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Quantitative analysis of the influence of actuator limitations on control quality

***Abstract**. This paper aims to develop a method for estimating the effect of the limited values of an actuator output on the quality of control. The proposed approach is to use dynamic systems models for quantitative studies of control quality. The paper compares the performance of an actuator with saturation at the output and the device without such a limitation. For a quantitative description, the author proposed indicators measuring saturation performance and a method for calculating the dependence of control quality on these indicators.*

***Streszczenie.** . Celem pracy jest opracowanie metody oszacowania wpływu ograniczonej wartości wyjścia urządzenia wykonawczego na jakość regulacji. Zaproponowane podejście polega na wykorzystaniu modeli układów dynamicznych do badań ilościowych jakości regulacji. W pracy porównuje się działanie urządzenia wykonawczego z nasyceniem na wyjściu i urządzenia bez takiego ograniczenia. W celu opisu ilościowego zaproponowano wskaźniki mierzące działanie nasycenia i metodę obliczania zależności jakości regulacji od tych wskaźników.(**Ilościowa analiza wpływu elementu wykonawczego na jakość regulacji**).*

Keywords: control system, control quality indicator, linear dynamic system, control signal,.
Słowa kluczowe: układ regulacji, wskaźnik jakości regulacji, układ dynamiczny liniowy, sygnał sterujący.

Introduction

During the synthesis of the control system, important steps include obtaining a correct plant model [1, 21] and selecting a controller that meets the assumptions regarding the control quality [1, 2, 3, 25]. The work assumes that we have a reasonably accurate model of the controlled plant.

It is usually not a problem to select a suitable controller. Numerical simulations allow us to verify obtained control quality. The results of simulations may correspond to real signals when the actuator output is not limited. However, in real systems we can very often notice such a limitation.

This situation occurs when we have significant energy consumption e.g. in plasma arc equipment [5-11] or heating of buildings [24]. Another example is the limitation of the weight of the actuator e.g. in UAVs (unmanned aerial vehicles) [12-19, 20] or limited amount of energy in electric vehicles [22].

It seems to be valuable to be able to calculate the dependence of control quality on the abovementioned limitations. The typical control quality indicators are overshoot A_m, settling time t_r and steady state error e_u whereas the limitations of actuator output $u(t)$ will be represented by saturation.

We can observe the maximum value of $u(t)$ using computer simulations. This value determines the requirements for the actuator. Knowledge of these requirements is beneficial, albeit three problems arise.

Firstly, one has to consider what maximum power of the actuator is justified. What it means is that further increasing the maximum value of the actuator output signal will not improve control quality. This involves such a situation when we have unlimited funds to buy and operate the actuator.

Secondly, one has to answer the question what control quality is achievable with the limited funds or with other limitations. The funds limitations concern the cost of purchase and operation of the actuator. The other limitations may constitute technical limitations such as: low available supply power, acceptable size or weight of the actuator, the space available for mounting or lifting power of the UAV on which the actuator will be mounted. This problem boils down to the fact that we know the maximum value of the actuator output which cannot be changed.

Thirdly, with the imposed necessary control quality one has to answer the question what the power of the actuator has to be to guarantee a given necessary control quality. All these questions can be answered using the method developed in the work.

General formulation of the developed method

Within the framework of the developed method, we assume a control system as in Fig.1. We assume that there are not disturbances in the control system whereas the controller allows us to guarantee zero steady state error for a constant signal $w(t)$.

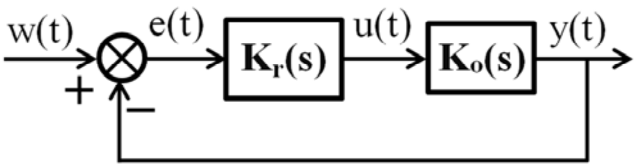

Fig.1. Block scheme of the considered control system

Evaluation of the control quality will be carried out on the basis of the quality indicators calculated from the signal $y(t)$. We will use the following quality indicators: overshoot Am, settling time t_r and steady-state error e_u. Fig.2 presents the definitions of the abovementioned indicators.

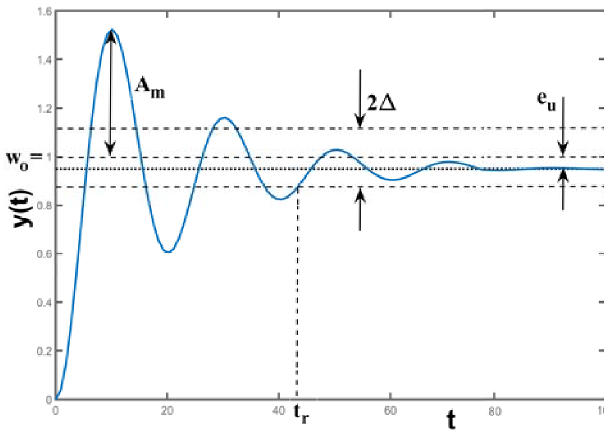

Fig.2. Output signal $y(t)$ with marked parameters A_m, t_r, and e_u

In the considered examples e_u will not impact the control quality analysis. It is caused by the assumption that the controller guarantees zero steady-state error.

