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Modelling of a single-phase induction motor with a 50Hz/150Hz magnetic frequency converter

Abstract. This paper presents the structure of a single-phase motor powered by a 50/150 Hz magnetic frequency converter and selects the arrangements of its windings to obtain favourable parameters. A physical model of the motor is described and its mathematical model is proposed, which has been adapted and realised in the PSpice application. The results of the computer simulation are provided in the form of characteristic curves of the speed and the electromagnetic torque as a function of time.

Streszczenie. W artykule przedstawiono strukturę silnika jednofazowego zasilanego z magnetycznego przekształtnika częstotliwości 50/150 Hz i dobrano układy połączeń jego uzwojeń tak, aby uzyskać korzystne jego parametry. Opisano model fizyczny silnika i zaproponowano jego model matematyczny, który został zaadoptowany i zrealizowany w aplikacji PSpice. Zamieszczono wyniki symulacji komputerowej w postaci charakterystyk prędkości i momentu elektromagnetycznego w funkcji czasu. (Modelowanie jednofazowego silnika indukcyjnego z magnetycznym przemiennikiem częstotliwości).

Keywords: single-phase induction motor, magnetic frequency converter, modelling.

Słowa kluczowe: jednofazowy silnik indukcyjny, magnetyczny przekształtnik częstotliwości, modelowanie.

Introduction

Electric machines are the most common devices used in everyday life and the number of their types is increasing with the development of science, engineering and technology. The development of electrical machines over the next few years will be mainly related to the evolution of computer hardware, consumer appliances and public transport applications and systems (land, sea and air), micro-electromechanical systems, special applications such as nuclear technologies, weapons systems and others [1, 2, 3]. The development of special machines with unconventional topologies is also foreseen, such as written pole motors, transverse flux motors, hybrid, piezoelectric, resonant, oscillating, superconducting, rotary-linear, etc. [1,2,3,4,5]. This group of unconventional machines includes a single-phase motor with a 50Hz/150Hz magnetic inverter. An analysis of the basic structures of magnetic systems for frequency multiplication, as well as an evaluation of models built in Poland and abroad, indicates the possibility of building high-speed single-phase or three-phase motors.

The data available in the literature and the results of the author's research allow us to conclude that it is possible to build single-phase and three-phase asynchronous motors with a synchronous speed of 9000 rpm when supplied from the 50 Hz mains [6, 7, 8]. This paper presents the structure of the magnetic circuit of a single-phase motor and selects arrangements for connecting its windings to obtain more favourable performance.

The magnetic circuits of these high-speed induction motors are more complex than those of conventional solutions. However, they can be made in traditional sheet metal structures, which impose privileged directions of magnetic flux flow. Advances in magnetic materials technology mean that powder composites may be a suitable material for magnetic circuits, particularly for low power, high speed motors.

There is also no sufficiently developed theory for such motors. Due to the non-linearity of the magnetic circuits, it is expedient to formulate a circuit description of the models taking into account the motion equations of the rotor.

This makes it easier to study the influence of the choice of stator and rotor magnetic circuits on the operating parameters of the motors under consideration.

At this stage, a mathematical model of a single-phase induction motor is presented in terms of the state variable

method and the solution of these equations is proposed in the PSpice application.

Physical model of the single-phase motor

The models of high speed motors built so far using the magnetic structures of frequency converters are projecting pole machines [9], which is mainly determined by technological considerations and by the possibility of easily adapting the magnetic circuit of the converter to generate a magnetic field in the air gap of the motor. Projecting pole motors are characterised by a non-sinusoidal distribution of the magnetic field in the motor gap. The harmonic fields of the rotor and stator interact to produce unfavourable asynchronous and synchronous drive torques. In order to achieve a more uniform magnetic field distribution across the rotor surface, the design of a slotted stator model has been considered. The magnetic circuit of the motor model, which also acts as a frequency converter, has the structure shown in Figure 1.

