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"Fractionalization": A new approach for comparing different approximation methods of fractional order systems and disturbances Rejection in PID Control

Abstract. *Recently, many research works have focused on fractional order systems and their approximation methods. It has been shown to be a useful tool for enhancing plant dynamics in terms of time and frequency performance. In this paper we propose a new approach for comparing between the different approximations methods of fractional order systems and disturbance rejection in PID control of DC motor by fractionalizing an integer order derivative operator in the original integer system. The implementation of the fractionalized terms is realized by mean of the well established approximation methods and in order to determine the best method, the responses of original integer system are compared to those of fractionalized systems. Illustrative simulations examples show that the fractionalization approach give the best decision (selected method) ,a good tool for comparison between different approximation methods and it give the good rejection of disturbances in PID control of DC motor . This approach can also be generalized to others numerical approximation methods and it can also be used in the area of systems control.*

Streszczenie. *Ostatnio wiele prac badawczych koncentrowało się na systemach rzędu ułamkowego i metodach ich aproksymacji. Wykazano, że jest to przydatne narzędzie do zwiększania dynamiki instalacji pod względem wydajności czasowej i częstotliwościowej. W tym artykule proponujemy nowe podejście do porównywania różnych metod aproksymacji systemów ułamkowego rzędu i odrzucania zakłóceń w sterowaniu PID silnika prądu stałego poprzez frakcjonowanie operatora pochodnej rzędu całkowitego w oryginalnym układzie całkowitym. Implementacja wyrazów ułamkowych jest realizowana za pomocą dobrze znanych metod aproksymacyjnych iw celu wyznaczenia najlepszej metody porównuje się odpowiedzi oryginalnego układu całkowitoliczbowego z odpowiedziami układów ułamkowych. Ilustracyjne przykłady symulacyjne pokazują, że podejście frakcyjne daje najlepszą decyzję (wybrana metoda), jest dobrym narzędziem do porównywania różnych metod aproksymacyjnych i zapewnia dobre odrzucanie zakłóceń w regulacji PID silnika prądu stałego. Podejście to można również uogólnić na inne metody aproksymacji numerycznej, a także można je stosować w obszarze sterowania systemami.* ***(„Frakcjonalizacja": nowe podejście do porównywania różnych metod aproksymacji systemów ułamkowego rzędu i zakłóceń Odrzucanie w regulacji PID***)
ami.

Keywords: Fractional order systems, Approximation methods, performances Analysis, DC Motor, disturbance rejection.
Słowa kluczowe: Systemy ułamkowe,Metody aproksymacyjne, Analiza występów, silnik prądu stałego,odrzucanie zakłóceń

Introduction

Although fractional calculus has a 300-year history, its current widespread use is largely due to its applications in science and engineering [1,2]. Specifically, control theory and fractional order systems, with a rapidly expanding volume of theoretical and experimental research production [3].

Fractional differentiators and integrators are important in fractional feedback and filter-based signal processing management of chaotic and complex systems [4,5]. An infinite-dimensional filter with an irrational continuous time transfer function is a fractional order system in the s-domain [6]. The applications of fractional order differentiation have attracted the attention of researchers from vide variety of science disciplines especially from the fields of applied sciences [7].

In order to apply the fractional order systems in different domains we must use the approximation methods such as the Oustaloup, Charef, and Matsuda approximations [8].

This paper's goal is to propose a novel strategy to comparing between various approximation methods of fractional order systems by introducing fractional order filters to the original system's classical design without modifying the equivalent transfer function generally.. If we consider a transfer function of integrator element G(s)=(1/s) of the initial integer system , The purpose is to "fractionalize" it as follows:

(1) $$\mathrm{G}(s) = G_{\alpha}(s).G_{1-\alpha}(s) = \left(\frac{1}{\mathrm{s}^{\alpha}}\right).\left(\frac{1}{\mathrm{s}^{(1-\alpha)}}\right)$$

where α : real number ($0 < \alpha < 1$).

Our objective is not to change the original global system but to use it as a new tool of comparison between various approximation methods of fractional order systems, because fractional order systems have interesting properties. Even though this concept appears to be fairly straightforward, we expect that it will provide a new tool for design engineers working with plants in actual industrial settings.

The remaining sections are arranged as follows: Basic definitions of the fractional order system are provided in Section 2, the various Approximation methods of a fractional order Transfer function is given in Section 3. The proposed fractionalized systems (integrator and derivative operators, Fractionalized first order system and fractionalized second order system) are introduced in section 4. In section 5, simulation results is given for comparison between three approximations methods (Oustaloup, Matsuda and Charef) using the original integer systems and fractionalized systems. and lastly, the conclusion along with future perspectives of the study is given in section 6.

