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A mathematical model of an electric drive system including GTO thyristors

Abstract. *A mathematical model of an electrical unit is developed that consists of an absolutely permanent source which supplies electric power to the primary winding of a power transformer, while some asynchronous drives, a resistant load, and a passive power capacitor including GTO thyristors are connected to the secondary winding of the transformer. Electric processes in the load unit are analysed on the basis of the model. A concept of stabilising the unit's voltage with various angles of thyristor opening and closure is demonstrated. Equations describing the drive system are integrated using the fourth-order Runge-Kutta method. The results of computer simulations are shown in a graphic format and analysed.*

Streszczenie. *W pracy opracowano model matematyczny zespołu elektrycznego, który składa się z absolutnie sztywnego źródła energii elektrycznej, z którego zostało zasilane pierwotne uzwojenie transformator mocy, a do wtórnego uzwojenia tego transformatora zostały podłączone napędy asynchroniczne, obciążenie rezystancyjne oraz kompensator mocy biernej z tyrystorami GTO. Na podstawie tego modelu analizowane są procesy elektryczne w układzie napędowym. Pokazano koncepcję stabilizacji napięcia zespołu drogą różnych kątów otwierania i zamykania tyrystorów. Równania stanu eklektycznego całkowane są metodą Runge-Kutta czwartego rzędu. Przedstawiono wyniki symulacji komputerowej w postaci rysunków, które są opisane i analizowane. (**Model matematyczny elektrycznego układu napędowego z uwzględnieniem tyrystorów GTO**).*

Keywords: mathematical modeling, passive power compensation, drive systems, GTO thyristor, Runge-Kutta method.
Słowa kluczowe: modelowanie matematyczne, kompensacja mocy biernej, układy napędowe, tyrystor GTO, metoda Runge-Kutta.

Introduction

The issue of stabilising the voltage of an electric drive unit is among the most pressing problems of applied power electronics. In steady states, voltage is stabilised by compensating passive power, whereas in transient states changing the excitation current of synchronous machines also produces the effect of regulation and thus of stabilising the voltage of a drive system. The background of voltage stabilisation is well-known and its physical sense can be described with the external characteristic curves of a synchronous generator. Under a resistant induction load, voltage across generator armature terminals reduces, thus the voltage supplied to a drive system declines as well. A great majority of a drive system electric elements are dead load: power transformers, asynchronous electric motors, reactors, etc. Under a resistant capacitive load, on the other hand, a drive system's voltage rises. The voltage needs to be stabilised in the event. There's a range of methods of that stabilisation. Including compensation batteries of static capacitors or other elements into a drive system is one of the best known.

An energetic analysis of an electric drive system is presented here using a unique type of a compensation device that consists of two GTO thyristors in parallel (cf. Fig.1). GTO thyristors are semiconductor elements that, like conventional thyristors, open with a control impulse and close with another control signal – closing signal.

Electric engineering theory knows ordinary rectifiers as equipment receiving passive power in a steady state. This is because rectified currents are generally non-periodic functions, which causes a phase shift between current and voltage as they are distributed into Fourier series. The situation changes where GTO thyristors are used. The different methods of controlling them make the system in Fig. 1 a resistant induction or resistant capacitive load. In the case of a resistant induction load (the system receives passive power), the thyristor is turned on with a controlling signal and blocked as the momentary current is equal to zero. In the case of a resistant capacitive load, on the other hand (the system generates passive power), the reverse obtains. The turn-on takes place at zero current and the turn-off is decided by the control signal supplied to the thyristor gate.

The issue of using GTO thyristors is very broad. The literature offers a great number of publications. We will review several briefly.

The need for and effectiveness of bi-operational semiconductor power thyristors (GTO) and transistors (IGBT, MOSFET) as Flexible Alternating Current Transmission Systems (FACTS) for the purpose of quality regulation of power flow in energy transmission and distribution systems in line with the Smart Grid concept are analysed in [1].

[2, 3] discuss the characteristics of voltage converters used in Microgrid and Smart Grid distributed generation systems. The great importance of the particular resources to the regulation of active and passive power flows and the resultant energy efficiency of a grid are emphasised. It's noted the progress in the development of power electronic technology is the fundamental part and determinant of Smart Grids.

The analysis of global literature does suggest the use of GTO thyristors as regulators allows in some cases for replacing the compensation batteries of static capacitors [4, 5, 6].