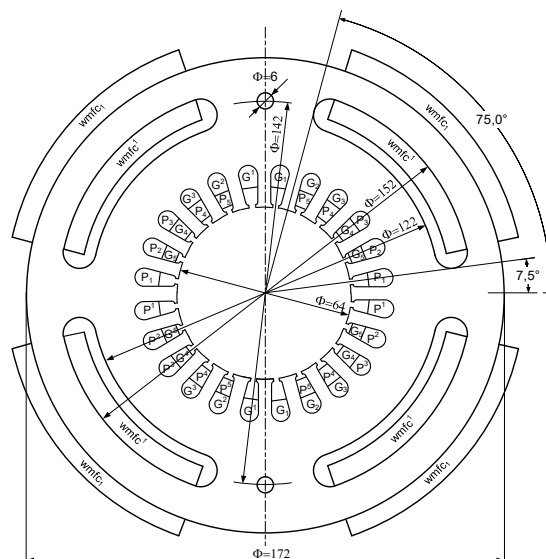


Fig. 1. Cross-section of the integrated magnetic circuit of the 50/150Hz frequency converter and the motor. Markings: wmfc - windings of the magnetic frequency converter, $G_1, G_1', \dots, G_5, G_5'$ - main motor winding coils $P_1, P_1', \dots, P_5, P_5'$ - auxiliary motor winding coils

The frequency converter/motor magnetic circuit and its components are shown in Figure 2.

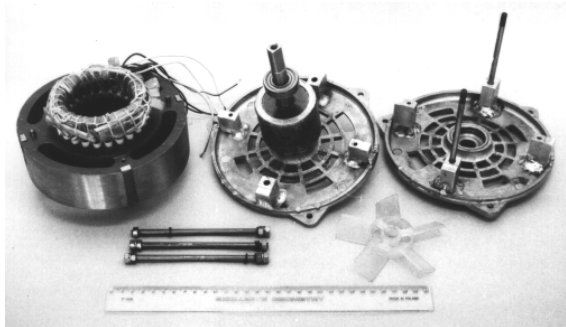


Fig. 2. Components of a single-phase motor model with a frequency converter; main and auxiliary winding wound, frequency converter winding missing

The magnetic circuit is made of 96 EP23 sheets with a thickness of 0.5 mm. The length of the stator plate pack is 48 mm. The inner part of the stator has 24 grooves in which the main winding and the auxiliary winding with their axes offset by an angle of $\pi/2$ are placed, serving to obtain the starting torque. The windings are made as two-phase symmetrical so as to ensure a near sinusoidal distribution of the magnetic field in the motor slot. The number of grooves per pole and phase is 6. The main and start-up phase windings were wound with DNE wire \varnothing 0.45 mm in the form of two strands each, with five coils and the following numbers of turns: 69, 64, 55, 42, 36 and 41, 38, 33, 25, 21 respectively. In the outer part of the magnetic circuit, there are elements that operate in magnetic saturation and are used to obtain the third harmonic flux. The non-linear elements are magnetised by the current flowing through 4 series-connected windings of 250 turns each, which can be called the frequency converter windings.

State variable mathematical model of a single-phase induction motor

An equivalent diagram of a single-phase induction motor fed from a magnetic frequency converter is shown in Figure 3. The non-linear part where the third harmonic flux is generated is replaced by a non-linear inductance L_p .

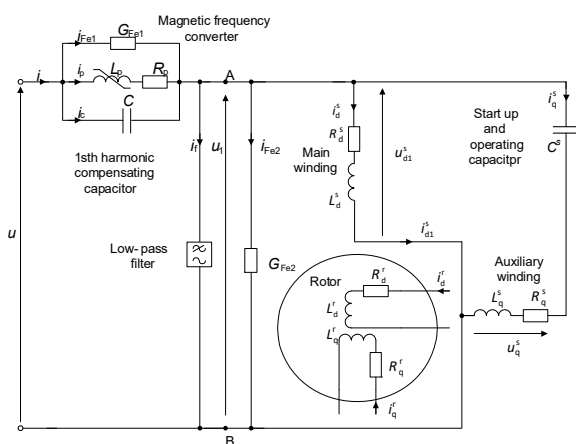


Fig. 3. Equivalent circuit diagram of a single-phase induction motor with a magnetic frequency converter

A capacitor is connected in parallel to it to compensate for the mains frequency component of the current. A capacitor is connected in parallel to it to compensate for the mains frequency component of the current. The iron loss in the outer part of the magnetic circuit can be

approximated by introducing an empirically determined conductance.

For the description of the linear part of the motor, a d - q model was adopted in which the stator currents and voltages ($i_d^s, i_q^s, u_d^s, u_q^s$) correspond to the real quantities, while the replacement of the variables i_d^r, i_q^r by real variables, requires only a transformation.