Fractional Order Systems

It is worth mentioning that the real-world applications of fractional calculus grow rapidly owing to the fact that these mathematical aspects would help to express a system more precisely when compared with conventional classical methods. Factually, the real objects are found generally fractional, although fractionally is observed to be very low for most of them [9,10]. The lack of solutions for fractional differential equations is one of the main reasons to study integer-order models. To date, many studies are available regarding the use of fractional calculus in many practical applications e.g. in controllers, capacitors, control theory and circuit analysis [11,12].

The Riemann-Liouville definition can be expressed mathematically as:

(2) $$f(t) = \frac{d^n}{dt^n}\int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau$$

It is worth-mentioning that as far as the problems of real-world applications are concerned, specifically applications from the field of physical and engineering sciences, the Grünwald-Letnikov and Riemann-Liouville concepts are taken as equivalent [13,14,15].

Approximation methods of a fractional order Transfer function

Literature suggested that there are several approximations are available for fractional order transfer functions. All the available approximations demonstrate some advantages over the others in context of few characteristics. It is bit difficult to say which approximation would deliver the best results. There are several characteristics that would decide the relative merits of any approximation such as differentiation order, frequency behavior or time responses etc. Several approximations are discussed here with respect to their comparative analysis with others [20]. The available approximations belongs to two different domains i.e. frequency and time domains which are specified as s-domain and z-domain, respectively. The approximations in frequency and time domains are also termed as continuous and discrete approximations [21].

Oustaloup approximation method

The Oustaloup method is based on the function approximation from as;

(3) $$G_f(s) = S^{\alpha} \qquad ,\alpha \in R^{+}$$

By taking into account the rational function [15,16,22]:

(4) $$G_f(s) = K\prod_{k=1}^{N}\frac{s+w_k^{'}}{s+w_k}$$

However, the poles, zeros, and gain can beevaluated as;

$$w_k^{'} = w_b.w_u^{(2k-1-\gamma)/N} \quad , \ w_k = w_b.w_u^{(2k-1+\gamma)/N} \ , K = w_h^{\gamma}$$

Where w_u represents the unity gain in frequency and the central frequency in a geometrically distributed frequency band. Let $w_u = \sqrt{w_h w_b}$, where w_h and w_b represents the upper and lower frequencies, respectively. γ and N are the orders of derivative and filter, respectively.

Matsuda's approximation method

Matsuda's method is based on the continuing fraction technique (CFE) [17], which allows for the approximation of an irrational function by a rational one. Assuming that the selected points are sk , k = 0,1,2, … N , the approximation takes on the form:

(5) $$\hat{G}(s) = a_0 + \frac{s-\omega_0}{a_1+\frac{s-\omega_1}{a_2+\frac{s-\omega_2}{a_3+\cdots}}}$$

Where $a_i = f(\omega_i)$, $f_0(\omega) = G(s)$, $f_{i+1}(s) = \frac{s-\omega_i}{f_i(s)-a_i}$

Charef 's transfer approximation method

The Charef 's transfer method given in [18,19] is based on approximation of the function as;

(6) $$G_f(s) = s^{\alpha} \qquad ,\alpha \in R^{+}$$

These processes in the frequency domain can be described by an approximation in the Laplace domain such as:

(7) $$G_f(s) = s^{\alpha} \approx \left(1+\frac{s}{P_T}\right) \approx \frac{\prod_{i=0}^{N}\left(1+\frac{s}{P_i}\right)}{\prod_{i=0}^{N-1}\left(1+\frac{s}{Z_i}\right)}$$

Where $P_i = (ab)^i.P_0 \quad , \ i = 1,2,3,\dots,N$

$Z_i = (ab)^i.a.P_0 \quad , \quad i = 2,3,\dots,N-1$

and $P_0 = P_T.10^{\frac{ep}{20\beta}}, \alpha = 10^{\frac{ep}{10(1-\beta)}}, b = 10^{\frac{ep}{10\beta}}, \beta = \frac{\log(a)}{\log(a.b)}$

Fractionalized Systems

Fractionalized First Order system

The First Order system is converted to a First Fractionalized Order system as follow:

(8) $$H(s) = \frac{K}{1+\tau s} = \frac{1}{1+\tau\ s^{\alpha}s^{1-\alpha}} \qquad \text{with}: \ 0 < \alpha < 1.$$

