A Methodology for Tuning and Optimization of a Quadruple Tank Control System Using Firefly Algorithm in Wavelet Space

Abstract. Using more efficient tuning techniques becomes imperative, due to the increasing competitiveness in the industry. With this propose, meta-heuristics, such as Firefly Algorithm (FA), can be used to obtain the parameters of the controller according to a cost function, which should encode how good a controller is, adequately expressing the desired specifications, so that the metaheuristic employed can find the desired controller that is able to reach the response wanted. The methods traditionally used for automatic tuning of controllers present difficulties in expressing the desired specifications, being able to mapping the desired search space and allowing that the algorithm finds the proper answer. These difficulties is more evident when more complex controllers are required, as for Multiple Input Multiple Output (MIMO) problems. Aiming to solve these difficulties, a methodology using wavelet transform to describe the behavior of a controller response and its use for obtain better performance of the optimization algorithm. A case study will be done using the quadruple tank system, showing the efficiency of the methodology proposed.

Streszczenie. Stosowanie bardziej wydajnych technik strojenia staje si ˛e koniecznością ze wzgl˛ędu na rosn˛ą ˛acy konkurencyjno´s´c w branży. Dzięki tej propozycji meta-heurystyk, takie jak Firefly Algorithm (FA), mogą być użyte do uzyskania parametrów kontrolera zgodnie z funkcją kosztu, która powinna kodować, jak dobry jest kontroler, adekwatnie wyrażając pożądané specyfikacje, tak aby zastosowana metaheurystyka może znaleźć żądané parametry, który jest w stanie osiągnąć żadaną odpowiedź. Metody tradycyjnie stosowane do automatycznego dostrajania sterowników stwarzają trud- ności w wyrażeniu pożądanych specyfikacji, możliwości odwzorowania pożądané przestrzeni wyszukiwania i umożliwienia algorytmowi znalezienia właściwéj odpowiedzi. Trudności te są bardziej widoczne, gdy wymagane są bardziej złożone kontroly, jak w przypadku problemów z wieloma wej´s- ciami i wieloma wyjściami (MIMO). Mając na celu rozwiązanie tych trudności, opracowano metodologię wykorzystującą transformację falową do opisu zachowania si˛e odpowiedzi sterownika i jej zastosowanie w celu uzyskania lepszej wydajno´s´ci algorytmu optymalizacji. Zostanie przeprowadzone studium przypadku z wykorzystaniem systemu poczwórnego zbiornika, pokazujące skuteczność proponowanej metodologii. (Metodologia dostraja- nia i optymalizacji poczwórnego systemu sterowania zbiornikiem za pomocą algorytmu Firefly w przestrzeni falowej)

Keywords: Optimization, Optimization Algorithm, Firefly Algorithm, Wavelet, MIMO, Process Control
Słowa kluczowe: optymalizacja, algorytm optymalizacji, algorytm Firefly

Introduction

The optimization of controllers become a relevant issue due to the increasing competitiveness of the industry and the search for more efficiency of production, fetching a less use of environmental resources and a greater profitability. In the scope of study, the optimization algorithm consists in a numerical techniques that minimize a numerical function, in other words, a heuristic technique that minimize a evaluate index in function of the controller parameters. Thus, we seek to find the point with the lowest evaluation index of a meta- surface that describes the evaluation index as a function of the controller parameters, in a space generated by the index and the controller parameters to be tuned.

Due to the practical impossibility of obtaining the evaluation index for all possible tunnings, it is necessary to use techniques that do not require prior knowledge of the derivative of the function to be minimized, so that the group of heuristics called metaheuristics stands out. These are characterized by seeking the minimum of a function avoiding stagnating local minima, without presenting the need for prior knowledge of the function, as well as the value of its derivative. With this objective, metaheuristics evaluate a set of possible solutions, that is, it only evaluates points of the optimized function iteratively, looking more intensely for the global minimum in the region of space near the most promising possible solutions.


In this context, one of the biggest challenges in control optimization is numerically, through a evaluation function, representing how a controller response fits the intended purpose. An inappropriate choice of an evaluation function may lead the optimization algorithm to converge to a result that is supposed to be optimal, but does not constitute what is really desired. Usually, valuation indices based on system response error are used, which is not always a good strategy.

This work seeks a new index to be minimized that presents a better expression of the real problem, taking into account the behavior of the response, information partially lost with the encoding of the system response as a function of error, and through this better representation, facilitate the pursuit of the minimum by differentiating between best and worst candidates for minimums and thus better representing the metasurface of the function one wants to find the minimum.