In the area of the rotor and stator parts, which form the electromechanical converter, a linear magnetisation characteristic was assumed, which means a constant value for the own and mutual inductances of the stator and rotor windings.

A description of the motor model is presented using the state variable method. The relationships between the quantities can be given in the form of a system of differential equations representing the rotation equation of the rotor and the relationships between the voltages, currents and magnetic fluxes in the system.

The state variables chosen are the instantaneous values of the fluxes associated with the phase windings of the equivalent two-phase machine. In the case of symmetrical windings, the time derivatives of the flux components in the d and q axes associated with the motor phase windings and the currents in the windings are of the form:

$$\begin{aligned} (1) \quad \frac{d\Psi_d^s}{dt} &= u_d^s - a_1\Psi_d^s + a_1K_r\Psi_d^r \\ (2) \quad \frac{d\Psi_q^s}{dt} &= u_q^s - a_1\Psi_q^s + a_1K_r\Psi_q^r \\ (3) \quad \frac{d\Psi_d^r}{dt} &= b_1K_s\Psi_d^s - b_1\Psi_d^r - \omega^r\Psi_q^r \\ (4) \quad \frac{d\Psi_q^r}{dt} &= b_1K_s\Psi_q^s - b_1\Psi_q^r + \omega^r\Psi_d^r \\ (5) \quad i_d^s &= c_1\Psi_d^s - c_2\Psi_d^r \\ (6) \quad i_q^s &= c_1\Psi_q^s - c_2\Psi_q^r \\ (7) \quad i_d^r &= c_3\Psi_d^r - c_2\Psi_d^s \\ (8) \quad i_q^r &= c_3\Psi_q^r - c_2\Psi_q^s \end{aligned}$$

where: $\Psi_d^s, \Psi_q^s, \Psi_d^r, \Psi_q^r$ – instantaneous values of the fluxes associated with the phase windings of the equivalent two-phase machine, ω^r – rotor angular velocity, u_d^s, u_q^s – voltages in the d and q axes of the motor stator model, $i_d^s, i_q^s, i_d^r, i_q^r$ – stator and rotor currents in the d and q axes of the motor model.

The coefficients in the above equations are of the form

$$\begin{aligned} (9) \quad K_s &= \frac{M_{sr}}{L_s}, \quad K_r = \frac{M_{sr}}{L_r}, \quad a_1 = \frac{R_s}{\sigma L_s}, \quad b_1 = \frac{R_r}{\sigma L_r} \\ c_1 &= \frac{1}{\sigma L_s}, \quad c_2 = \frac{K_r}{\sigma L_s}, \quad c_3 = \frac{1}{\sigma L_r} \end{aligned}$$

where: M_{sr} – mutual inductance between stator and rotor phase windings, L_s, L_r – self-inductances of the rotor and stator windings in the d - q model of a symmetrical machine, R_s, R_r – resistances of stator and rotor windings in the d - q model, σ – dissipation factor in the d and q axes.

The part of the hybrid motor circuit is a frequency converter. The following are assumed as further state

variables – voltage on the converter capacitor u_c , voltage on the start-up phase capacitor u_{cs} , and current in the filter choke i_f

$$(10) \quad \frac{du_c}{dt} = \frac{G_{Fe2}}{C} u - \frac{G_{Fe1} + G_{Fe2}}{C} u_c + \frac{c_1}{C} (\Psi_d^s + \Psi_q^s) - \frac{c_2}{C} (\Psi_d^r + \Psi_q^r) - \frac{f(\Phi_p)}{z_p C} + \frac{i_f}{C}$$

$$(11) \quad \frac{du_{cs}}{dt} = \frac{c_1}{C^s} \Psi_q^s - \frac{c_2}{C^s} \Psi_q^r$$

$$(12) \quad \frac{d\Phi_p}{dt} = \frac{u_c}{z_p} - \frac{R_p}{z_p^2} f(\Phi_p)$$

$$(13) \quad \frac{di_f}{dt} = \frac{u - u_c}{L}$$

$$(14) \quad u_d^s = u - u_c, \quad u_q^s = u_d^s - u_{cs}$$

where: Φ_p, Ψ – magnetic flux and linkage flux respectively in the frequency converter, C – capacitor for compensation of first harmonic current, C^s – starting phase capacitor force in the frequency converter ($z_p i_p = f(\Phi_p)$), L – filter choke, f – function relating the magnetic flux to the magnetomotive inductance, G_{Fe1}, G_{Fe2} – conductances that represent the power losses in the magnetic circuits of the frequency converter and the motor, respectively.

