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A mathematical model of an ultrahigh voltage transmission line taking into account overhead ground wires

Abstract. *The development of a mathematical model of a three-phase ultrahigh voltage transmission line with two overhead ground wires is presented in the paper. The mathematical model is based on differential equations of state of a long line with partial derivatives in the matrix-vector form. The boundary conditions of the second kind (Neumann's conditions) are used, which enables obtaining functions of unknown coordinates of the overhead ground wires mode at the beginning and at the end of the voltage transmission line to solve the transmission line equation. Some results of computer simulations of transients during switching on and a single-phase short circuit of the line are presented in the form of corresponding graphic functions of its mode coordinates.*

Streszczenie. *W artykule przedstawiono model matematyczny trójfazowej linii elektroenergetycznej ultra wysokiego napięcia z dwoma przewodami odgromowymi. Model matematyczny wykorzystuje równania różniczkowe o pochodnych cząstkowych elektromagnetycznej długiej linii w postaci macierzowo-wektorowej. Jako warunki brzegowe wykorzystano warunki drugiego rodzaju (warunki Neumanna), co z kolei pozwoliło obliczyć przebiegi napięcia i prądu na początku i na końcu linii elektroenergetycznej. Wyniki symulacji komputerowej stanów nieustalonych podczas załączania linii oraz zwarcia jednofazowego w jej końcu przedstawiono w postaci rysunków.* ***(Model matematyczny linii przesyłowej wysokiego napięcia z uwzględnieniem napowietrznych przewodów uziemiających).***

Keywords: mathematical model, power line, overhead ground wires, high voltage, distributed parameters, long line equation.
Słowa kluczowe: model matematyczny, linia energetyczna, przewody odgromowy, wysokie napięcie, parametry rozłożone, równania linii długiej.

Introduction

The paper concerns ultrahigh voltage transmission lines, which are the components of main electric power networks with a voltage of 220 kV and higher. One of the important factors to be considered is the peculiarities of transient electromagnetic processes during emergency and switching modes, due to the significant lengths and distribution of the parameters of such lines [1, 2, 3, 4]. The reproduction and analysis of these processes is an integral part of the design, planning, maintenance of the modes and the configuration of relay protection and controls.

An effective way to study power lines is to use a mathematical simulation tool, which makes it possible not only to reproduce transient electromagnetic processes with high adequacy, but also to avoid the use of expensive field experiments. To obtain reliable results of power lines simulation, it is necessary to take into account as many factors of influence as possible, in particular, the mutual effects of overhead ground wires and power line phases on the coordinates of transient electromagnetic processes. Their reasonable consideration involves the presentation of ground wires and some appropriate elements of the mathematical model of a line. Work is in progress, in particular, to study direct lightning strikes and their impact on the operation of power transmission lines with grounded and ungrounded overhead wires [5, 6].

Another important factor that is overlooked is the simulation of the line as a system with distributed parameters, which is relevant for ultrahigh voltage class lines, since, due to significant lengths, they have wave processes that can be fully taken into account only by solving the transmission line equation [7]. Instead, such lines are considered as circular equivalent circuits. We cannot say that such an approach would be wrong, but it is clear that the physical essence of the very transmission line equation is lost here. This means that the problems are shifted from the field to the circular statement, thus reducing a priori the degree of adequacy of the line model, and hence the degree of adequacy of the model of the whole object. Thus, for example, in paper [8] a digital model of a 400 kV transmission line in the *EMTP-RV* software package is developed and in [9] a model of an electrical network with power lines with overhead ground wires in the *PSCAD* software is presented. The lines there are considered in the form of circular equivalent circuits. What is more, there are works [10] in which the simulations are performed for a single-circuit equivalent circuit, i.e., a line is considered in a simplified single-phase version with no active resistances and conductivities, which also reduces the adequacy of the results of transient studies.

Taking the above into account, **our research objective** is to develop an approach to developing a mathematical model of a long three-phase ultrahigh voltage transmission line with two overhead ground wires taking into account the peculiarities of the formation of boundary conditions for the study of electromagnetic processes in the natural coordinates of the line's phases and wires.

The presentation of basic data

We propose to develop a mathematical model of a long three-phase ultrahigh voltage transmission line with two ground wires, taking into account the distribution of its parameters, based on differential equations of the state of a long line with partial derivatives in a matrix-vector form and using boundary conditions of the second kind (Neumann's conditions).