It is therefore our **objective** to develop a mathematical model of an electric load unit including a power transformer, asynchronous drives, resistant load, and passive power regulator that includes GTO thyristors.

Mathematical model

A drive system is analysed that encompasses a power transformer, asynchronous motors, resistant load, and compensation equipment, which includes GTO thyristors, cf. Fig. 1.

The mathematical model of an electromechanical system is based on the theory of non-linear electromagnetic circuits employing Kirchhoff's first and second laws to describe the structural equations of electrical drive system. The voltage supplied to the drive system is determined on the basis of the mathematical model of electric load unit.

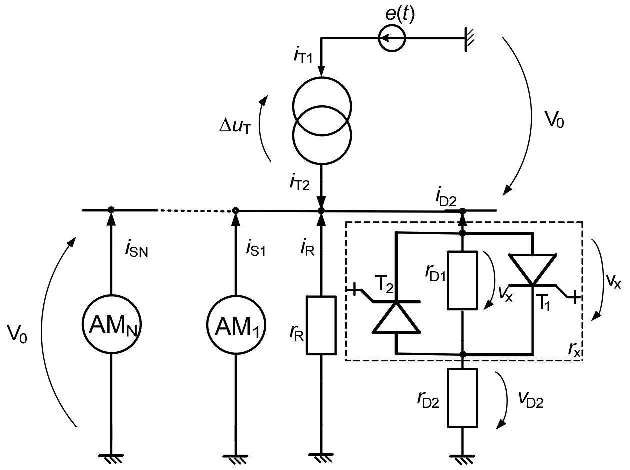

Fig.1. The diagram of the drive system analysed

Based on both Kirchhoff's laws, a mathematical model of an electric load centre (unit) will be constructed [7 – 11] with a compensating device made of two GTO (double function) thyristors in counter-parallel that shunt the resistance element r_{D1}, see Fig. 1.

$$\text{(1)}\qquad \frac{d\mathbf{i}_{T1}}{dt}=\mathbf{A}_{11}(\mathbf{e}-\mathbf{r}_1\mathbf{i}_1)+\mathbf{A}_{12}(\mathbf{V}_0-\mathbf{r}_2\mathbf{i}_2)$$

$$\text{(2)}\qquad \frac{d\mathbf{i}_{T2}}{dt}=\mathbf{A}_{21}(\mathbf{e}-\mathbf{r}_1\mathbf{i}_1)+\mathbf{A}_{22}(\mathbf{V}_0-\mathbf{r}_2\mathbf{i}_2)$$

$$\text{(3)}\qquad \frac{d\mathbf{i}_{S,k}}{dt}=\mathbf{A}_{S,k}(\mathbf{V}_0-\mathbf{r}_{S,k}\mathbf{i}_{S,k})+\mathbf{A}_{SR,k}(-\mathbf{\Omega}_k\mathbf{\Psi}_{R,k}-\mathbf{r}_{R,k}\mathbf{i}_{R,k})$$

$$\text{(4)}\qquad \frac{d\mathbf{i}_{R,k}}{dt}=\mathbf{A}_{RS,k}(\mathbf{V}_0-\mathbf{r}_{S,k}\mathbf{i}_{S,k})+$$
$$+\mathbf{A}_{R,k}(-\mathbf{\Omega}_k\mathbf{\Psi}_{R,k}-\mathbf{r}_{RL,k}\mathbf{i}_{R,k})+\mathbf{\Omega}_k\mathbf{i}_{R,k}$$

$$\text{(5)}\qquad \mathbf{\Pi}=\frac{2}{\sqrt{3}}\left|\begin{array}{|c|c|}\hline \sin(\gamma+120°) & -\sin\gamma \\ \hline \sin\gamma & -\sin(\gamma-120°) \\ \hline\end{array}\right|,\ \mathbf{\Omega}_k=\mathbf{\Pi}_k^{-1}\frac{d\mathbf{\Pi}_k}{dt}$$

$$\text{(6)}\qquad \mathbf{\Psi}_{R,k}=\mathbf{L}_{\sigma R,k}\mathbf{i}_{R,k}+\mathbf{\psi}_{R,k},\ \ \mathbf{\psi}_{R,k}=L_{m,k}(\mathbf{\Pi}_k^{-1}\mathbf{i}_{S,k}+\mathbf{i}_{R,k})$$