The parametrisation of the signal $u(t)$ established by the author boils down to calculating the maximum value of the signal $u(t)$, denoted as U_{max}, during the control process. U_{max} determines how 'powerful' actuator is indispensable for obtaining a given quality control in the case without saturation.

Our goal is to examine how the saturation value affects the control quality. To achieve it, we will examine the control system with different values of the saturation shown in Fig.3.

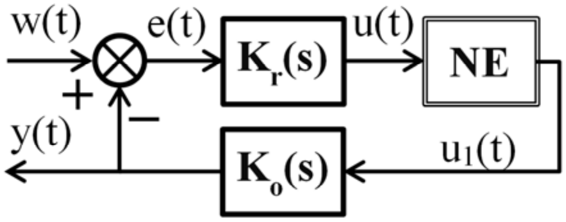

Fig.3. Block scheme of the considered control system with nonlinear element – saturation

The insertion of a nonlinear element NE in Fig.3, representing saturation, corresponds to the situation when the maximum value of the actuator output is lower than U_{max} [23]. In other words, we have too 'weak' actuator. Equation (1) describes the operation of NE.

$$(1) \qquad u_1(t) = \begin{cases} -U_{max1} & for \quad u(t) \leq -U_{max1} \\ u(t) & for \quad |u(t)| < U_{max1} \\ U_{max1} & for \quad u(t) \geq U_{max1} \end{cases}$$

For the control system with saturation, we assume the quality indicators as presented in Fig.2, but denoted: A_{m1}, t_{r1} and e_{u1}. We know that in this control system the maximum value of u(t) is less than or equal to U_{max1} because of saturation.

We will use two indicators to quantitatively assess the impact of saturation on control quality. The first is the ratio of the settling time in the control system with saturation to the settling time in the control system without any limitations on $u(t)$. We denote this parameter as t_{r1}/t_r. The values of t_{r1}/t_r greater than 1 indicate deterioration in control quality due to saturation. The second indicator is the ratio of U_{1max}/U_{max}. The values of U_{1max}/U_{max} lower than 1 indicate that the actuator in the system is too 'weak.'

The proposed method consists of the following stages:

Stage 1: general formulation of assumptions and controller selection for the control system;

Stage 2 : calculation of the acceptable (stable) range of the controller parameters;

Stage 3: analysis of $y(t)$ and calculation of parameters A_m, t_r and e_u for various controller parameters for the control system without saturation;

Stage 4: analysis of $u(t)$ and calculation of U_{max} for various controller parameters for the control system without saturation;

Stage 5: analysis of $y(t)$ and calculation of parameters A_m, t_r and e_u for various controller parameters for the control system with saturation (for various U_{max1} values);

Stage 6: the calculation and analysis of control quality indicators:

a) relative control quality indicator for settling time equal to t_{r1}/t_r as a function of U_{max}/U_{max1};

b) exponential control quality indicator for overshoot equal to $10^{(Am1-Am)}$ as a function of U_{max}/U_{max1}.

The above graphs for different values of U_{max1} allow for quantitative analysis of the relationship between U_{max1} and loss of control quality.

Proposing various control quality indicators results from the fact that settling time never achieves zero value whereas overshoot may equal zero.

In the example presented, the use of developed method of analysis will be discussed in detail, together with the interpretation of the indicators proposed by the author.

Application of the developed method to example 1

Example 1 presents the stages of the developed method which allow us to obtain a quantitative description of the changes in control quality depending on the control signal limitations for the first order plant .

Stage 1: general formulation of assumptions and controller selection for the control system.

In this example we will examine the control system from Fig.1 with the object described as the first order transfer function (2).

$$(2) \qquad K_o(s) = \frac{k}{(s \cdot T + 1)}.$$

The values of parameters are equal respectively: $k=2$, $T=7$.

The conditions which the control system has to meet are as follows: a) error equal to zero for constant $w(t)$, b) the form of signal $w(t)$ is $w(t)=w_o \cdot 1(t)$ and $w_o=const$, c) the shortest settling time is required, d) the lowest overshoot is required. In view of the above requirements, we choose PI controller with the transfer function (3).

$$(3) \qquad K_r(s) = k_r \frac{s \cdot T_c + 1}{s \cdot T_c}$$

We assume $T_c=T$ according to the rule of the reduction of dominant constant time (pole zero cancellation method). The open loop transfer function assumes the form presented in (4).

$$(4) \qquad K(s) = K_o(s) \cdot K_r(s) = k_r \frac{s \cdot T_c + 1}{s \cdot T_c} \frac{k}{(s \cdot T + 1)} = \frac{k_r k}{T_c} \frac{1}{s}$$

Stage 2: calculation of the acceptable (stable) range of the controller parameters

The first step to examine the properties of the closed loop will be the analysis of the root locus plot assuming that $k_r>0$.

Root locus plot for (4) consists of one root locus branch which starts at the open loop pole (zero) and end at minus infinity. The whole branch of the root locus lie on the negative part of real axis. It causes that the control system is stable and works without oscillations for $k_r>0$. Summing up, there are no any limitations of k_r due to control system stability. Moreover, bigger k_r results in better control quality.