This alternative way to measure the performance of a controller for optimization algorithms is a necessity, since data such as overshoot, rise time and settling time are not formalized when the reference signal is not a step, and indexes based on error cannot encode information contained in these step-response characteristics. Thus, a possible solution for this issue using wavelet transform will be examined in this work.

The wavelet transform is a powerful mathematical tool, which has the ability to present information from a signal in frequency and time simultaneously. This tool is consolidated to analysis when the change of the signal frequency as a function of time is relevant and is widely used in the identification of patterns and behaviors due it possibility to describe signal in frequency and time simultaneously. In this context, [6] used wavelet for spectral analysis of vibration processes at hydropower units, [7] apply to detecting faults in power transformers, [8] employ wavelet analysis for damage detection of long-span bridge cable structure and [9] proposed a non-decimated wavelet based multi-band ear recognition using principal component analysis and [10] used wavelet transforms for partial discharge signal detection in generators.
In this work, analysis in a wavelet space will be applied to compare the behaviors of the controlled system with the intended one, using wavelet analysis proposed is use the frequency analysis and pattern recognition simultaneously, according to the methodology presented in [11]. The tuning and optimization of controller system for a four-coupled tank system will be used as a case of study.

The algorithm used for the optimization is the metaheuristic called Firefly Algorithm (FA), meta-heuristics are able to be applied in tuning and optimization of controller due they do not require gradient information, that is usually unknown. The FA was proposed in 1995 by Yang and Xin-She [12], this algorithm is particle swarm based algorithm, where the particles are fireflies flying considering the brightness of their bioluminescence, with which each one attracts the others.

In the second section, the classic methodology of the controller project using decouplers and proportional–integral–derivative (PID) controllers for the MIMO system like the quadruple tank will be presented. In the third, the optimization using FA. In fourth, the wavelet discrete transform and its proposed use in the optimization of controllers will be discussed. The fifth, sixth and seventh sections will show the case of study, the methodology used, results and conclusions respectively.

**Project of MIMO control system using decoupler and PID controllers**

In order to implement the intended automatic tuning and optimization of the controllers, understand the traditional approach for the project of control for MIMO systems is required. In this work will be used decouplers, that aim to minimize the effects that one variable exerts on the others, allowing to treat practically independent each set of input and output. This makes possible to develop a controller PID for each input and output pair.

The decoupling techniques can be ideal decoupler, simplified decoupler and inverted decoupler. The choice of one of these methods is a relatively complex task since each technique has its advantages and limitations [13]. The ideal decoupler give a perfect decoupling of system, but it needs matrix inversion to be projected. The simplified decoupler loses precision, but it does not need matrix inversion, however it usually leads to complex elements in the equivalent decoupled systems. In this work, the inverted decoupler proposed by Shinkey [14] will be used since it avoids the numerical issues of matrix inversion and complex number in automatic tuning.

The system decoupling is implemented considering a matrix $C(s)$ formed by the controllers for each input and output pair and with a decoupling block or compensator $D(s)$, which aims to make the controllers observe the equivalent process $Q(s)$, which consists of a set of independent processes. Thus, the transfer function matrix $D(s)$ aims to introduce dynamics that cancel the interactions between the process variables, allowing an independent control to be made for each of the system’s meshes [15]. The inverted decoupler present the arrangement of this elements in the decoupler structure shown in Figure 1. Thus $D(s)$ and $Q(s)$ can calculated as:

\[
D(s) = \begin{bmatrix} d_1(s) \\ d_2(s) \end{bmatrix} = \begin{bmatrix} C_{11}(s) \\ C_{21}(s) \\ C_{12}(s) \\ C_{22}(s) \end{bmatrix}
\]

(2)

\[
Q(s) = \begin{bmatrix} q_1(s) \\ q_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) \\ G_{12}(s) \\ G_{21}(s) \\ G_{22}(s) \end{bmatrix}
\]

(1)

![Fig. 1. Structure of an inverted decoupler for a system with two inputs and two outputs.](image)

**Tuning and optimization using FA**

The use of meta-heuristics for tuning and optimization as proposed is advantageous over the approach shown in the previous section, since this has some limitations such inaccuracy of system representation including those introduced by the linearization and the subsequent decoupling for project of the control system. This inaccuracy can result in an unexpected and inadequate result in some cases. Metaheuristic can be used to overcome these limitations.