$$\text{(7)}\qquad \mathbf{V}_0=-\mathbf{r}_R\mathbf{i}_R$$

$$\text{(8)}\qquad \mathbf{V}_0=-\mathbf{v}_x-\mathbf{r}_{D2}\mathbf{i}_{D2}$$

$$\text{(9)}\qquad \mathbf{i}_{T2}+\sum_{k=1}^{N}\mathbf{i}_{S,k}+\mathbf{i}_R+\mathbf{i}_{D2}=0\,,$$

where V_0 – the voltage of electric load unit, e – SEM of the unit's power supply, $\mathbf{i}_{T1}$, $\mathbf{i}_{T2}$ – currents across the primary and secondary transformer windings, $\mathbf{r}_1$, $\mathbf{r}_2$ – the resistance of transformer windings, $\mathbf{i}_{S,k}$, $\mathbf{i}_{R,k}$ – the currents of asynchronous motor stator and rotor, $\mathbf{r}_{S,k}$, $\mathbf{r}_{R,k}$, – the resistance of motor stator and rotor windings, $\mathbf{\Psi}_{R,k}$ – full coupled rotor fluxes, $\mathbf{L}_{\sigma R,k}$, $L_{m,k}$ – the inductances of motor rotor dispersion and magnetisation, $\mathbf{\Pi}_k$ – oblique transformation matrix, Ω_k – the matrix of transformed rotor rotational speeds, $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$, $\mathbf{A}_{22}$ – coefficients dependent on the inductances of power transformer dispersion and magnetisation, $\mathbf{A}_{S,k}$, $\mathbf{A}_{SR,k}$, $\mathbf{A}_{RS,k}$, $\mathbf{A}_{R,k}$, – the same for asynchronous motors, k – the number of asynchronous drives.

The electromagnetic moments of asynchronous motors will be computed traditionally [7, 9]:

$$\text{(10)}\qquad M_{E,k}=\sqrt{3}p_{0,k}L_{m,k}(i_{SB,k}i_{RA,k}^{\Pi}-i_{SA,k}i_{RB,k}^{\Pi})\,,$$

where $p_{0,k}$ – the number of pole pairs of asynchronous motors, $^{\Pi}$ – the power indicates a transformed system of oblique coordinates.

A compensating device consisting of two dual operation GTO thyristors T_1 and T_2, in parallel as a bridge, see Fig. 1, and a shunting resistor r_{D1} are important parts of the electric load unit. To limit the current i_{D2} across the compensation branch, resistance r_{D2} is in series to the thyristor capacitor. As the design of GTO thyristors allows for opening and closing across a quite broad range, the maximum power compensation will serve as an example, namely, a control angle of 90 electric degrees. Two cases (experiments) are addressed here. In the first one, the opening and operation angles of thyristors T_1 $\alpha\epsilon(90^0;\ 180^0)$ and T_2 $\alpha\epsilon(270^0;\ 360^0)$. GTO works as an ordinary thyristor, therefore. In the second experiment, the opening and operation angles of T_1 $\alpha\epsilon(0^0;\ 90^0)$ and of T_2 $\alpha\epsilon(180^0;\ 270^0)$. Thus, GTO worked in the closing state.

To visualise the work process of both the thyristors in Figures 2a and 2b, a time diagram of their operation will be shown [12]. The integrated compensation device can then be treated as a classic non-linear element (r_x), cf. Fig. 1. It's presented for phase A here only, for instance.

1. Experiment one: $\alpha_{T1}\epsilon(90^0;\ 180^0)$, $\alpha_{T2}\epsilon(270^0;\ 360^0)$.

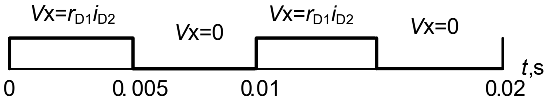

Fig.2a. Time diagram of the operation of non-linear element r_x for the angle of thyristor release $\alpha\epsilon(0^0;\ 90^0)$

2. Experiment two: $\alpha_{T1}\epsilon(0^0;\ 90^0)$, $\alpha_{T2}\epsilon(180^0;\ 270^0)$.

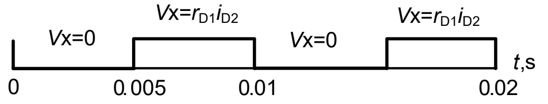

Fig.2b. Time diagram of the operation of non-linear element r_x for the angle of thyristor release $\alpha\epsilon(0^0;\ 90^0)$

The traditional formula is utilised to calculate the rms value of the voltage function for the period 0.02 s

$$\text{(11)}\qquad V_{0SK}=\frac{1}{0,02}\sqrt{\int_{t_0}^{t_0+0,02}V_0(\tau)d\tau}\,,$$

where V_{0SK} — the rms value of the voltage function, t_0 – the time of entry in the unit's steady state.