Figure 1 shows the fractionalized First Order system:

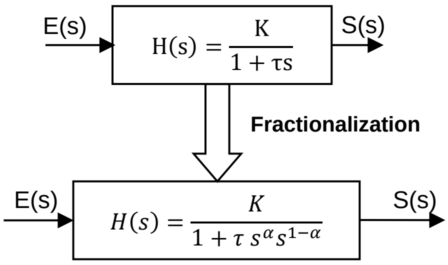


Fig.. 1 Proposed Fractionalized First Order system

Fractionalized Second Order system

Figure 2 shows the fractionalized second Order system

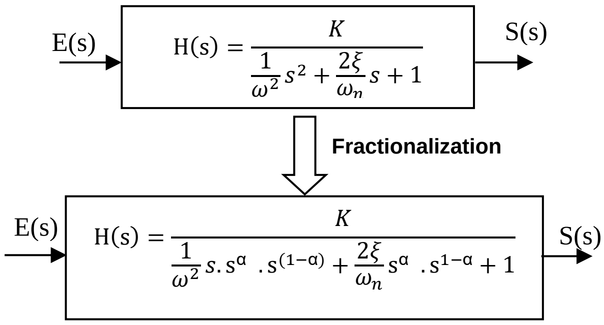


Fig. 2 Proposed Fractionalized Second Order system

The second Order system is converted to a Fractionalized second Order system as follow:

(9) $$H(s) = \frac{K}{\frac{1}{\omega^2}s^2+\frac{2\xi}{\omega_n}s+1} = \frac{K}{\frac{1}{\omega^2}s.s^{\alpha} .s^{(1-\alpha)}+\frac{2\xi}{\omega_n}s^{\alpha} .s^{1-\alpha}+1}$$

Results and Discussion

The Proposed Approximatior design using "Fractionalization" approach is given by the following figure:

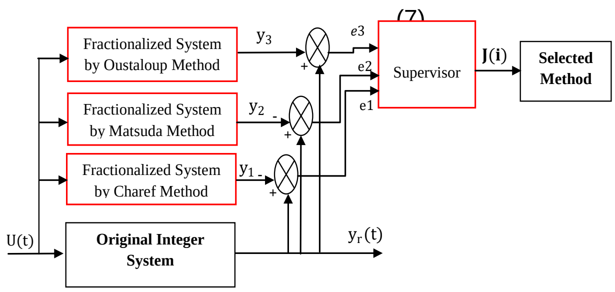


Fig..3 Proposed Approximation strategy design using " Fractionalization" approach

The supervisor architecture, given in [23], is illustrated by figure 4:

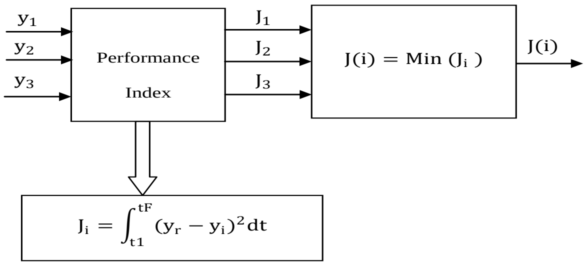


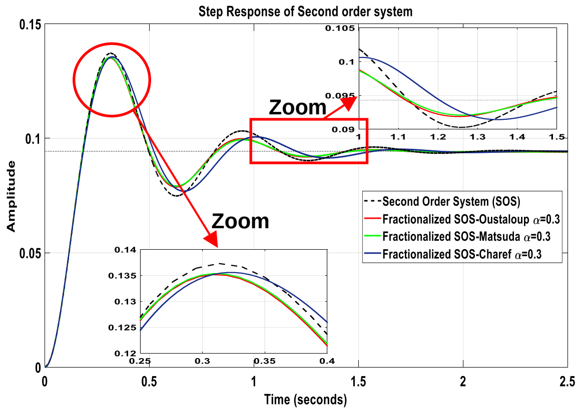
Fig. 4 Supervisor Architecture

System: Fractionalized Second Order System (FOS)

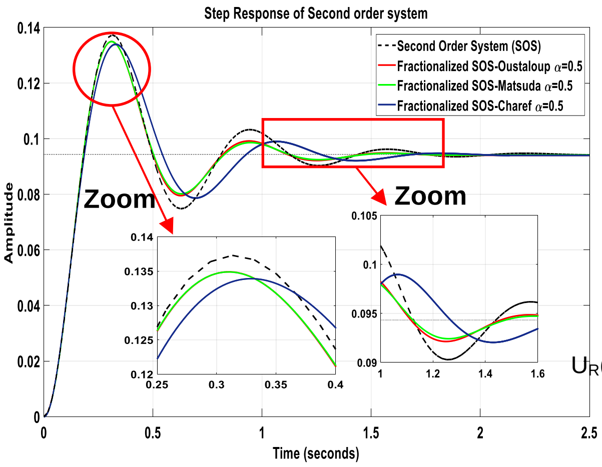
The Original Integer Second order System is:

