over time using formation control. Two types of agents' velocities are considered for tracing the moving source: A gradient-based approach to derive the velocity called gradient velocity, and a proposed combination of gradient-based and a constant velocity called desired velocity. Under both velocities, it is shown that the agents can reach the source, which moves either linearly or sinusoidally. It is also shown by numerical examples that in some cases the desired velocity may perform better, i.e., giving a shorter seeking time for tracing the source.

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