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Convex optimization model for Network Reconfiguration of Smart Grids

***Abstract**. This study proposed a smart grid reconfiguration strategy that takes technical aspects into account. Convex optimization is used to answer the strategy. We find original quadratically constrained and second-order cone approximations to power flow in radial networks during the derivation of each model. Using standard commercial software, the proposed formulation guarantees global optimality with reliable and efficient outcomes. We use IEEE 33 and add DGs to model active distribution systems to evaluate the proposed method. The simulation findings show that the proposed method is capable of solving reconfiguration efficiently.*

***Streszczenie**. W badaniu tym zaproponowano strategię rekonfiguracji inteligentnej sieci, która uwzględnia aspekty techniczne. Optymalizacja wypukła służy do odpowiedzi na strategię. Znajdujemy oryginalne kwadratowe ograniczenia i przybliżenia stożka drugiego rzędu do przepływu mocy w sieciach promieniowych podczas wyprowadzania każdego modelu. Przy użyciu standardowego oprogramowania komercyjnego proponowana formuła gwarantuje globalną optymalizację z niezawodnymi i wydajnymi wynikami. Używamy IEEE 33 i dodajemy DG do modelowania aktywnych systemów dystrybucji w celu oceny proponowanej metody. Wyniki symulacji pokazują, że proponowana metoda jest w stanie skutecznie rozwiązać problem rekonfiguracji. (**Model optymalizacji wypukłej dla rekonfiguracji sieci inteligentnych sieci**)*

Keywords: Network Reconfiguration, Convex Optimization, Second Order Cone Programming, and Smart Grids.
Słowa kluczowe: Rekonfiguracja sieci, optymalizacja wypukła, programowanie stożkowe drugiego rzędu i inteligentne sieci

Introduction

Power distribution reconfiguration is an essential component of modern power system engineering because it supports the efficient and effective delivery of electrical power to end users. The technique involves changing the configuration of electrical power distribution systems by selecting open or closed switch combinations that enhance specific performance criteria while maintaining a radial network topology [1], [2]. Branch exchange procedures had been employed for handling reconfiguration [1]. The main goal of power distribution reconfiguration is to increase the efficiency and reliability of the power system. Owaifeer et al. [3] divided reconfiguration optimization into three categories: heuristics algorithms, soft computing (SC), mathematical programming, and mathematical programming.

Distribution network reconfiguration can be carried out using a heuristic algorithm, as demonstrated in studies conducted by [4]-[6]. In these studies, the selection of switches is used to determine the best configuration by opening all the switches, then closing the switches one by one and calculating the objective function. As a result, the best configuration is selected based on the best objective function. This method is simple and does not require complicated computations. However, extensive computational processing is still necessary because the load flow needs to be calculated at every switching step.

Soft computing techniques are widely used in power systems, with meta-heuristic optimization methods being particularly popular for network optimization [7]-[13]. One advantage of meta-heuristic optimization is that it can provide a global optimum multi-objective solution while searching for the best local solution in each iteration. However, when applied to power systems with numerous constraints that must be met, achieving global optimality is not always possible, and extensive computational time may be required.

Mathematical Programming (MP) was rarely used to handle reconfiguration problems prior to the development of sophisticated solvers and high-speed processors. This is due to the fact that finishing reconfiguration optimization with MP takes more computational time than heuristic and soft computing methods. Nonetheless, MP has a substantial advantage over a direct method to finding the best solution. A direct method to the reconfiguration issue entails defining the objective function's mathematical equation, power flow, and constraints. The MP technique used is determined by the objective function. In [14], for example, mixed-integer programming (MIP) was used to determine the minimum blackout and maximum power values for the de-energized region while accounting for losses and reactive power.

Cavalcante et al. used two kinds of MP algorithms in [15]: one used Mixed-Integer Linear Programming (MILP) to find the optimal configuration value, and the other used nonlinear programming (NLP) to find the optimal load shedding value. [3] used MILP in 2017 to determine the minimum value of reconfiguration costs while accounting for reactive power and losses but not as an objective function. However, radiality constraints are imposed in distribution network optimization issues, which are nonlinear problems [16]. R.A. Jabr [17] used mixed-integer nonlinear programming to solve this issue. Some recent research has successfully converted the distribution network reconfiguration problem, which was initially a MINLP, into a convex optimization [18]-[19]. [20] effectively modeled expansion planning using mixed-integer quadratically-constrained programming. They suggests using mixed-integer second-order cone programming (MISOCP) to solve the mathematical model formulation quickly for an optimal solution.