Stage 3: analysis of $y(t)$ and calculation of parameters A_m, t_r and e_u for various controller parameters for the control system without saturation.

We can conduct an analysis of $y(t)$ using the root locus plot we have discussed in the previous stage. There are no oscillations what causes that $A_m=0$ for $k_r>0$. At the same time, increasing k_r moves the closed-loop pole to the left side of the real axis, resulting in a shorter settling time t_r.

It was assumed in the paper that the width of the acceptable range of $y(t)$ is equal to $2\Delta=2\cdot10\%w_o$ and $wo=1$. Thus, the value of $y(t)$ that corresponds to t_r is $y(t_r)=0.9$.

The dependence of $t_r(k_r)$ obtained under such assumptions is shown in Fig.5 (curve 1.). As can be seen in Fig.5 as k_r increases, there is a decrease in t_r. For small k_r this decrease is rapid and for larger k_r it is slower.

Stage 4: analysis of $u(t)$ and calculation of U_{max} for various controller parameters for the control system without saturation.

Examples of $u(t)$ are shown in Fig.4. They show that the maximum value of $u(t)$, denoted as U_{max}, is determined by the value of $u(t=0)$. At this stage, U_{max} was also calculated for stable values of k_r.

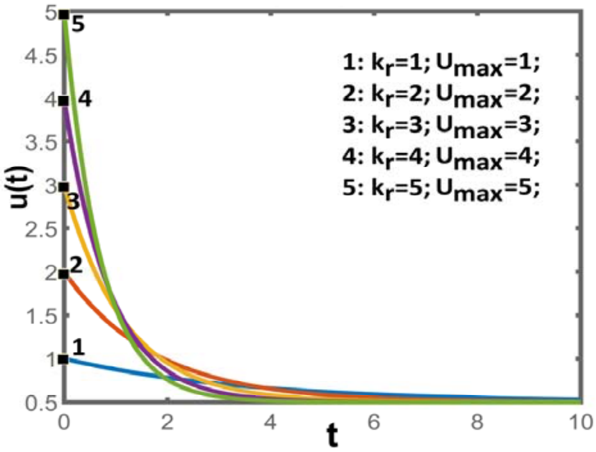

Fig.4. Control signal $u(t)$ for various values of k_r

Stage 5: analysis of $y(t)$ and calculation of parameters A_m, t_r and e_u for various controller parameters for the control system with saturation (for various U_{max1} values).

Stage 5 is focused on studying the behaviour of the nonlinear system shown in Fig.3. In this system, we have taken into account the limited value of the actuator output.

This limitation was modelled by a saturation type element. The maximum output value of the saturation block is denoted as U_{max1}. To study the influence of the U_{max1} value on the t_r obtained in the nonlinear system, three different values of U_{max1} were assumed.

We will denote the maximum value of $u(t)$ in the system without saturation, considering the $u(t)$ transitions for all stable k_r, as U_{max}, and assume U_{max1} as $10\%U_{max}$, $30\%U_{max}$ and $50\%U_{max}$, respectively.

Fig.5 shows the dependence of the settling time on k_r for linear system (Fig.1) and nonlinear systems (Fig.3) at three different values of U_{max1}.

We denote the settling time for a linear system as t_r (curve 1.) while we denote the settling time t_r1 in a nonlinear system as t_r110 (curve 4.), t_r130 (curve 3.), t_r150 (curve 2.) for saturation equal to 10%, 30% and 50%, respectively.

The points marked in Fig.5. denote the following values of k_r and t_r:

point a.: $k_r=10.7$, $t_r=0.7537$;
point b.: $k_r=6.74$, $t_r=1.197$;
point c.: $k_r=3.06$, $t_r=2.657$.

These points correspond to the value of k_r above which the settling time is greater in a system with saturation than in a system without saturation. As we can notice in Fig.5 the larger value U_{max1}, the larger is the k_r range for which the curves for systems with and without saturation overlap.

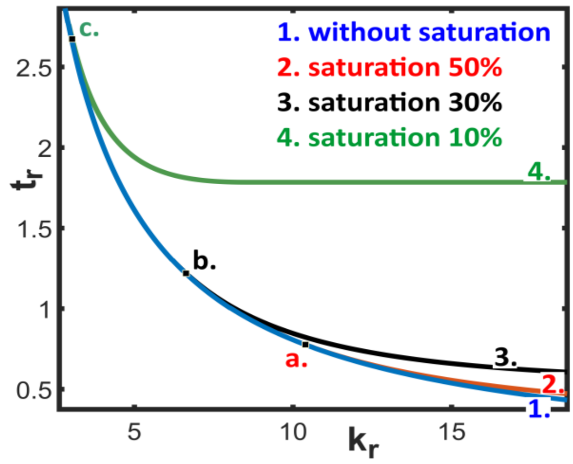

Fig.5. The dependence of the settling time t_r on k_r for control systems with and without saturation

At this point we obtain the first result, which can be helpful in choosing an actuator. For example, if we are satisfied with $t_r=2$ then an actuator with a saturation of $10\%U_{max}$ is sufficient. Whereas if we want $t_r=1$ then an actuator with a saturation of $30\%U_{max}$ is necessary.