In this context, the firefly algorithm is a metaheuristic based on the flight of fireflies considering the brightness of their bioluminescence. The information to be optimized is encoded in the form of positions in a multidimensional space, each firefly being contained not only in that space but also in motion in it. Each firefly is evaluated according to a predefined cost function that will be expressed by its attractiveness or intensity of its brightness. The algorithm assumes three premises: 1) Each firefly can be attracted by all other fireflies; 2) The attractiveness of each firefly is proportional to its brightness, being attracted to the one it perceives to be brighter than it, but the perception of the intensity of the brightness reduces with the increase of the distance between them; 3) If there are no brightest fireflies, the movement will be random.

The Firefly Algorithm operation can be understood observing the diagram expressed in Figure 2. The update for each pair of fireflies is given by Equation 3.

\[
x_i(n+1) = x_i(n) + \beta e^{-\gamma r_{ij}} (x_j(n) - x_i(n)) + \alpha \epsilon(n)
\]

(3)

In the iteration, $x_i$ and $x_j$ are the positions of the fireflies $i$ and $j$ respectively, forming a pair of fireflies,$\epsilon$ is a random vector with Gaussian distribution and $\alpha$, $\beta$ and $\gamma$ are parameters that determine the speed of convergence, the factor of luminosity perception reduction according to distance and the factor of randomness respectively. While $r_{ij}$ is the Euclidean distance between fireflies $i$ and $j$.

**0.1 Classic Evaluation of Controllers**

The evaluation indexes establish a criterion to evaluate the performance of controllers. When used in optimization, this index should correctly indicate if one solution is better than the other, allowing to optimization algorithm find the inteded response. Several indexes are found in the literature for this
The discrete wavelet transform has redundancies that generate a high computational cost, [18] optimized the representation removing the redundancies of the pyramidal coding that form the wavelet space, allowing the transform to be implemented by a bank of low-pass and high-pass filters [16]. The most usual choice is to make a dyadic sampling of the coefficients, that is, choose \( a = 2 \) and \( b_0 = 1 \). Where the output of each filter is subsampled by a factor of two. This process can be described by Equation 5.

\[
\begin{align*}
    a(n) &= x(n) * g(n) = \sum_{k=-\infty}^{\infty} x(k)g(2n-k) \\
    d(n) &= x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(2n-k)
\end{align*}
\]

Where \( d(n) \) and \( a(n) \) are the high-pass and low-pass filter outputs subsampled by a factor of 2 respectively, with passband \( g(n) \) and the high pass \( h(n) \). The sequences \( d(n) \) and \( a(n) \) are also known as detail and approximation of the \( x(n) \) signal respectively.

Fig. 3. Block diagram of the filter that implements the discrete wavelet transform.

The ability to show the frequency of a signal over time can be used in the evaluation and design of controllers since transient and steady state have different frequencies. Thus, the smoothness of the response can be evaluated. Thus, we can propose an evaluation function \( W \), given by:

\[
W = k_0 \cdot \sum_{r=1}^{l} \left| C_{a_r} - C_{a_y} \right| + k_1 \cdot \sum_{h=1}^{l} \left| C_{d_{n_h}} - C_{d_{n_y}} \right|
\]

In this, \( l \) is the number of analysed levels, \( C_{a_r} \) are the approximation coefficients and \( C_{d_n} \) are the detail coefficients of the system reference signal; \( C_{d_{n_h}} \) are the coefficients of detail of the output signal in \( n \) level \( C_{d_{n_h}} \), those of the reference signal; and \( k_0 \) and \( k_1 \) are weights used in weighting the relevance of the coefficients in \( n \) level. In choosing these weights, we must take into consideration, the difference of magnitude, the intended behavior. This strategy is mathematically similar to that used to measure the similarity of signals of different natures comparing the descriptors of them as in [19][20][21][22]. Since only transient and steady state are analyzed \( l = 1 \):

\[
W = k_0 \cdot \sum_{r=1}^{1} \left| C_{a_r} - C_{a_y} \right| + k_1 \cdot \sum_{h=1}^{1} \left| C_{d_{n_1}} - C_{d_{n_y}} \right|
\]

If \( k_1 \) is zero, \( W \) will present a value corresponding to the IAE. Increasing the number of levels analyzed open...
other possibilities as analysis of the rejection of measurement noise and disturbance if it is possible to isolate the frequency band of its occurrence, this will be studied in future works.

**Quadruple Tank System**

Storing a liquid in a tank and pumping it to another tank for processing are routines in industry [23]. The methodology proposed is quite adequate to be used in the process of quadruple coupled tanks, since it is often used as a benchmark and it is well known and has well known modeling, on the other hand it is relatively complex because it is a nonlinear MIMO system, what enables system simulations and justifies the use of more elaborate strategies respectively. This work aim to obtain the level control of the bottom tanks in the system show in Figure 4 with constants shown in Table 1.