This research concentrates on radial network reconfiguration to optimize performance using MISOCP, which ensures global optimality. The models can be easily solved and simulated using Python and generally available, powerful solver software.

The remainder of this paper is organized as follows. We formulate in Section **Model and Problem Formulation** to solve the Distribution Network Reconfiguration problem. We simulate and analyse in Section **Simulation and Result**. We conclude in Section **Conclusions**.

Model and Problem Formulation

This formulation assumes that the network is a balanced three phases system. Figure 1 illustrates the line flow of a network.

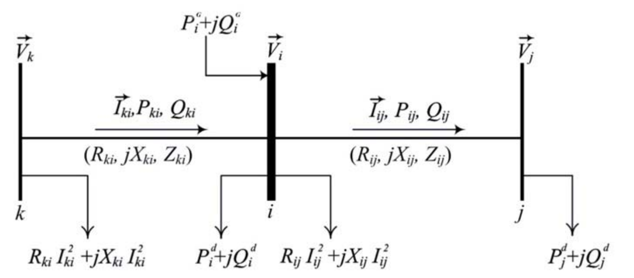

Figure 1. Illustration of line flows in a network

The reconfiguration model is presented in this section. Two parts will be modeled mathematically. The objective function and constraints will be discussed.

1. Objective Function

The objective function is given by Equation (1). That is minimum losses.

(1) $\mathrm{Min}\ \sum_{i=0}^{n} P_{lossses}\ \mathrm{Min}$

2. Constraints

The active power balance constraint is given by Equation (2). The active electricity flowing into bus *i* is represented by P_{ki}. P_{ji}, on the other hand, indicates the active power flowing out of bus *i*. P_i^G represents active power production at bus *i*. The resistance multiplied by the current squared $(R_{ij} I_{ij}^{sqr})$ is also used to calculate operational losses.

(2) $\sum_{ki\in\Omega_l} P_{ki} - \sum_{ij\in\Omega_l}\left(P_{ij} + R_{ij} I_{ij}^{sqr}\right) + P_i^G = P_i^D\,(-y_i) \quad \forall i \in \Omega_b$

The reactive power balance constraint is given by equation (3). Q_{ki} denotes the reactive power flows that become bus *i*. Q_{ji}, on the other hand, depicts the reactive power flows from bus *i*. Q_i^G represents the active power production at bus *i*. Reactive losses are determined by multiplying reactance by current squared $(X_{ij} I_{ij}^{sqr})$.

(3) $\sum_{ki\in\Omega_l} Q_{ki} - \sum_{ij\in\Omega_l}\left(Q_{ij} + X_{ij} I_{ij}^{sqr}\right) + Q_i^G = Q_i^D\,(1 - y_i) \quad \forall i \in \Omega_b$

The voltage drops are given by Equation (4). b_{ij} is an auxiliary variable that takes different values when the circuit *ij* is open or closed.

(4) $V_i^{sqr} - V_j^{sqr} = 2\left(P_{ij} R_{ij} + Q_{ij} X_{ij}\right) + Z_{ij}^2\, I_{ij}^{sqr} + b_{ij} \quad \forall i \in \Omega_b$

For each line, equation (5) reflects Kirchhoff's second law (KVL). In (5), the initial relationship is transformed into the nonlinear model-satisfying second order conic constraint. Those are accurate according to [20], considering that the objective function is linear and convex. Aside from that, the network graph is connected, and the **issue is feasible.**

(5) $V_j^{sqr}\, I_{ij}^{sqr} \ge P_{ij}^2 + Q_{ij}^2 \quad \forall ij \in \Omega_l$

The operational limit of voltage is given in equation (6) and, the current limit is given in equation (7).

(6) $\underline{V}^2 \le V_i^{sqr} \le \bar{V}^2 \quad \forall i \in \Omega_b$

(7) $|I_{ij}^{sqr}| \le I_{ij}^2 x_{ij} \quad \forall ij \in \Omega_l$

When some parts of the network cannot be recovered, take care of the topology radial. Assumptions are made by employing fake substations at nodes, artificial nodes, and artificial branches, resulting in an unreal power flow as in constraint. (8).

The fictitious power source restriction is present in constraints (9)-(12). The unreal substation flow generates fictitious electricity. (9). As a result, constraint (10) is a fictitious power transfer restriction. Constraints (11) and (12) require that no active or reactive electricity be present at a fictitious substation.