Note the voltage function cannot be treated as a root mean square value as the latter only applies to harmonic processes. In thyristor systems, the harmonicity of voltage, current, power, and other functions is moving. It can only be applied for the first harmonic of Fourier series here.

Computer simulation results

A major idea of using dual operation thyristors in electric load units involves passive power compensation [12, 13]. Note this concept applies exclusively to steady processes and is of a purely calculation nature. It a priori makes no sense in transient processes. It can be concluded then passive power compensation in its physical sense consists in stabilising the voltage of an electric load unit [7]. Since it relates to passive power compensation, to visualise the process of voltage stabilisation, the mathematical models of transformer and asynchronous motors can well be presented as resistance induction loads. Calculating the stabilisation of a load unit voltage in transient states is our question. It addresses Runge-Kutta and Simson methods [7].

Simulations are used to simplify the calculations for the resistance induction load and the compensation device, based on the use of GTO thyristors, which as an initial approximation can serve as prototype electric load units, Fig. 1 [12]. These are the electric system's parameters: $e(t)=310\sin(314t)$, $r_T=0.5\ \Omega$, $r_{S1}=1\ \Omega$, $r_{S2}=1\ \Omega$, $r_{D2}=0.5\ \Omega$, $r_{D1}=0.5\ \Omega$, $r_x=0.5\ \Omega$, $r_R=0\ \Omega$, $L_{T1}=0.001$ H, $L_{S1}=0.01$ H, $L_{S2}=0.01$ H. Given these parameters, the integration time of the system's differential state equations (1) – (9) is quite short, which enables entry into the steady state during fewer than five periods (0.01 s). This means a steady process is fully established in a centre within the time range (0.08; 0.1).

The computer process simulation continued for two stages, beginning with the calculation of unit voltage, cf. (11), t_0=0.08 s. Two practical experiments are conducted. Experiment one: GTO worked as ordinary thyristors as per the time operation diagram in Fig. 2A; experiment two: GTO thyristors operated in the signal closing state as per the time operation diagram in Fig. 2b.

Experiment one. Figure 3 contains an operation graph of the non-linear element r_x. The time diagram of both the GTO thyristors is depicted in Figure 2a. The thyristors work in the ordinary state that is, supplying a signal to the control electrodes causes the lights to switch on, whereas the switch-off (closure) is executed physically, by the current function passing across zero.

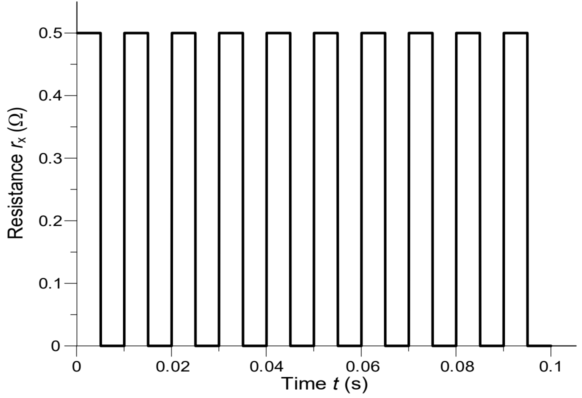

Fig.3. The time operation diagram of the non-linear element r_x (experiment one)

The momentary voltage of the electric load unit is shown in Figure 4. Ultimately, the Figure should be analysed together with Fig. 3. For instance, the first thyristor is closed in the time $t \in (0;\ 0.005)$ s and the voltage drop across the non-linear element r_x is not equal to zero, cf. 2a. During the time $t \in (0.005;\ 0.01)$ s, on the other hand, the thyristor is open and shunts r_x; in other words, the current flows across the thyristor. This means the voltage drop across the non-linear element r_x is equal to zero. The time operation diagram for the second thyristor is similar.

Experiment two. Figure 5 presents a dynamic operation diagram of r_x. The time operation diagram of both the GTO thyristors is included in Figure 2b. The thyristors work in a state where closing the control signal, that is, supplying a signal to the control electrodes, usually causes the thyristors to switch on, whereas the switch-off (closure) is executed by supplying the control signal. This means the device works the other way round.