The computational scheme of a real ultrahigh voltage line is presented in Fig. 1, which uses the conventional designations of its components and coordinates of the mode. As can be seen from Fig. 1, the ground wires (hereinafter wires $T1$, $T2$) are connected to each other at the end of the line and disconnected at its beginning, that is, they form an open circuit with grounding of the wire $T2$ at one point at the beginning of the line. Given that this paper will focus on the formation of boundary conditions for the equations of state of wires $T1$, $T2$, the three-phase transmission line will be considered symmetric with the same inherent and mutual parameters of split phases and wires and with given symmetric phase voltage systems of its beginning and end. The split phases are represented as single-wire ones with corresponding equivalent radii. That is why such a line can be considered as a five-wire one – cables $T1$, $T2$ and phases A, B, C. The computational scheme of the line adopted in this paper enables to obtain the functions of the unknown coordinates of the ground wire mode at the beginning and at the end of the power transmission line, taking into account the peculiarities of their execution.

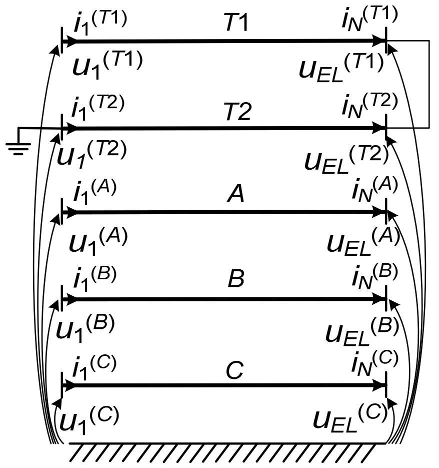

Fig. 1. A computational scheme of a long transmission line with distributed parameters

In our case, the coordinates of the mode are some vectors of unknown voltages relative to earth and currents of wires and phases of the line with the length l with distributed parameters as functions of independent arguments (variables) – distances x from the beginning of the line ($0 \le x \le l$) and time t ($t \ge 0$):

(1) $$\mathbf{u} = \mathbf{u}(x,t) = \left(u^{(T1)}, u^{(T2)}, u^{(A)}, u^{(B)}, u^{(C)}\right)_t ;$$

(2) $$\mathbf{i} = \mathbf{i}(x,t) = \left(i^{(T1)}, i^{(T2)}, i^{(A)}, i^{(B)}, i^{(C)}\right)_t .$$

In [7, 11], it is noted that for a more optimal description of physical processes in long lines and for improved efficiency of implementing digital models, it is advisable to use the vector function of voltages (1) as a generalised coordinate of the mode. That is why to develop a mathematical model we will use the differential equation of the state of a second-order long line with partial derivatives in respect of the vector function of voltages (1), which in a matrix-vector form will look as follows:

(3) $$\frac{\partial^2 \mathbf{u}}{\partial t^2} = (\mathbf{LC})^{-1}\left(\frac{\partial^2 \mathbf{u}}{\partial x^2} - (\mathbf{Lg}+\mathbf{rC})\frac{\partial \mathbf{u}}{\partial t} - \mathbf{rgu}\right),$$

where: $\mathbf{r}$, $\mathbf{L}$ –square matrices of specific (per unit length) inherent and mutual contour parameters (active supports, inherent and mutual inductances) of the system of five independent contours of the wires and line phases – ground, respectively; $\mathbf{g}$, $\mathbf{C}$ – square matrices of specific (per unit length) inherent and mutual nodal parameters (active conductivity and capacitance) of the system of five independent nodes of wires and line phases relative to the ground, respectively.