(10) $H(s)=\frac{10}{s^2+5\,s+106}$

Figure 5 and figure 6 show the steps responses comparison between the original second order system and the approximated fractionalized second order system with Oustaloup, Matsuda and Charef approximations methods for α=0.3 and α=0.5 respectively



Fig/ 5 Step response comparison of the original second order system with the fractionalized second order system approximated by Oustaloup, Matsuda, and Charef approximation methods ($\alpha=0.3$).



Fig/ 6 Step response comparison of the original second order system with the fractionalized second order system approximated by Oustaloup, Matsuda, and Charef approximation methods($\alpha=0.5$).

The performance Index used by the supervisor defined to evaluate the performance of various approximations methods, it is given by the following equation:

(11) $J_i=\int_{t1}^{tF}(y_r-y_i)^2dt$

where $i=1,2,3$ for three errors approximating methods (Oustaloup, Matsuda and charef respectively):

$e_1=y_r-y_1, e_2=y_r-y_3$ and $e_3=y_r-y_3$

The criterion for quadratic error when we use the steps responses of first order system are given in Table1:

Table 1. Criterion of quadratic error for steps responses first order system

(α,1- α)	(0.1,0.9)	(0.2,0.8)	(0.3,0.7)	(0.4,0.6)	(0.5,0.5)
J_1	0.1300	0.25	0.7044	0.0031	0.0049
J_2	**0.0041**	**0.0051**	**0.0040**	**0.0022**	**0.00031**
J_3	0.0109	0.043	0.062	0.011	0.0016
Min (J_i)	Matsuda	Matsuda	Matsuda	Matsuda	Matsuda

In table 1, we remark that when we use the first order system,

- The frequency response fitting by Oustaloup, Matsuda and the Charef filter is likewise good, as can be observed.
- Good results of all approximation methods because Ji is minimal $(i=1,2,3)$,
- The selected method is Matsuda approximation for $\alpha=0.1, 0.2, 0.3, 0.4$ and $\alpha=0.5$.

Table 2. Criterion of quadratic error for steps responses Second order system

(α,1- α)	(0.1,0.9)	(0.2,0.8)	(0.3,0.7)	(0.4,0.6)	(0.5,0.5)
J_1	0.0087	0.007	**0.00549**	0.003	0.002
J_2	0.0061	0.005	0.00693	**0.002**	**0.001**
J_3	**0.0035**	**0.002**	0.0092	0.011	0.1027
Min (J_i)	Charef	Charef	Oustaloup	Matsuda	Matsuda

we remark in the table 2 that when we use the second order system:

- The frequency response fitting by Oustaloup, Matsuda and the Charef filter are likewise good, as can be observed.
- Charef approximation method is the best when α= 0.1 and α= 0.2.
- For α= 0.5 the selected method is Oustlaoup approximation.
- Matsuda method is the best when α= 0.4 and α= 0.5.

The Fractionalized Order PID Controller

The feedback control loop of a fractionalized of the integer order system is shown in Figure 7.

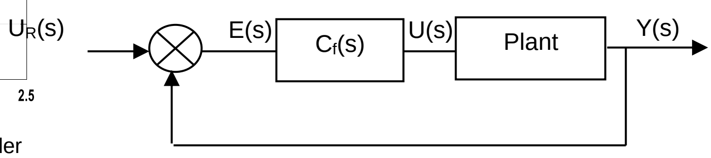


Fig.7 Fractionalized order PID Controller

Where, $C_f(s)$: Fractionalized Controller Transfer Function

The Fractionalized of the integer-order PID controller to be designed is in the following form:

(12) $C_f(s)=K_p\left(1+\frac{1}{T_i s}+T_d s\right)$

$$C_f(s)=\frac{1}{s}\left(\frac{k_pT_ds^2+k_pT_is+k_p}{T_i}\right)$$

$$C_f(s) = \frac{1}{s^{\alpha}} \frac{1}{s^{(1-\alpha)}} \left(\frac{(k_p T_d s^2 + k_p T_i s + k_p}{T_i} \right)$$