To sum up, the higher the k_r gain, the shorter settling time, but for each saturation there is a limiting t_r below which is impossible to go down. Here we see a dependence that the weaker the actuator, the greater the limit t_r below which is impossible to go, even by increasing the gain k_r.

Stage 6: the calculation and analysis of control quality indicators.

Stage 6 contains an analysis of the control quality indicators proposed by the author. These are two indicators that take into account the influence of actuator limitations on control quality. These have been named as:

a) a relative control quality indicator for settling time equal to t_{r1}/t_r as a function of U_{max}/U_{max1};

b) an exponential control quality indicator for overshoot equal to $10^{(Am1-Am)}$ as a function of U_{max}/U_{max1}.

Indicator a) corresponds to the parameter t_r, which never takes the value zero, while indicator b) was developed for the parameter Am, which can take value 0.

The exponential control quality indicator for A_m will be illustrated in example 2 while in example 1 we will focus on the relative control quality indicator.

The relative control quality indicator for the examined system is shown in Fig.6. Here, a non-linear system was examined for three values of saturation U_{max1} equal to 10%, 30% and 50% U_{max}, respectively.

As we can see in Fig.6 on the vertical axis is the ratio of t_{r1} (t_r in the system with saturation) to t_r (t_r in the system without saturation).

Our goal is to describe how the saturation value U_{max1} influences the deterioration of the control quality. Therefore, on the horizontal axis we have the ratio of U_{max} to U_{max1}.

A value of $U_{max}/U_{max1} < 1$ means that saturation does not cause any limitation - the actuator is adequately strong. A value of this ratio greater than 1 describes how much the actuator is too weak.

In Fig.6 three curves are shown for saturation values of 10%, 30% and 50%. The following points are marked on them:

a.=b.=c.: $U_{max}/U_{max1}=1$, $t_{r1}/t_r=1$;
d.: $U_{max}/U_{max1}=9.42$, $t_{r1}/t_r=4.17$;

e.: $U_{max}/U_{max1}=3.33$, $t_{r1}/t_r=1.47$;
f.: $U_{max}/U_{max1}=2$, $t_{r1}/t_r=1.12$.

The individual points correspond to the respective curves:

a. and d. – curve 1., b. and e. – curve 2., c. and f. – curve 3.

b.

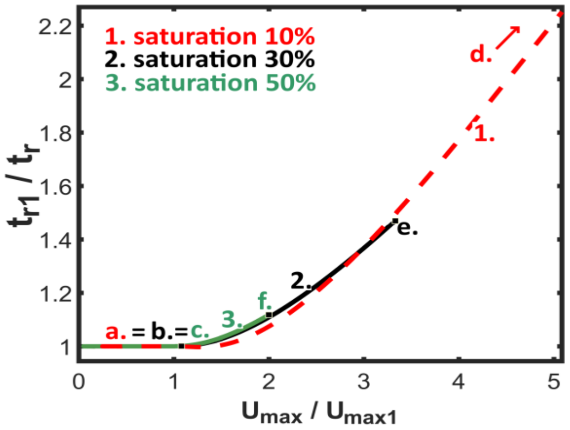

Fig.6. Relative control quality indicator for t_r for saturation $U_{max1}=10\%U_{max}$, $U_{max1}=30\%U_{max}$ and $U_{max1}=50\%U_{max}$

The values on the horizontal axis to the left of points a., b. and c, respectively, correspond to the case when there is no deterioration in the control quality. The size of this range is identical for all cases.

On the other hand, as U_{max}/U_{max1} increases, the following dependencies are observed:

a) the smaller the saturation value, the greater the U_{max}/U_{max1} value achieved,

b) the smaller the saturation value, the greater the possible deterioration of the t_{r1}/t_r ratio. In other words, a saturation of $10\%U_{max}$ results in a maximum value of $u(t)$ that is 9.42 times smaller than that obtained in a system without saturation (more than four times the increase in t_r), etc.

Thus, we get data to help answer the question "what longest t_r is acceptable?".

This data also allows us to decide: "what weakest actuator is sufficient". This allows us to select an actuator that provides the necessary quality of control, which is also the cheapest and requires the least energy.

Application of the developed method to example 2

Example 2 presents the stages of the developed method which allow us to obtain a quantitative description of the changes in control quality depending on the control signal limitations for the third order plant.

Stage 1: general formulation of assumptions and controller selection for the control system

In this example we will examine the control system from Fig.1 with the object described as the third order transfer function (5).

(5) $$K_o(s) = \frac{k}{(s\cdot T_1+1)(s\cdot T_2+1)(s\cdot T_3+1)}$$

The values of parameters are equal respectively: $k=0.7$, $T_1=0.4$, $T_2=0.6$, $T_3=1$.

The conditions which the control system has to meet are as follows: a) error equal to zero for constant $w(t)$, b) the form of signal $w(t)$ is $w(t)=w_o\cdot 1(t)$ and $w_o=const$, c) the shortest settling time is required, d) the lowest overshoot is required. In view of the above requirements, we choose *PI* controller with the transfer function (6).

