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doi:10.15199/48.2022.09.24

Six-phase Two-level VSI Control based on Polar Voltage Space Vectors

***Abstract**. The paper recommends polar voltage space vectors of the six-phase two-level inverter as a useful mathematical tool for vector control of the inverter. The inverter model is described using two mathematical tools: voltage state vectors and voltage space vectors. The polar voltage space vectors are used for inverter control. They are defined using the standard voltage space vector transformation and are determined by the usage of the same binary digits as the numbers defining state vectors. The simulation experiment described in this paper shows the results of the assumed control strategy and its advantages compared to the PWM method.*

***Streszczenie.** W artykule przedstawiono wektory przestrzenne napięć biegunowych dwupoziomowego falownika 6-fazowego jako użyteczne narzędzie matematyczne do sterowania takim falownikiem. Model falownika jest opisany za pomocą dwóch narzędzi matematycznych, którymi są wektory stanu i wektory przestrzenne napięć. Wektory stanu są opisane przez sześciocyfrowe liczby binarne i pozwalają określić napięcia fazowe wszystkich stanów falownika. Wektory przestrzenne napięć są wynikiem standardowej transformacji i są opisane tymi samymi cyframi jak odpowiadające im wektory stanu. Wyniki badań symulacyjnych przyjętej metody sterowania zostały zaprezentowane i porównane z wynikami sterowania według metody PWM. (**Sterowanie sześciofazowym dwupoziomowym VSI oparte na biegunowych wektorach przestrzennych napięcia**).*

Słowa kluczowe: dwupoziomowy falownik 6-fazowy; wektor stanu, wektor przestrzenny; sterowanie falownika; napięcie biegunowe.
Keywords: two-level six-phase inverter; state vector, space vector; inverter control; polar voltage.

Introduction

Due to the continuous development of electrical engineering, there has been an expansion of new solutions in drives control systems, among which the topic of high interest is multi-phase converters. Their varied topologies, as well as control strategies, are the subjects of many research papers, for example [1-7]. Multiphase inverters provide high reliability, limited DC-link voltage ripples, increased power density, fault tolerance and are distinguished by their redundant structure [5]. The multiphase inverter input current ripple can be reduced by increasing the phase number which is observed significantly when the number of phases is greater than five but not more than nine [8]. The inverter's possible switching states increase exponentially and amounts to 2^n, which means that a two-level six-phase voltage source inverter (VSI) is indicated by 64 switching state combinations. However, along with the increase in phase number, the switching control strategies become more advanced and complex.

Many researchers' works focus on both various PWM-based modulation methods used in multi-phase VSI [9-10] and vector control techniques [11-13, 14-15]. The paper presents the effectiveness of the usage of polar voltage space vectors in controlling six phase 2-level inverters, which has been proved by the conducted simulation tests. Achieved results, as well as proposed analytic expressions, allow concluding that the main idea of the proposed system can be expanded and adopted to n-phase and n-level inverters in an easy way.

State and Space Vectors of a Six-Phase Two-Level Voltage Source Inverter

Figure 1 depicts a simplified diagram of the six-phase two-level converter. The proposed model consists of six two-state switches denoted as K_a, K_b, K_c, K_d, K_e, K_f that are connected to the U_D voltage source and associated with the corresponding phases.

According to the model in Figure 1, the six-phase two-level voltage source inverter (VSI) has 64 diverse switching states, denoted by decimal index k = 0, 1, ... 63 respectively. Particular k-states determine the six phase-to-phase voltages that appear at the output of the inverter. The 6-phase VSI state vector is defined as a matrix row composed of six elements:

$$(1)\quad V_k = \left[U_{abk}\; U_{bck}\; U_{cdk}\; U_{dek}\; U_{efk}\; U_{fak}\right]$$

where k = 0, 1, 2, …, 63.

Assuming $U_{abk}+U_{bck}+U_{cdk}+U_{dek}+U_{efk}+U_{fak} = 0$ the state vector V_k is determined by a set of five U_{xy} phase-to-phase voltages under the condition that *x* and *y* denote the neighbouring phases: *x≠y* and *x,y=a,b,c,d,e,f*.

The phase-to-phase voltage is formed by a difference between the particular phase outputs' potentials. The switch $K_{a,b,c,d,e,f}$ attaches the related phase output to one pole of the voltage source U_d. The state of the $K_{a,b,c,d,e,f}$ is determined by the selected vector V_k.