To solve the equation (3), we propose to use boundary conditions of the second kind, in particular, the differential equation of state of a first-order long line with partial derivatives with regard to the change of the vector function of voltages (1) by argument x for time t, which in a matrix-vector form will look as follows:

(4) $$-\frac{\partial \mathbf{u}}{\partial x} = \mathbf{L}\frac{\partial \mathbf{i}}{\partial t} + \mathbf{ri} .$$

Discretizing by step Δx the argument x of equations (3), (4) using the method of straight lines with N nodes and employing the concept of the central derivative [12], we get the following for the j-th node of the line:

(5) $$\frac{d\mathbf{v}_j}{dt} = (\mathbf{LC})^{-1}\left(\frac{1}{(\Delta x)^2}\left(\mathbf{u}_{j+1} - 2\mathbf{u}_j + \mathbf{u}_{j-1}\right) - (\mathbf{Lg}+\mathbf{rC})\mathbf{v}_j - \mathbf{rgu}_j\right);$$

(6) $$\frac{d\mathbf{u}_j}{dt} = \mathbf{v}_j , \quad \frac{d\mathbf{i}_j}{dt} = \mathbf{L}^{-1}\left(\frac{1}{2\Delta x}\left(\mathbf{u}_{j-1} - \mathbf{u}_{j+1}\right) - \mathbf{ri}_j\right), \quad j = 1,\ldots,N .$$

An analysis of equations (5), (6) shows that to find the voltages of the first $\mathbf{u}_1$ ($j = 1$) and the last $\mathbf{u}_N$ ($j = N$) sampling nodes and currents in the first $\mathbf{i}_1$ and last $\mathbf{i}_N$ discrete branches of the line, we must first find the unknown voltages in fictitious nodes $\mathbf{u}_0$ and $\mathbf{u}_{N+1}$.

According to the assumptions made in our specific case, for the equations of state of the line phases we have boundary conditions of the first kind in the form of known (given) phase voltages of its beginning and its end ($u_1^{(A)}$, $u_1^{(B)}$, $u_1^{(C)}$, $u_{EL}^{(A)}$, $u_{EL}^{(B)}$, $u_{EL}^{(C)}$). For the equation of the state of the wire $T2$ grounded at the beginning of the line, we also have one boundary condition of the first kind, $u_1^{(T2)} = 0$.

The situation is quite different for the equations of state of the wire $T1$, disconnected at the beginning of the line, and the wires $T1$ and $T2$, connected to each other at its end. Therefore, in our case, to form boundary conditions it is necessary to find voltages of fictitious nodes at the beginning of the line for the wire $T1$ ($u_0^{(T1)}$) and at its end – for wires $T1$ ($u_{N+1}^{(T1)}$) and $T2$ ($u_{N+1}^{(T2)}$). To this end, it is proposed to consider the first (at the beginning) and the last (at the end) discretes of the line in circular variants based on the corresponding straight *Г*-shaped equivalent circuits (hereinafter referred to as the straight *Г*-circuits).

The boundary conditions for the equations of state of the $T1$ wire at the beginning of the line are determined according to the straight *Г*-circuit of the first discrete of the line, which for this wire is presented in Fig. 2 (only a fragment of the equivalent circuit of the first discrete of the $T1$ wire is shown)

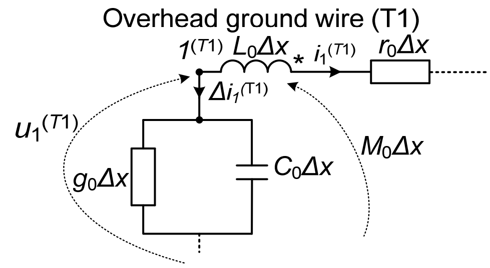

Fig. 2. Straight equivalent *Г*-circuit of the first discrete of the wire $T1$ at the beginning of the line

Let us write down the equation according to Kirchhoff's first law for the first sampling node of the wire $T1$ (Fig. 2):

(7) $$-i_1^{(T1)} - \Delta i_1^{(T1)} = 0 ,$$

where: $\Delta i_1^{(T1)}$ is the total leakage current from the wire $T1$ to the wire $T2$, phases A, B, C of the line and to ground Z, respectively.

The total leakage current $\Delta i_1^{(T1)}$ in (7) is calculated using the partial leakage currents as follows:

(8) $$\Delta i_1^{(T1)} = \sum_{m=T2}^{Z}\left(\Delta x g_{T1,m} u_1^{(T1,m)} + \Delta x C_{T1,m}\frac{du_1^{(T1,m)}}{dt}\right), \ =T2, A, B, C, Z$$

where: $g_{T1,m}$, $C_{T1,m}$ are specific (per unit length) partial inherent and mutual active conductivities and capacitances between the wire $T1$ and the wire $T2$, phases A, B, C of the line and ground Z; $u^{(T1,m)}$ – voltages between wire $T1$ and wire $T2$, phases A, B, C of the line and ground Z of the first sampling node of the line, respectively.