(6) $$K_r(s) = k_r\frac{s\cdot T_c+1}{s\cdot T_c}.$$

We assume $T_c=T_3$ according to the rule of the reduction of dominant constant time (pole zero cancellation method). The open loop transfer function assumes the form (7).

(7) $$K(s) = K_o(s)\cdot K_r(s) = = k_r\frac{s\cdot T_c+1}{s\cdot T_c}\frac{k}{(s\cdot T_1+1)(s\cdot T_2+1)(s\cdot T_3+1)} = \frac{k_r k}{T_c}\frac{1}{s(s\cdot T_1+1)(s\cdot T_2+1)}$$

Stage 2: calculation of the acceptable (stable) range of the controller parameters

The first step to examine the properties of the closed loop will be the analysis of the root locus plot assuming that $k_r>0$.

From the root locus plot, shown in Fig.7, we can see that for $0<k_r<=0.42$ control system works without oscillations whereas for $0.42<k_r<5.95$ control system is stable, however, the oscillations appear. For $k_r>5.95$ the control system is unstable. Therefore further examination will be carried out for $0<k_r<5.95$.

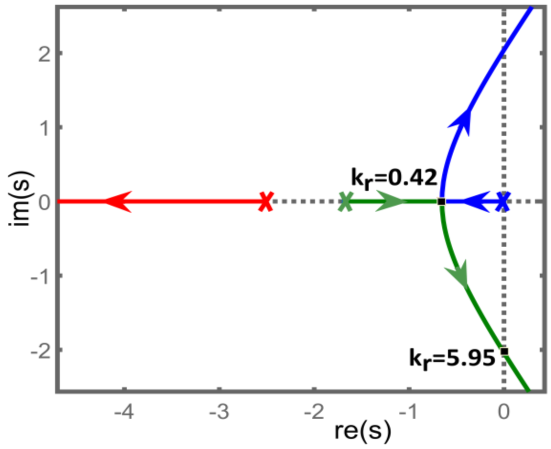

Fig.7. Root locus plot for the example 2

Stage 3: analysis of $y(t)$ and calculation of parameters A_m, t_r and e_u for various controller parameters for the control system without saturation.

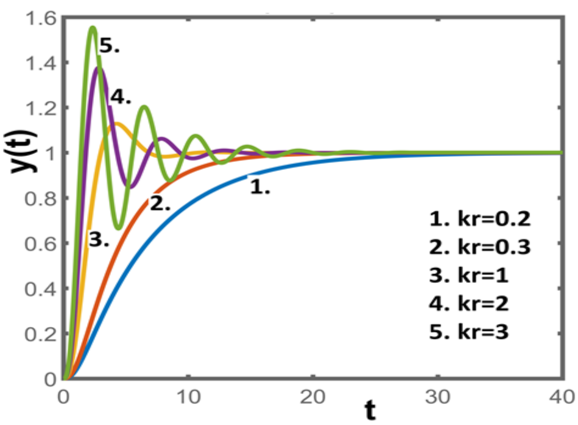

Fig.8. Output signal $y(t)$ for various values of k_r for example 2

The examples of $y(t)$ are presented in Fig.8. As we can see in this figure, for low values of k_r (e.g. $k_r=0.2$, $k_r=0.3$) $y(t)$ is aperiodic and increasing k_r will shorten settling time t_r whereas further increase of k_r (e.g. $k_r=1$, $k_r=2$, $k_r=3$) causes occurrence of increasing oscillations.

It was assumed, similar to the example 1, that the width of the acceptable range of $y(t)$ is equal to $2\Delta=2\cdot10\%w_o$ and $w_o=1$. Thus, the values of $y(t)$ that corresponds to t_r are 0.9 and 1.1.

The dependence of $t_r(k_r)$ obtained under such assumptions is shown in Fig.10 (curve 1.).

As it can be seen in Fig.10 (curve 1.) there is a minimum for $k_r=0.9$. Assuming an "acceptable" increase in t_r relative to the minimum value as 100% of the minimum t_r, we obtain an "acceptable" range of k_r from $k_r=0.44$ to $k_r=2.32$.

At the same time, it is noteworthy that for k_r smaller than 0.9 the decreasing of t_r with the increase of k_r is very fast. It is otherwise on the opposite side of the minimum t_r. Here, for k_r larger than 0.9, the rise of t_r has a much smoother progress.

The dependence of overshoot A_m on k_r is shown in Fig.11 (curve 1.) where we can see that for $k_r\leq0.42$ overshoot is equal to zero while further increasing k_r results in increasing overshoot A_m.

Stage 4: analysis of $u(t)$ and calculation of U_{max} for various controller parameters for the control system without saturation.

Examples of $u(t)$ are shown in Fig.9. They show that, similar to $y(t)$, there is also a certain range of small k_r values for which there are no oscillations in $u(t)$.

In this case, the maximum value of $u(t)$, denoted as U_{max}, is determined by the steady-state value of $u(t)$.