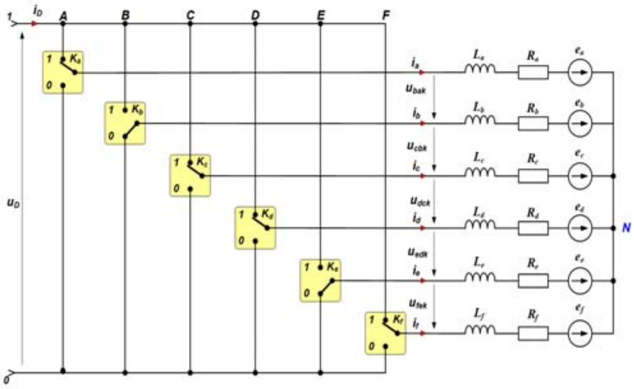

Figure 1. The model of the 6-phase 2-level inverter with the star connected load

The decimal vector index *k* can be reconverted to the binary number system and written as a binary number

$$(2)\qquad k = (a_k, b_k, c_k, d_k, e_k, f_k)_2$$

where: a_k,b_k,c_k,d_k,e_k,f_k=0,1.
The phase output potentials related to the negative pole of the voltage source U_D are defined using the relevant binary symbol. So, the phase output polar voltages are:

$$(3)\quad \begin{bmatrix} u_{a0_k} = a_k U_D \\ u_{b0_k} = b_k U_D \\ u_{c0_k} = c_k U_D \\ u_{d0_k} = d_k U_D \\ u_{e0_k} = e_k U_D \\ u_{f0_k} = f_k U_D \end{bmatrix}$$

It allows defining six-phase two-level VSI state vector using the binary symbols of the decimal index k. The transpose of the matrix V_k allows presenting in one column all six phase-to-phase voltages:

$$(4)\quad V_k = \begin{bmatrix} u_{ab_k} \\ u_{bc_k} \\ u_{cd_k} \\ u_{de_k} \\ u_{ef_k} \\ u_{fa_k} \end{bmatrix} = \begin{bmatrix} u_{a0_k} - u_{b0_k} \\ u_{b0_k} - u_{c0_k} \\ u_{c0_k} - u_{d0_k} \\ u_{d0_k} - u_{e0_k} \\ u_{e0_k} - u_{f0_k} \\ u_{f0_k} - u_{a0_k} \end{bmatrix} = U_D \begin{bmatrix} (a_k - b_k)_2 \\ (b_k - c_k)_2 \\ (c_k - d_k)_2 \\ (d_k - e_k)_2 \\ (e_k - f_k)_2 \\ (f_k - a_k)_2 \end{bmatrix}$$

The symbols $a_k, b_k, c_k, d_k, e_k, f_k$ may be equal only to 0 or 1, so any phase-to-phase voltage may assume one of these voltages: $0, +U_D, -U_D$.

In the considered load circuit, the phase voltage can be evaluated by the binary symbols of the decimal vector index k in the way presented in [16]. The phase voltages: $u_{ak}, u_{bk}, u_{ck}, u_{dk}, u_{ek}, u_{fk}$ of the two-level six-phase VSI are:

$$(5)\quad \begin{bmatrix} u_{a_k} \\ u_{b_k} \\ u_{c_k} \\ u_{d_k} \\ u_{e_k} \\ u_{f_k} \end{bmatrix} = \frac{U_D}{6} \begin{bmatrix} 5a_k - b_k - c_k - d_k - e_k - f_k \\ 5b_k - a_k - c_k - d_k - e_k - f_k \\ 5c_k - a_k - b_k - d_k - e_k - f_k \\ 5d_k - a_k - b_k - c_k - e_k - f_k \\ 5e_k - a_k - b_k - c_k - d_k - f_k \\ 5f_k - a_k - b_k - c_k - d_k - e_k \end{bmatrix}$$

The considered six-phase VSI has 64 voltage states determined as state vectors. Each vector represents its own phase connection to the supply voltage U_D and resulting in particular polar phase voltage. Two examples illustrating in detail reliance between polar phase voltages and phase-to-phase voltages are presented in Fig. 2.

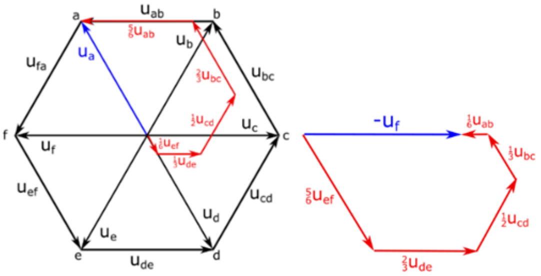

Figure 2. The dependencies between voltages vectors U_a, $-U_f$ and phase-to-phase voltage vectors.