The electromagnetic torque is expressed as a function of the fluxes associated with the phase windings

$$(15) \quad M_e = \frac{K_r}{\sigma L_s} (\Psi_q^s \Psi_d^r - \Psi_d^s \Psi_q^r)$$

The equation of the mechanical system is of the form:

$$(16) \quad M_e = J \frac{d\omega^r}{dt} + D_\phi \omega^r + M^r$$

where: M^r – externally applied torque, J is the total polar moment of the rotor inertia, D_ϕ – coefficient of friction in rotary motion.

Single-phase motor model in PSpice application

The circuit model of the hybrid motor in the PSpice application convention is shown as two segments in Figures 4 and 5. It includes passive elements R, L, C, ABM functional elements and controlled voltage and current sources. The values of the coefficients appearing in the equations of state (1) - (16) and the parameters in the calculation model were determined from the values of the inductance and resistance of the windings of the converter, motor and rotor. The values of these coefficients are given in Table 1. The frequency converter is represented by elements E1, E7, which are controllable voltage sources whose outputs include signals containing the dominant third harmonic.

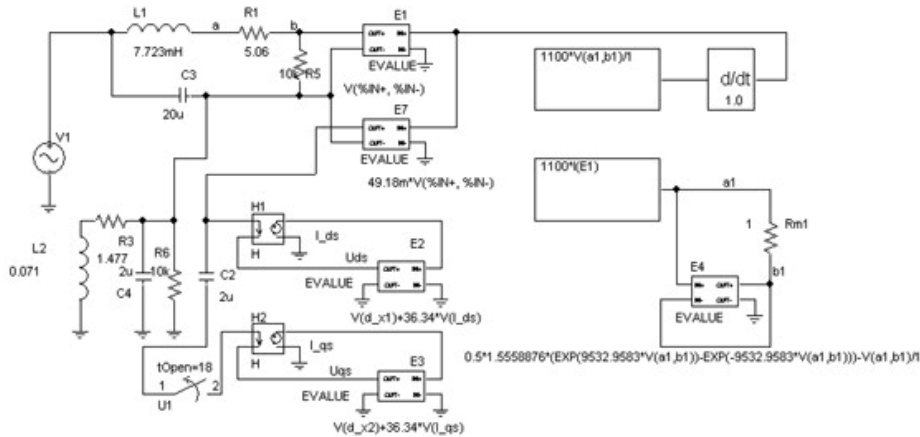


Fig. 4. Single-phase hybrid induction motor model in PSpice, segment 1

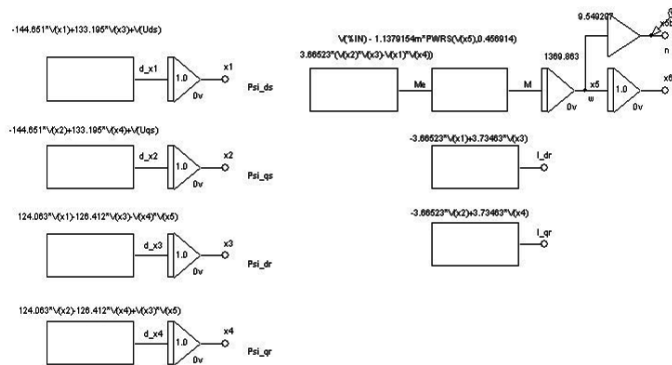


Fig. 5. Single-phase hybrid induction motor model in PSpice, segment 2

Table 1. Values of the coefficients in equations (1)-(11)

a_1	K_r	$a_1 \cdot K_r$	b_1	K_s	$b_1 \cdot K_s$
1/s	-	1/s	1/s	-	1/s
144,651	0,92081	133,195	126,412	0,981419	124,063
c_1		c_2		c_3	
1/H		1/H		1/H	
3,98048		3,66523		3,73463	

The non-linearity of the magnetic core is described by the magnetising characteristic defined by the hyperbolic function $H = \text{ash}(\beta B)$, where $\alpha = 4.32191 \text{ A/m}$, and $\beta = 4.57582 \text{ 1/T}$.