On the other hand, when further increasing k_r, oscillations appear in $u(t)$ and U_{max} is equal to the maximum amplitude of oscillations.

At this stage, U_{max} was also calculated for stable values of k_r.

Stage 5: analysis of $y(t)$ and calculation of parameters A_m, t_r and e_u for various controller parameters for the control system with saturation (for various U_{max1} values);

In this stage, similar to Example 1, we will study the behaviour of a nonlinear system with saturation. To study the influence of the value of U_{max1} on the achieved t_r and A_m in the nonlinear system, three different values of U_{max1} were assumed: $25\%U_{max}$, $40\%U_{max}$ and $60\%U_{max}$.

Fig.10 shows the dependence of t_r on k_r for the system without saturation and three systems with saturation for three different values of U_{max1}. Curve 1. in Fig.10 corresponds to the relationship of $t_r(k_r)$ in the system without saturation, whereas curves 2., 3. and 4. correspond to the systems with successively increasing saturation.

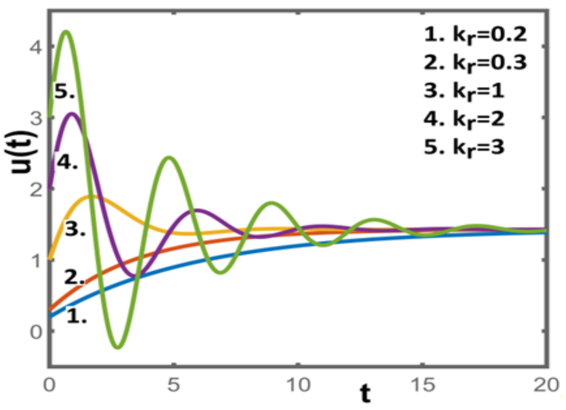

Fig.9. Control signal $u(t)$ for various values of k_r

Point a) on curve 1. corresponds to the minimum value of t_r for the system without saturation. Points b), c) and d) on curves 2., 3. and 4. respectively, show the values of k_r above which the values of t_r for the system with saturation stop overlapping with the values of t_r for the system without saturation. As can be seen from Fig.10, the greater the value of U_{max1} (the stronger the actuator), the greater the range of k_r for which the curves for the linear and nonlinear system overlap.

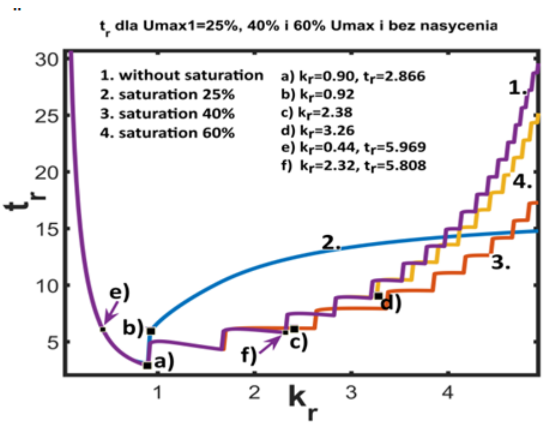

Fig.10. The dependence of the settling time t_r on k_r for control systems with and without saturation

At the same time, we get here the first conclusion of value in practical terms: the actuator with a saturation of 25% guarantees the achievement of $t_r=t_{rmin}$ therefore the use of a stronger actuator is pointless - at least for the minimization of t_r.

Fig.11, in turn, shows the dependence of A_m on k_r for the same systems as described for the analysis of t_r in Fig.10. Also in this case, curve 1. corresponds to the relationship $A_m(k_r)$ in the system without saturation and the subsequent curves correspond to the systems with successively increasing saturation.

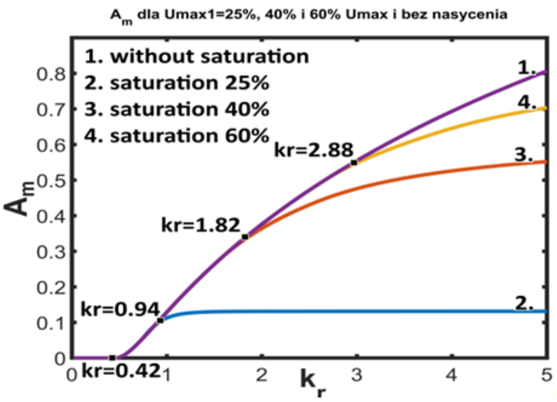

Fig.11. The dependence of the overshoot A_m on k_r for control systems with and without saturation

And in this case, also, we see that for increasing values of U_{max1} we have longer and longer sections of the curves for the system with saturation overlapping with the curve for the system without saturation. However, we must remember that we want to achieve overshoot A_m equal to zero. Thus, we get another conclusion of value in practical terms: the actuator with saturation equal to 25% of U_{max} is completely sufficient to achieve $A_m=0$.

From Fig.10 and Fig.11 we can infer that to achieve minimum t_r we need k_r equal to 0.9 while to achieve $A_m=0$ we need k_r no greater than 0.42. Thus, it is not possible to achieve minimum t_r and zero A_m at the same time.