Every polar phase voltage may be determined by a set of five phase-to-phase voltages U_{xy} according to the state vector definition (1): $V_k = [U_{abk}\ U_{bck}\ U_{cdk}\ U_{dek}\ U_{efk}]$.

Using the standard space voltage transformation it is possible to define the polar voltage space vector using symbols of the index k binary expansion:

$$(6)\quad \bar{V}_k = \frac{5}{6} U_D \begin{pmatrix} a_k + b_k e^{j\frac{\pi}{3}} + c_k e^{j\frac{2\pi}{3}} \\ + d_k e^{j\pi} + e_k e^{j\frac{4\pi}{3}} + e_k e^{j\frac{5\pi}{3}} \end{pmatrix}$$

This definition confirms an evident correlation between the space and state vectors as they are described using the same digits.

Applying the Euler's formula and introducing interdependences among the angles as well as symbols $a_k, b_k, c_k, d_k, e_k, f_k$ the space vector is given as follows:

$$(7)\quad \bar{V}_k = \frac{5}{6} U_D \begin{bmatrix} a_k - d_k + \frac{1}{2}(b_k - c_k - e_k + f_k) \\ + j\frac{\sqrt{3}}{2}(b_k + c_k - e_k - f_k) \end{bmatrix}$$

After the transformation the space vector is situated as a complex number, therefore it may be represented by the modulus M_k and the argument φ_k. But in the case of the six-phase inverter the situation is more demanding in comparison to three-phase inverters due to a greater number of variables. The expression allowing calculation of the space vector modulus is the following:

$$(8)\quad M_k = K\sqrt{\gamma_0 + \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5}$$

where K denotes a constant of proportionality. Its value depends on the voltage U_D supplying the inverter and may differ depending on the assumed canon. The coefficients $\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5$ are as follows:

$$(9)\quad \gamma_0 = a_k^2 + b_k^2 + c_k^2 + d_k^2 + e_k^2 + f_k^2$$
$$(10)\quad \gamma_1 = a_k(b_k - c_k - 2d_k - e_k + f_k)$$
$$(11)\quad \gamma_2 = b_k(c_k - d_k - 2e_k - f_k)$$
$$(12)\quad \gamma_3 = c_k(d_k - e_k - 2f_k)$$
$$(13)\quad \gamma_4 = d_k(e_k - f_k)$$
$$(14)\quad \gamma_5 = e_k f_k$$

If the constant K equals 5/6 U_D, as it was assumed in (6) then, according to (8), the modulus M_k may reach four diverse values: 0, $5/6U_D$, $5/3U_D$, $5/6\sqrt{3}U_D$.

The vectorial argument φk denotes an angle between vector and real axis and is determined using trigonometrical functions. In some cases it might be necessary to use both functions: *arctg* and *arcsin* due to diverse periodicity of sine and tangent. Then the argument is determined if $\phi_k(\text{arcsin}) = \phi_k(\text{arctg})$. The formulas (15) and (16) allow calculating the ϕ_k value:

$$(15)\quad \phi_k = \arcsin \frac{\sqrt{3}(b_k + c_k - e_k - f_k)}{2M_k},$$

$$(16)\quad \phi_k = \text{arctg}\frac{\sqrt{3}(b_k + c_k - e_k - f_k)}{2(a_k - d_k) + b_k - c_k - e_k + f_k}$$

The denominator of the expressions (15) and (16) cannot equal zero. So, the results of ϕ_k is indeterminate for $a_k = b_k = c_k = d_k = e_k = 0$ or $a_k = b_k = c_k = d_k = e_k = 1$. The corresponding two vectors $V_{0(000000)}$ and $V_{63(111111)}$ are habitually named zero vectors because they do not have any impact on the phase currents.

Figure 3 present the polar voltage space vectors of the six-phase inverter. The constant K of the moduli used in this presentation was assumed $K = \frac{5}{6}U_D$. The vectors are presented as points on the complex plane (α-jβ), where α denotes the real axis and β — the imaginary one. The diagram presents their position. Each marked point denotes at least one sense of a vector. According to the assumption $K = \frac{5}{6}U_D$, the vector moduli reach only four values: 0,1,√3, 2.

Apart from two zero vectors in the middle of 62 vectors there are four groups of active vectors : six vectors with moduli 2, twelve with moduli √3, thirty six with 1 and also eight active vectors which formally have zero-dimensional moduli and indeterminate angles. The further down in tables 1 and 2, two sets of the binary expansions defining these relevant vectors are indicated. Among eight vectors two of them: V21 as well as V42 can be considered as representing two three-phase voltage systems (Table 1).