(8) $\sum_{ki\in\Omega_l\cup\Omega_h} H_{ki} - \sum_{ki\in\Omega_l\cup\Omega_h} H_{ij} + H_i^G = y_i \quad \forall i \in \Omega_b$

(9) $H_i^G = 0 \quad \forall i \in \Omega_b\,, i \ne S^f$

(10) $|H_{ij}| \le M\, x_{ij} \quad \forall ij \in \Omega_l \cup \Omega_h$

11) $P_{S^f}^G = 0$

(12) $Q_{S^f}^G = 0$

To improve the solution process, additional constraints have been introduced. Limitation (13) is required to accelerate the optimization procedure. This is accomplished by attaching the load node to a single circuit. Constraint (14) states that when no branch is closed, the active power in one branch must be zero, and its value must be less than the highest apparent power in the branch. The same is true for reactive strength. (15). Constraint (16) states that if the circuit is operational, the values y_i and y_j must have the same value. Constraints (17) and (18) reflect the decision variables' binary nature. Constraints (19) ensure feasibility (4), and the circuit must work in accordance with Kirchhoff's second law. If this is not the case, an arbitrary number must be used to satisfy.

(13) $\sum_{ij\in\Omega_l\cup\Omega_h} x_{ij} + \sum_{ki\in\Omega_l\cup\Omega_h} x_{ki} \ge 1 \quad \forall i \in \Omega_b$

(14) $|P_{ij}| \le \bar{V}\bar{I}_{ij} x_{ij} \quad \forall ij \in \Omega_l$

(15) $|Q_{ij}| \le \bar{V}\bar{I}_{ij} x_{ij} \quad \forall ij \in \Omega_l$

(16) $|y_i - y_j| \le (1 - x_{ij}) \quad \forall ij \in \Omega_l$

(17) $x_{ij} \in \{0,1\} \quad \forall ij \in \Omega_l \cup \Omega_h$

(18) $y_i \in \{0,1\} \quad \forall i \in \Omega_b$

(19) $|b_{ij}| \le (\bar{V}^2 - \underline{V}^2)(I - x_{ij}) \quad \forall ij \in \Omega_l$

Simulation and Result

This section presents case studies on modified IEEE 33-bus distribution networks. The computational tasks were performed on a personal computer with an Intel Core i5 Processor (2.70 GHz) and 8-GB RAM, and the code was implemented using Python and solved via the GUROBI solver.

The following criteria are established: The substation voltage was set to 1 pu, with minimum and highest voltages of 0.95 pu and 1.05 pu, respectively. There is a 4.00 pu maximal current flow. To compare the voltage obtained from the optimization result with the loadflow in order to verify that the proposed technique is accurate. The mean square error (MSE) (20) was used to evaluate validity accuracy. magnitudes of voltage reconfiguration were compared with AC power flow. The results are very similar and the MSE value of reconfiguration for the 33-bus test system is 1.2e-11.

(20) $\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(\hat{x_i} - x_i)^2$

The IEEE-33 bus radial distribution scheme is used in this test system. The load is made up of a substation, 33 buses, and 37 branches, according to the data from reference [21]. On buses 24, 16, and 31, three DGs have been added as modifications. 32 sectionalizing switches are represented by straight lines in Fig. 2, and five tie switches are marked by dashed lines. With a total active capacity of 3715 kW and a total reactive power load of 2300 kVar, the voltage and MVA bases are 12.66 kV and 1 MVA, respectively. Each DG has a 300 kVA rating.

Figure 3 shows the VP results for each bus before and after NR under the assumption of a typical load scenario. The lowest voltage level is 0.950 p.u. before and after the network change, correspondingly. According to the findings, the real power loss is reduced by 69.68 kW when compared to the base case determine of 83.25 kW. This means that 13.56 kW of real electricity can be saved when compared to the current network topology. When compared to total power loss at normal load, the percentage decrease is 44.42%. The MISOCP method closed all tie switches while leaving the sectionalizing switches S6-7, S8-14, S10-11, S27-28, and S31-32 open. The optimization calculation took 29 seconds in total. The results are summarized in Table 1.