![Quadruple tank](image)

**Fig. 4. Quadruple tank**

**Table 1.** Constants used in modeling the coupled tank system.

<table>
<thead>
<tr>
<th>Símbolo</th>
<th>Descrição</th>
<th>Valor</th>
<th>Unidade</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{m1}$</td>
<td>Constant of pump 1 flow</td>
<td>3.3</td>
<td>$cm^3/s$</td>
</tr>
<tr>
<td>$K_{m2}$</td>
<td>Constant of pump 2 flow</td>
<td>4.6</td>
<td>$cm^3/s$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>Tank 1 outlet area</td>
<td>0.178</td>
<td>$cm$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>Tank 2 outlet area</td>
<td>0.178</td>
<td>$cm$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>Tank 3 outlet area</td>
<td>0.178</td>
<td>$cm$</td>
</tr>
<tr>
<td>$a_4$</td>
<td>Tank 4 outlet area</td>
<td>0.178</td>
<td>$cm$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>Cross-sectional area of tank 1</td>
<td>15.25</td>
<td>$cm^2$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>Cross-sectional area of tank 2</td>
<td>15.25</td>
<td>$cm^2$</td>
</tr>
<tr>
<td>$A_3$</td>
<td>Cross-sectional area of tank 3</td>
<td>15.25</td>
<td>$cm^2$</td>
</tr>
<tr>
<td>$A_4$</td>
<td>Cross-sectional area of tank 4</td>
<td>15.25</td>
<td>$cm^2$</td>
</tr>
<tr>
<td>$g$</td>
<td>gravity</td>
<td>981</td>
<td>$cm/s^2$</td>
</tr>
</tbody>
</table>

Since $x_1$, $x_2$, $x_3$ and $x_4$ are the fluid height, $u_1$ and $u_2$ are the input signals for the pumps and the outputs are chosen, $y_1$ and $y_2$ are the levels of the two bottom tanks. By mass balance the following equations can be obtained:

\[
\begin{align*}
\frac{dx_1(t)}{dt} &= \frac{K_{m1}}{A_1} u_1(t) - \frac{a_1}{A_1} \sqrt{2 g x_1(t)} \\
\frac{dx_2(t)}{dt} &= \frac{K_{m2}}{A_2} u_2(t) + \frac{a_2}{A_2} \sqrt{2 g x_1(t)} - \frac{a_3}{A_3} \sqrt{2 g x_3(t)} \\
\frac{dx_3(t)}{dt} &= \frac{K_{m3}}{A_3} u_3(t) - \frac{a_3}{A_3} \sqrt{2 g x_3(t)} \\
y_1 &= x_4 \\
y_2 &= x_2
\end{align*}
\] (8)

Linearizing the above system with the operating point where $u_1(t) = u_{10}$, $u_2(t) = u_{20}$, $x_1(t) = x_{10}$, $x_2(t) = x_{20}$, $x_3(t) = x_{30}$ and $x_4(t) = x_{40}$, then, the system can be expressed as:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\] (9)

\[
A = \begin{bmatrix}
-\frac{a_2}{A_2} & 0 & 0 & 0 \\
\frac{a_1}{A_1} \sqrt{\frac{g}{2 x_{10}}} & -\frac{a_2}{A_2} \sqrt{\frac{g}{2 x_{20}}} & 0 & 0 \\
0 & 0 & -\frac{a_3}{A_3} \sqrt{\frac{g}{2 x_{20}}} & 0 \\
0 & 0 & 0 & -\frac{a_4}{A_4} \sqrt{\frac{g}{2 x_{40}}}
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
k_{m1} & 0 & 0 & k_{m2} \\
k_{m1} & 0 & k_{m2} & k_{m1} \\
k_{m1} & k_{m1} & 0 & 0 \\
k_{m1} & k_{m1} & k_{m1} & 0
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
0 & 0 & 0 & 1
\end{bmatrix}
\]

D = 0

**Implementation and Results**

In order to optimize the control system, a computational routines of FA used to optimize the of the controllers $C_1(s)$ and $C_2(s)$ was implemented in Matlab, both are PI controllers, in addition to the parameters denominated $p_{11}$, $p_{12}$, $p_{13}$, $p_{21}$, $p_{22}$ and $p_{23}$, which are the parameters of the decouplers $d_1(s)$ and $d_2(s)$. To find suitable values for these parameters and optimize them it is necessary to initially encode them as positions in a multidimensional space of possible solutions.