Controller tuning requires deciding which control quality indicator is more important for us. Knowledge of the control system operation, shown in Fig.10 and Fig.11, makes it possible to select k_r providing sufficiently small t_r and acceptable A_m.

Based on Fig.10 and Fig.11, we can also infer that an actuator with U_{max1} equal to $25\%U_{max}$ provides the technical conditions for guaranteeing control quality with $t_r=t_{rmin}$ or $A_m=0$.

Of course, in the case of plants for which the actuators use very large amounts of energy, it would be valuable to conduct further research starting from 25% down so as to find the minimum value of U_{max1} that guarantees appropriate control quality.

Stage 6: the calculation and analysis of control quality indicators.

Stage 6 for Example 2 analyses two control quality indicators proposed by the author: a) relative control quality indicator for settling time and b) exponential control quality indicator for overshoot.

The general features of indicator a) are discussed in Example1. Indicator b) was developed for the parameter A_m, which can assume value 0.

The relative control quality indicator for the examined system is shown in Fig.12. Here, a non-linear system was examined for three values of saturation U_{max1} equal to 25%, 40% and 60% U_{max}, respectively. The curves corresponding to the subsequent U_{max1} are denoted as: 1., 2. and 3. Three intervals can be distinguished on each curve:

1) the range for which $t_{r1}=t_r$ and $U_{max1} \geq U_{max}$,

2) the range in which there is an increase in t_{r1}/t_r for increasing U_{max}/U_{max1},

3) the range for which t_{r1}/t_r decreases with oscillations to a value less than 1.

For the individual curves, the ranges can be described as follows:

1) curve 1.: from a. to b. , curve 2.: from e. to f. , curve 3.: from i. to j;

2) curve 1.: from b. to c. , curve 2.: from f. to g. , curve 3.: from j to k.;

3) curve 1: from c. to d. , curve 2.: from g. to h. , curve 3.: from k to L.

The numerical values for the individual points are as follows:

point a.: $U_{max}/U_{max1} = 0.884$, $t_{r1}/t_r = 1$;
point b.: $U_{max}/U_{max1} = 0.9976$, $t_{r1}/t_r = 1$;
point c.: $U_{max}/U_{max1} = 1.642$, $t_{r1}/t_r = 2.387$;
point d.: $U_{max}/U_{max1} = 4$, $t_{r1}/t_r = 0.4653$;

point e.: $U_{max}/U_{max1} = 0.5525$, $t_{r1}/t_r = 1$;
point f.: $U_{max}/U_{max1} = 0.9993$, $t_{r1}/t_r = 1$;
point g.: $U_{max}/U_{max1} = 1.32$, $t_{r1}/t_r = 1.068$;
point h.: $U_{max}/U_{max1} = 2.474$, $t_{r1}/t_r = 0.5708$;

point i.: $U_{max}/U_{max1} = 0.3683$, $t_{r1}/t_r = 1$;
point j.: $U_{max}/U_{max1} = 1.013$, $t_{r1}/t_r = 1$;
point k.: $U_{max}/U_{max1} = 1.143$, $t_{r1}/t_r = 1.007$;
point L.: $U_{max}/U_{max1} = 1.661$, $t_{r1}/t_r = 0.8149$;

As we can see in Fig.12 the ranges mentioned above depend on the saturation value as follows:

1) the higher the value of U_{max1} the wider the area of range 1),

2) the smaller the value of U_{max1} the greater the deterioration of settling time in range 2),

3) the smaller the value of U_{max1} the greater the improvement in settling time in range 3).

The first two conclusions seem obvious, while the third effect of shortening the settling time for small U_{max1} is caused by the fact that for large k_r the system becomes unstable.

In this case, increasing oscillations appear in the linear system. A weak actuator, in a non-linear system, in practice reduces the effective gain in the feedback loop and stabilises the system.

The exponential control quality indicator for the examined system is shown in Fig.13. Curves 1., 2. and 3. correspond to U_{max1} being equal to 25%, 40% and 60% of U_{max}.

Since A_m can take the value 0, the author proposed an exponential indicator. For $A_{m1}=A_m$ the indicator value is 1 - also for $A_m=0$. For $A_{m1}>A_m$ the indicator value is greater than 1 and for $A_{m1}<A_m$ the indicator value is less than 1.

As we can see in Fig.13 in example 2, we have an exponential indicator equal to or less than 1. This is the result of the 'weak' actuator reducing the amplitude of the oscillation.