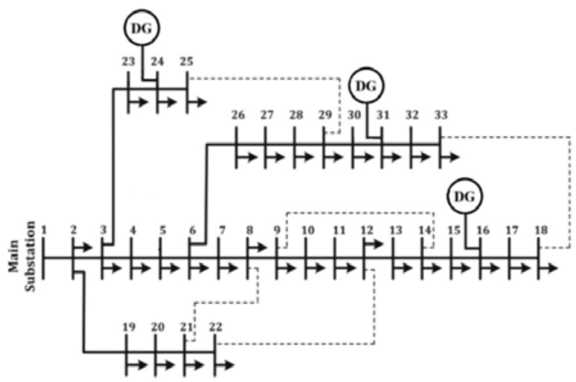

Fig.2. Modified IEEE *33-bus test system*

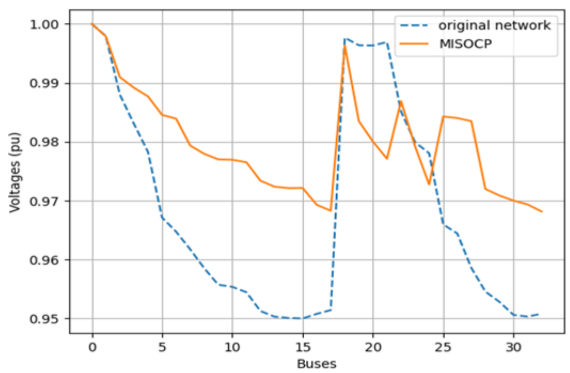

Fig.3. Voltage Profile before and after network reconfiguration for 33-bus test system

Table 1. Result Comparison of 33-bus system

	Original Network	Optimization using MISOCP
Opened Switch	$S_{1\text{-}43}$, $S_{13\text{-}21}$, $S_{15\text{-}46}$, $S_{50\text{-}59}$, $S_{27\text{-}65}$	$S_{6\text{-}7}$, $S_{8\text{-}14}$, $S_{10\text{-}11}$, $S_{27\text{-}28}$, $S_{31\text{-}32}$
Losses kW	83.25 kW	69.68 kW
Losses kVar	54.36 kVar	54.5 kVar
Min Voltage	0.95 pu	0.968 pu
Max Voltage	1 pu	1 pu
Time computing	12 s	29 s
Code	Python	Python
Solver	-	GUROBI

Conclusions

We proposed MISOCP-based algorithms to reduce losses while increasing efficiency and accuracy in feeder reconfiguration. Our algorithm has been shown to handle network reconfiguration issues optimally. Furthermore, we showed the efficacy of our algorithms using simulations on a 33-bus test system. Several constraints are included in the optimization method, and the fundamental studies conducted include load flow analysis and voltage profile improvement. According to the results, our suggested algorithm is efficient in terms of global optimality and computation time.

Nomenclature

Indices and sets

Ω_b	Sets of all bus
Ω_h	Sets of all fictitious line
Ω_l	Sets of all line

Variables

H_{ki}	The artificial power flow in line *ki*
H_i^G	The artificial generation at node *i*
I_{ij}^{sqr}	The square of the current flow magnitude in line *ij*
P_{ij}	The active power flows in line *ij*
P_i^G	The active power generation in node *i*
Q_{ij}	The reactive power flows in line *ij*
Q_i^G	The reactive power generation in node *i*
V_i	The voltage in node *i*
b_{ij}	The binary variable, the value is opposite of the value of x_{ij}
x_{ij}	The binary variable, the value = 1 if the circuit *ij* is closed and 0 if the circuit is open
y_i	The binary variable, the value = 1 if the demands at node *i* are not supplied. and 0 if the demand fully met.

Prameters

M	A large multipliers
P_i^D	The active power demand in node *i*
Q_i^D	The reactive power demand in node *i*
R_{ij}	The resistance in line *ij*
X_{ij}	The reactance in line *ij*
Z_{ij}	The impedance in line *ij*
$\bar{I}_{ij}$	The upper bound of current
$\bar{V}$	The upper bound of voltage
$\underline{V}$	The lower bound of current
$\$LS$	The cost of load shedding
$\$Gen$	The cost of generation
$\$Cur$	The cost of load curtailment

Acronym

DG	Distributed Generation
KVL	Kirchhoff's Voltage Law
MILP	Mixed-Integer Linear Programming
MINLP	Mixed-Integer Non-Linear Programming
MISOCP	Mixed-Integer Second Order Cone Programming
MP	Mathematical Programming
MSE	Mean Square Error
NLP	Non Linear Programming
SC	Soft Computing
VP	Voltage Profile

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