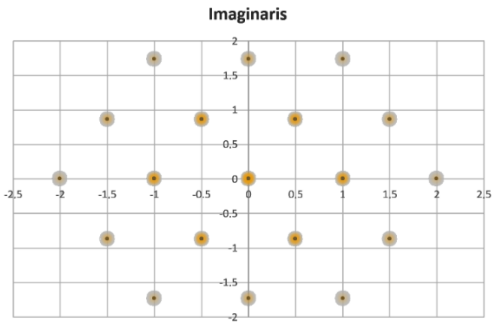

Figure 3. The polar voltage space vectors of the two-level six-phase VSI

The first one could be determined by three voltages: V_b, V_d, V_f and the other - by voltages: V_a, V_c, V_e.

Table 1. The two vectors representing the three-phase voltage system

V_k	a	b	c	d	e	f
V_{21}	0	1	0	1	0	1
V_{42}	1	0	1	0	1	0

Another possibility to reach two three-phase voltage systems is accessible using the next six active vectors which formally have zero-dimensional moduli, that is: V_9, V_{18}, V_{27}, V_{36}, V_{45}, V_{54}. These ones present the symmetry $a_k,b_k,c_k = d_k,e_k,f_k$ in the binary expansion of the vector number. The vectors are collected below in moderately different sequences. The sequences a_k,b_k,c_k and d_k,e_k,f_k are suitable to control three-phase system. So, following the order indicated in Table 2 it is possible to control two twin three-phase AC drives. Formally these vectors' angle φ_k is indeterminate but in Table 2 the angle φ_k was attributed in theory for application in standard α-β transformation used routinely to control three phase AC drives.

Table 2. The active vectors guaranteeing the symmetry $a_k, b_k, c_k = d_k, e_k, f_k$

	3-phase AC drive			3-phase AC drive			
V_k	a	b	c	d	e	f	φ_k
V_{36}	1	0	0	1	0	0	0
V_{54}	1	1	0	1	1	0	60º
V_{18}	0	1	0	0	1	0	120º
V_{27}	0	1	1	0	1	1	180º
V_9	0	0	1	0	0	1	240º
V_{45}	1	0	1	1	0	1	300º

The control process of twin AC drives consists in switching of the successive vectors starting from vector V_{36}. The sequence of vectors is evident according to the attributed angle φ_k. The pairs of vectors V_{36}-V_{54},V_{54}-V_{18},V_{18}-V_{27},V_{27}-V_9,V_9-V_{45},V_{45}-V_{36} relate to the consecutive sectors of the stationary coordinate system on the complex plane α-jβ.

Simulation experiment

The described methodology of using polar voltage space vectors was compared with other strategies during simulation study. The adopted simulation test scenario was based on the fact that four control strategies were applied for two induction motors connected to a six-phase VSI. The first strategy is based on the use of PWM modulation (marked as pwm), the second is based on the use of vector control described in Table 2 (marked as 1), the third is based on the use of vector sequences presented in the form of a vector map in Figure 4 (marked as 2), the fourth is based on a simple concatenation of successive vectors for the sequence presented in Figure 4 and its repetition (marked as 3).

Figure 4 shows in a graphic form a vector map for the adopted vector control strategy. This is an incomplete sequence in which the vectors for sectors 1 and 2 are presented. This sequence is the basis for the next modification in which the presented vector sequence concept was repeated twice in each sector. It should also be noted that the graphical representations of the vectors in each sector shown in Figure 8 were achieved by mirroring. The vector sequences were selected according to the one switch per phase principle above.

Nr		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
sectors		1												2											
vectors		32	48	49	50	59	63	59	50	49	48	32	0	16	24	56	60	61	63	61	60	56	24	16	0
p	a	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	0	0	0
h	b	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0
a	c	0	0	0	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0
s	d	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
e	e	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
s	f	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0

Figure 4. The polar voltage space vectors of the two-level six-phase VSI and their binary expansions

Figure 5 shows the waveforms of the control signals corresponding to the individual control systems. The figures show the signals for phases a.