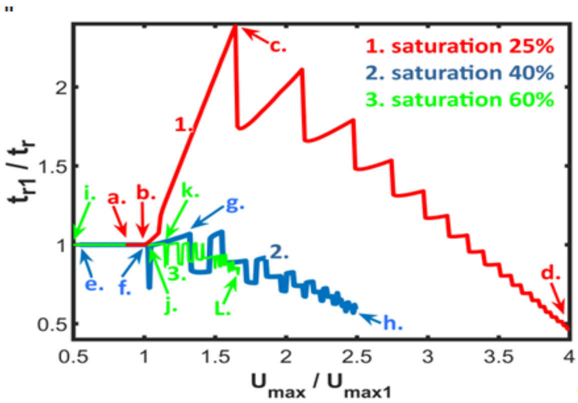

Fig.12. Relative control quality indicator for t_r for saturation $U_{max1}=25\%U_{max}$, $U_{max1}=40\%U_{max}$ and $U_{max1}=60\%U_{max}$

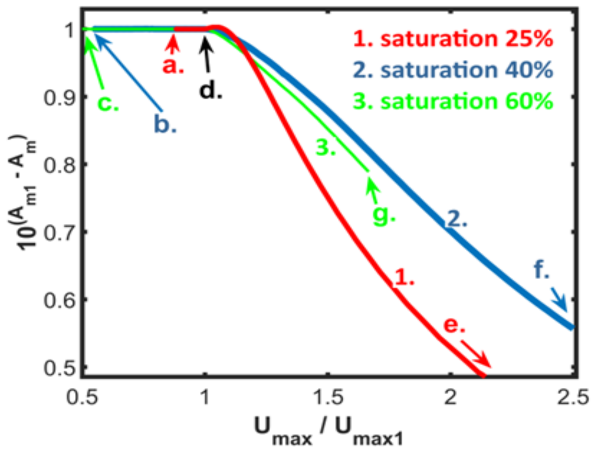

Fig.13. Exponential control quality indicator for A_m for saturation $U_{max1}=25\%U_{max}$, $U_{max1}=40\%U_{max}$ and $U_{max1}=60\%U_{max}$

In Fig.13 the curves corresponding to the subsequent Umax1 are denoted as: 1., 2. and 3. Two ranges can be distinguished on each curve:

1) the range for which $A_{m1}=A_m$ and $U_{max1} \geq U_{max}$,

2) the range for which $A_{m1}<A_m$ and $U_{max1}<U_{max}$.

In the first range, the exponential control quality indicator is equal to 1 whereas in the second range there is a decreasing control quality indicator for increasing U_{max}/U_{max1}. The decreasing effect of the exponential control quality indicator is due to the reduction of oscillations by decreasing saturation.

For the individual curves, the ranges can be described as follows:

1) Curve 1.: from a. to d. , curve 2.: from b. to d. , curve 3.: from c. to d.;

2) curve 1.: from d. to e. , curve 2.: from d. to f. , curve 3.: from d to g.;

The numerical values for the individual points are as follows:

point a.: $U_{max}/U_{max1} = 0.8838, \quad 10^{(Am1- Am)} = 1;$
point b.: $U_{max}/U_{max1} = 0.5527, \quad 10^{(Am1- Am)} = 1;$
point c.: $U_{max}/U_{max1} = 0.3685, \quad 10^{(Am1- Am)}) = 1;$

point d.: $U_{max}/U_{max1} = 1, \quad 10^{(Am1- Am)}) = 1;$
point e.: $U_{max}/U_{max1} = 4, \quad 10^{(Am1- Am)}) = 0.2113;$
point f.: $U_{max}/U_{max1} = 2.5, \quad 10^{(Am1- Am)} = 0.5566;$
point g.: $U_{max}/U_{max1} = 1.667, \quad 10^{(Am1- Am)}) = 0.7889;$

In summary, it can be concluded that in the system of example 2 for both indicators, relative and exponential indicator, there is a dependence: the larger the value of $Umax1$, the wider are the U_{max}/U_{max1} ranges in which the indicators have a value of 1.

An important conclusion is that these ranges for different $Umax1$ values partly overlap.

The proposed forms of the quality indicators make it possible to assess the effect of an increasingly powerful actuator in general and to obtain a quantitative dependence between control quality and actuator power.

Conclusion

Nowadays, it is technically possible to create control systems that provide very good control quality. Nevertheless, an extremely important issue is the economic and ecological aspect of such a control system. These problems are related to the amount of energy required to guarantee acceptable control quality. The element in the structure of the control system where a large amount of energy may be required is an actuator. The power of the actuator is directly related to the amount of energy used.

The aim of the work was to develop a method to obtain quantitative dependences between the power of an actuator and the achieved quality of control.

The assumption of the work was that the method should not require complex mathematical analysis, so an approach based on numerical simulations was proposed. The only requirement is to have a correct model of the controlled object.

The developed method has been formulated in the form of algorithm. This allows easy implementation for a wide range of industrial automation engineers.

The stages presented include both a preliminary analysis of the control system and a fundamental evaluation of the relationship between the power of the actuator and the control quality that can be achieved.

The paper proposes a relative control quality indicator and an exponential control quality indicator . They make it possible to illustrate the influence of actuator limitations on the achieved control quality.

The two examples in the paper illustrate the method of inference on both a general and a detailed level. They show that limiting the power of an actuator can sometimes have an unequivocally bad influence on control quality.

In example 1, the low power of the actuator makes it impossible to reduce the settling time. But in another case, example 2, excessive increases of power do not improve the control quality. With this approach, it is possible to select an actuator that provides an acceptable control quality and at the same time is not unnecessarily over-powered.

In this way, it becomes possible to limit the energy consumed and reduce the environmental impact of the system.

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