\[
d_1(s) = \frac{p_{11}}{p_{12} s + p_{13}}
\]

(11)

\[
d_2(s) = \frac{p_{21}}{p_{22} s + p_{23}}
\]

(12)

The parameters $\alpha = 0.05$, $\beta = 1$ and $\gamma = 1$ were chosen in FA. The FA was performed with 31 fireflies in the course of 100 iterations. The goal of the FA is find a response with a rapid rise and with fewer high-frequency oscillations in the transient using the analyse of wastel space for this. This analyse can be made using the sum of index proposed previously applied to each of the lower tanks as the cost function of FA. The index was implemented in Matlab using walet decompostion of Matlab with its standard discretization, using the Daubechies 1 family of filters. Since the system to be controlled is MIMO the index

For analysis the result will be compared with that found with the classic evaluation indexes shown previously. In addition, the influence of the values of the parameters $k_0$ and $k_1$
of the \( W \) index on the behavior of the systems will be studied. Figure 5 shows \( y_1 \), Figure 6 shows \( y_2 \) and Figure 7 and 8 show \( u_1 \) and \( u_2 \) respectively.

![Figure 5. Level of bottom tank \( X_2 \)](image)

![Figure 6. Level of bottom tank \( X_4 \)](image)

![Figure 7. Input in first pump \( u_1 \)](image)

For a better analysis, the output \( y_1 \) will be presented in Figure 9 for the time between 0s and 100s. For the same purpose, for the time between 200s and 250s, the output \( y_2 \) is shown in Figure 10. The output \( y_1 \) is expressed in Figure 11 for time values between 400s and 500s.

The output \( y_2 \) was expressed in Figure 12, 13 and 14 for the time values between 0s and 100s, between 200s and 300s, and between 400s and 500s respectively.

As rapid response with soft transient and quickly arriving at the reference was the intended, the \( W \) index shown better response after observing the graphs presented. Only ITAE response seems to present generally, for the most part of steps, a really slow and inadequate response.

![Figure 8. Input in second pump \( u_2 \)](image)

![Figure 9. Level of bottom tank \( X_2 \) between 0s and 100s.](image)

The increase of parameter \( k_0 \), taking into account the \((k_0; k_1) = (10; 400)\) led meta-heuristics to find, controllers that led to more oscillations of higher frequencies, while higher value of parameter \( k_1 \), which can be observed when \((k_0; k_1) = (1; 4000)\), generally led to soft response. Thus, in the case study, for the presented values, it was observed that larger weights, proportionally, for parameter \( k_1 \) inhibit a greater number of oscillations of higher frequencies, which, in general, led to the absence or small overshoot. However, this did not necessarily result in a slower response for higher values of \( k_1 \).

When \( k_1 \) this parameter was privileged the response found by the tuned controllers tended or to be slower so that the system response would slowly follow the reference without greater oscillations, or quickly follow the reference and remain stable on it. These slower responses also result in the reduction of high-frequency oscillations. This result is ex-

![Figure 10. Level of bottom tank \( X_2 \) between 200s and 250s.](image)
Fig. 11. Level of bottom tank $X_2$ between 400s and 500s.

Fig. 12. Level of bottom tank $X_4$ between 0s and 100s. Expected since the parameter $k_1$ corresponds to the coefficient of detail that presents the information of higher frequencies of the analyzed signal. The parameter $k_{10}$, on the other hand, take into account the general form of the wave, favoring the low frequencies.

The slower responses obtained for higher values of $k_1$ may not be adequate, as they take longer to follow the reference the high and low-frequency components of $k_1$ are expected to be worse, so to have a balance between these two parameters so that the response in high and low frequency, referring to reference tracking and high frequency, referring to fewer oscillations, are adequate.

**Conclusion**

According to what was observed, it is possible to observe the feasibility of using the wavelet transform and its multilevel analysis, already widely used in the most diverse applications of signal analysis, in evaluation for metaheuristics applied in the tuning of controllers. During the tests performed to compare different cost functions for evaluation for FA. The applied methodology demonstrated to allow the optimization taking into account the wanted behavior for the controller. From the results obtained, it can be observed that the proposed evaluation function managed to adequately map the performance of the controllers to the metaheuristics, pointing satisfactorily to the best controllers.

Although the study carried out in this work has been applied only to the analysis of the high-frequency oscillations, associated to the transient, it is expected that it can be extended to the analysis of rejection to the perturbation and the noise what will be analyzed in future works.

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