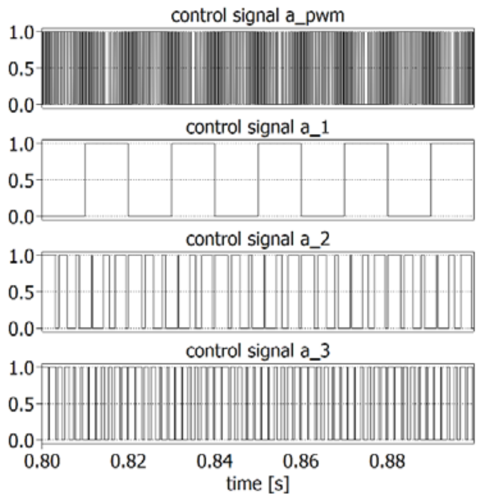

Figure 5. The comparison of the control signal waveforms for various control strategy and for phase a

Figure 6 shows the phase-to-phase voltage waveform u_{ba}.

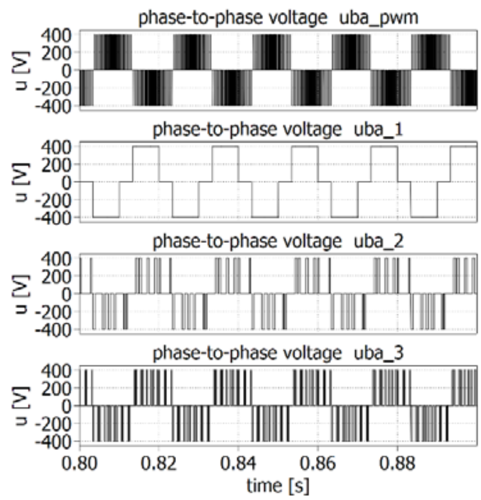

Figure 6. The comparison of the phase-to-phase voltage waveforms for various control strategies

Table 3 presents a summary of the most important parameters obtained during the simulation tests.

Table 3. The vectors' parameters of the six-phase VSI inverter

Strategy	RMS [A]	THD [%]
1 (pwm)	2.1	3.7
2 (2 x 3 phase)	2.8	15.1
3 (vectors)	1.5	18.3
4 (vectors2)	1.4	11.5

Figure 7 shows the phase current i_a obtained for different control strategies.

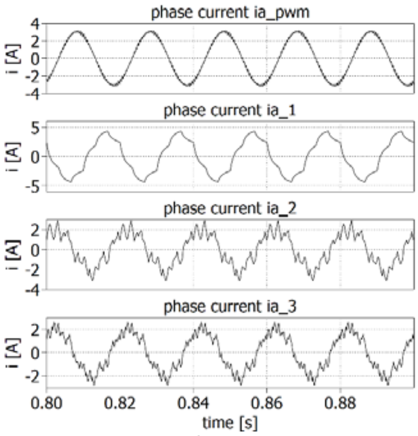

Figure 7. The comparison of the phase current waveforms for different control strategies

The control strategy number 3 has been chosen so that the switching of the keys in a given clock takes place only once. The control strategy 4 is a double repetition in a cycle of the control strategy 3. Using such a simple control method, very good results have been obtained for the content of higher harmonics, which can be seen from the THD values. At the same time, the adopted control also allows for maintaining a high rms value of the currents. Subsequent repetition of the adopted vector sequence would lead to results comparable to PWM control. However, the main advantage of the con-trol method described in the experiment over PWM is that it requires a much smaller amount of switching. However, the best results were obtained by selecting such vectors so that the six-phase inverter works as two three-phase inverters.

Conclusions

The aim of the paper was to prove that state and polar voltage space vectors are a useful mathematical tool in the analysis and control of a six-phase inverter. The whole control process could be restricted to the simplest concatenation of successive vectors. This approach permits limiting to a minimum the amount of switching and the number of zero vectors used, as well as avoiding problems resulting in parity of phases.

The conducted simulation tests have proved that the proposed solution might be indeed suitable in designing inverter control algorithms. The obtained results have shown that space vector method assures a significantly lower number of inverter switching in comparison to the PWM, which results also in higher efficiency of the inverter.

Both the notation and computational method described in this paper were used in previous works. However, only the inverters with an odd number of phases have been considered there. An even number of phases makes the computational process more complicated since the choice of vectors is limited by the phases' symmetry. Another problem is that even though some vector sequences are optimal in terms of the amount of switching, they are not implementable in the inverter due to the asymmetry of the output voltage and current waveforms. It can be stated that it is easier to obtain vector sequences which meet control expectations for inverters with an odd number of phases than for ones with an even number of phases.

This project is financially supported under the framework of a program of the Ministry of Science and Higher Education (Poland) as "Regional Excellence Initiative" in the years 2019–2022, project number 006/RID/2018/19, amount of funding 11 870 000 PLN.

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