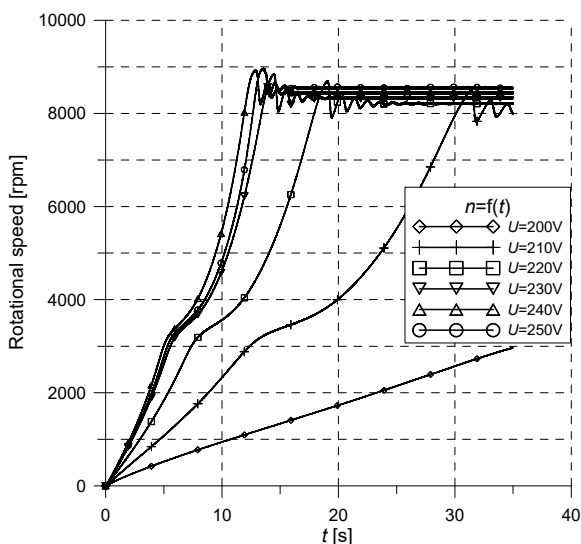


Fig. 6. Characteristics of the speed as a function of the starting time at different values of the supply voltage, $C_r = 2 \mu\text{F}$

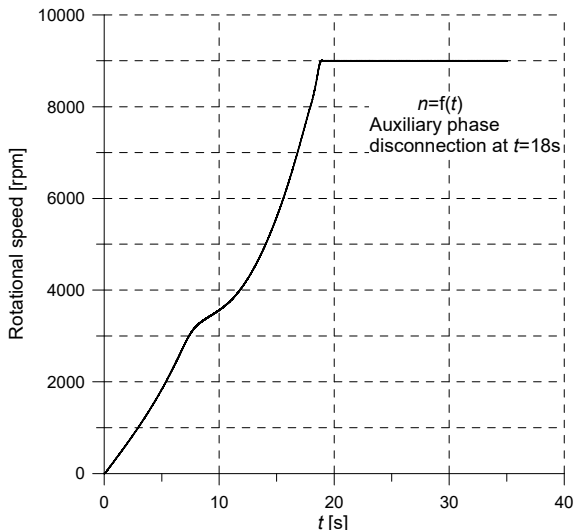


Fig. 7. Characteristics of the speed as a function of the starting time with the auxiliary phase disconnected at time $t = 18 \text{ s}$, $U = 220 \text{ V}$, $C_r = 2 \mu\text{F}$

The resistance of the converter windings and their leakage inductance are represented by elements $L1$, $R1$. The power dissipation in the magnetic core of the converter is represented by resistor $R5$ and in the internal part of the package by resistor $R6$. The motor circuits of the d and q axes are represented by the controlled voltage and current sources $E2$, $E3$ and $H1$, $H2$. A capacitor is connected in a series with the winding in the q -axis, together with a switch to turn off the starting phase. The drag torque, which represents the mechanical losses, is modelled by a

functional element and an appropriate mathematical formula. On the basis of experimental investigations of the physical model, it is assumed that the drag torque is described by relation (17), which has been modified in the calculation model due to the change in the speed unit, and is expressed as follows

$$(17) \quad M_s = 1,2379 \cdot 10^{-3} \omega^{0,4569}$$

where: M_s – drag torque expressed in $\text{N}\cdot\text{m}$, ω – angular velocity expressed in rad/s .

Start-up simulations of the motor model were carried out for different supply voltages and switching times. Selected calculation results are shown in Figures 6 and 7.

Summary

In general, an increase in the supply voltage results in a reduction in the starting time: for a change in voltage from 210 V to 240 V, these times are 32 s and 12.5 s respectively. However, if the voltage is increased to 250 V, the starting time is increased to 13 seconds.

The speed curves $n = f(t)$ show that there is an influence of the flux harmonic of the mains frequency on their shape change at speeds of (3000-5000) rpm. Initially this harmonic causes an increase in speed, but above 3000 rpm it slows down the movement of the rotor due to the change in sign of the torque it produces.

In the final phase of start-up, speed oscillations are damped due to the absence of load and the effects of sub-harmonic (50 Hz) and higher harmonic fluxes in the main and auxiliary windings.

Disconnecting the start-up phase reduces the occurrence of higher harmonics in the fluxes and increases the amplitude of the third harmonic, which helps to limit the speed oscillation and increase its steady-state value.

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