Were, $0 < \alpha < 1$

Disturbance Rejection in PID Control

This example shows the comparison of three DC motor control techniques (PID,Fractional PID and Fractionalized PID) for tracking setpoint commands and reducing sensitivity to load disturbances

The identified model for DC Motor voltage-speed is given by [5]:

(13) $G_{DC-motor}(s) = \frac{K_{DC-motor}}{\tau s+1} = \frac{0.25}{1.45s+1}$

Figure 8 , figure 9 and figure 10 shows the speed of DC Motor using the Integer adaptive PID Controller for α=0.1, α=0.3 and α=0.5 respectively with the following optimized parameters values:

$k_p = 966.6787, k_i = 426.8123, k_d = 48.3883$:

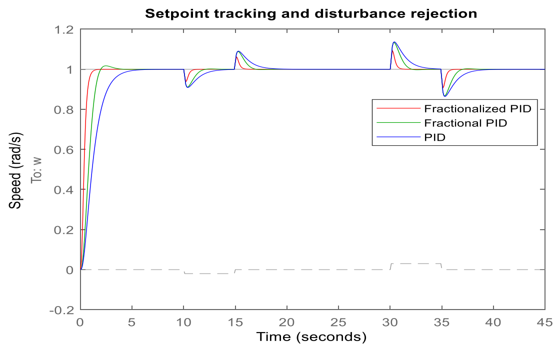


Fig 8 Speed of the DC Motor using the PID, Fractional PID (α=0.1) and Fractionalized PID Controllers with disturbance rejection.

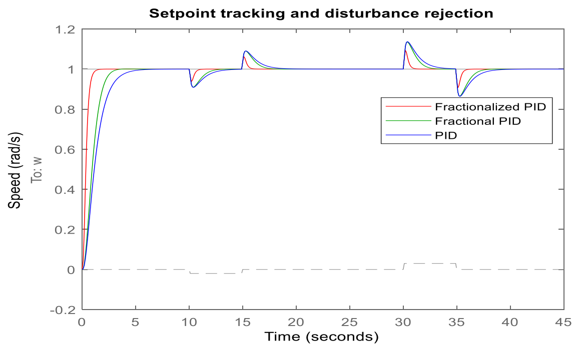


Fig 9 Speed of the DC Motor using the PID, Fractional PID (α=0.3) and Fractionalized PID Controllers with disturbance rejection.

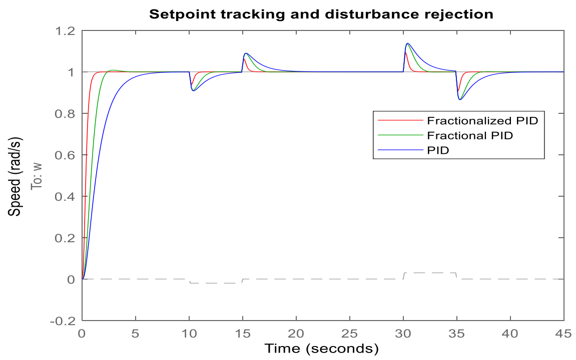


Fig 10 Speed of the DC Motor using the PID, Fractional PID (α=0.5) and Fractionalized PID Controllers with disturbance rejection.

The performance analysis of the proposed fractional order adaptive PID controller and the classical adaptive PID controller is given by the following table:

Table 3. Transient Response Stability Parameters of Aircraft System

Controllers	Overshoot [%]	Setting time [s]	Rise time [s]	Mean Absolute Error (Rad)
PID	1.7242	0.0595	0.0373	0.0012
FractionalPID	0.0533	0.0193	0.0106	0.0005
Fractionalized PID	**0.041**	**0.015**	**0.009**	**0.0001**

We remark that the fractional adaptive PID Controller for Aircraft system give the good improvement of overshoot, setting time , rise time and mean absolute error comparatively to the Integer adaptive PID Controller results.

Conclusion

In this paper, a new approach to comparing various approximation methods of fractional order systems and disturbance rejection by including fractional order filters in the original system's design, with no modification to the overall equivalent transfer function, is proposed. Based on the simple idea of introducing fractional order operators in the original integer system, the Fractionalization approach give the best decision (selected method) and presents a good tool for comparison between the various approximation methods of fractional systems (Oustaloup, Matsuda, and Charles approximations) and it give the good rejection of disturbances in PID control of DC motor. This has been illustrated by a set of simulation examples.

In future work, we will investigate the generalization of this fractionalization approach to more general integer order functions in order to improve their noise rejection and robustness in various control system approaches.

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