The momentary voltage of the electric load unit in the second experiment is presented in Figure 6. Ultimately, the Figure should be analysed together with Fig. 5. For instance, the first thyristor is open in the time $t \in (0;\ 0.005)$ s and shunts r_x. while the voltage drop across it is zero, cf. Fig 2b. During the time $t \in (0.005;\ 0.01)$ s, on the other hand, the thyristor is closed. This means the voltage drop across the non-linear element r_x is not equal to zero. The time operation diagram for the second thyristor is similar too.

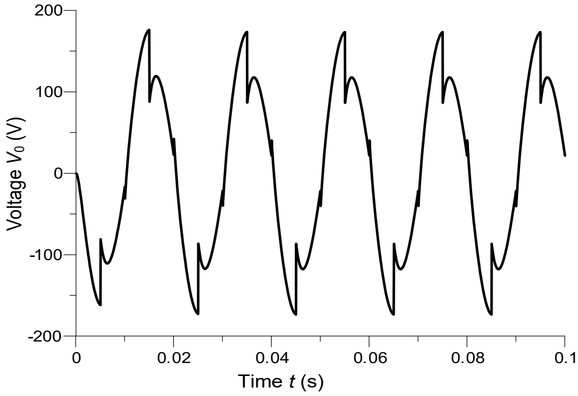

Fig.4. The momentary voltage of the electric load unit (experiment one)

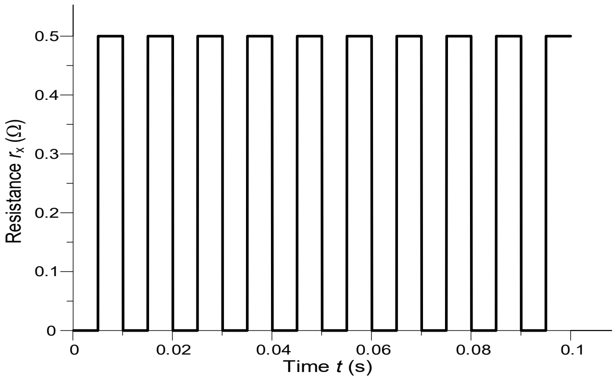

Fig.5. The time operation diagram of the non-linear element r_x (experiment two)

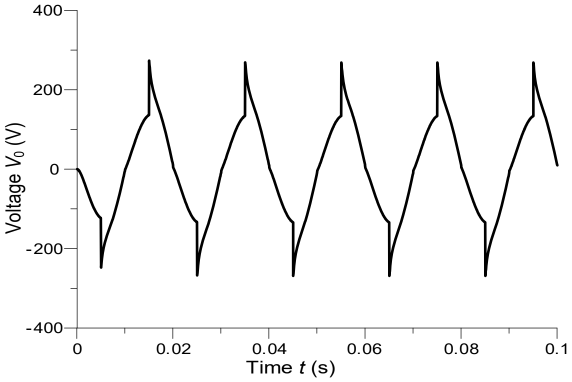

Fig.6. The momentary voltage of the electric load unit (experiment two)

An analysis of Figures 4 and 6 implies a highly non-linear course of oscillatory dynamic processes in the electric load unit [14]. This has already been mentioned. Such processes do not allow for the notions of rms values of voltage or current (this is only possible for the first harmonic of Fourier series). The periodic unit voltages are calculated according to (11), therefore, or as root mean squares. Their values are given below. *Experiment one* – V_{0SK} = 106.5 V. *Experiment two* – 119.3 V. The difference between the two experiments is significant, which fully corroborates the compensation effect of GTO thyristors

Conclusion

1. The mathematical modelling of electric processes in electric load systems helps to optimise the complicated processes of electromechanical energy conversion. It's also applied to the stabilisation of a unit's voltage. In the steady state, this applies to the passive power compensation. In non-linear systems, the rms values of voltages and currents are computed for the first harmonic of function distribution into Fourier series.
2. When semiconductor elements like thyristors are utilised in electric load units and key facilities, GTO-type elements should be used as target. The control effect remains virtually unchanged compared to ordinary thyristors, whereas the effect of passive power compensation (steady state) will emerge in load units).
3. In some cases, control equipment based on GTO thyristors can successfully replace compensation equipment like static capacitor batteries, synchronous drives, mechanical compensators, and the like.

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