For the same first node, we separate from the second matrix-vector equation (6) the elementary equation of state of the wire $T1$, which, taking into account the structure of the coordinate vector of the line mode accepted in (1), (2) and $j = 1$, will be expressed in the following coordinate form:

$$(9)\qquad \frac{di_1^{(T1)}}{dt} = \frac{1}{2\Delta x}\left(\sum_{k=1}^{5}\left(\Lambda_{1,k}u_0^{(k)}\right) - \sum_{k=1}^{5}\left(\Lambda_{1,k}u_2^{(k)}\right)\right) - \sum_{k=1}^{5}\left(K_{1,k}i_1^{(k)}\right), \qquad k = 1,2,3,4,5,$$

where $\mathbf{\Lambda} = \mathbf{L}^{-1}$; $\mathbf{K} = \mathbf{L}^{-1}\times\mathbf{r}$; k – number of columns of matrices and rows of vectors.

Next, we differentiate (7) and (8) over time, taking into account the initial conditions [11], and arrive at:

$$(10)\qquad -\frac{di_1^{(T1)}}{dt} - \frac{d\Delta i_1^{(T1)}}{dt} = 0;$$

$$(11)\qquad \frac{d\Delta i_1^{(T1)}}{dt} = \sum_{m=T2}^{Z}\left(\Delta x g_{T1,m}v_1^{(T1,m)} + \Delta x C_{T1,m}\frac{dv_1^{(T1,m)}}{dt}\right).$$

Considering that

$$(12)\qquad u^{(T1,m)} = u^{(T1)} - u^{(m)}, \qquad \frac{du^{(T1,m)}}{dt} = \frac{du^{(T1)}}{dt} - \frac{du^{(m)}}{dt},$$

$$\frac{dv^{(T1,m)}}{dt} = \frac{dv^{(T1)}}{dt} - \frac{dv^{(m)}}{dt},$$

the equation (11) will look as follows:

$$(13)\qquad \frac{d\Delta i_1^{(T1)}}{dt} = \Delta x\sum_{m=T2}^{Z}\left(g_{T1,m}\left(v_1^{(T1)} - v_1^{(m)}\right) + C_{T1,m}\left(\frac{dv_1^{(T1)}}{dt} - \frac{dv_1^{(m)}}{dt}\right)\right).$$

We apply (9) and (13) to the equation (10) and produce:

$$(14)\qquad -\frac{1}{2\Delta x}\left(\sum_{k=1}^{5}\left(\Lambda_{1,k}u_0^{(k)}\right) - \sum_{k=1}^{5}\left(\Lambda_{1,k}u_2^{(k)}\right)\right) + \sum_{k=1}^{5}\left(K_{1,k}i_1^{(k)}\right) - \Delta x\sum_{m=T2}^{Z}\left(g_{T1,m}\left(v_1^{(T1)} - v_1^{(m)}\right) + C_{T1,m}\left(\frac{dv_1^{(T1)}}{dt} - \frac{dv_1^{(m)}}{dt}\right)\right) = 0.$$

When we apply (5) to the equation (14) (for the first sampling node (only for ground wire $T1$)) and then derive from the expression the voltage of the fictitious node at the beginning of the line for ground wire $T1$ ($u_0^{(T1)}$), we obtain:

(15)

$$u_0^{T1} = \frac{2\Delta x}{2C_{11}P_{11} + \Lambda_{11}}\left\{\frac{1}{2\Delta x}\left(\sum_{k=1}^{5}\left(\Lambda_{1,k}u_2^{(k)}\right) - \sum_{k=2}^{5}\left(\Lambda_{1,k}u_0^{(k)}\right)\right) + \sum_{k=1}^{5}\left(K_{1,k}i_1^{(k)}\right) - \Delta x\left(G_{11}v_1^{(T1)} - \sum_{m=T2}^{C}\left(g_{T1,m}v_1^{(m)} + C_{T1,m}\frac{dv_1^{(m)}}{dt}\right)\right) - \Delta x C_{11}\left(\frac{1}{(\Delta x)^2}\left(\sum_{k=1}^{5}\left(P_{1,k}i_1^{(k)} + P_{1,k}u_2^{(k)}\right)\right) - \frac{2}{(\Delta x)^2}\sum_{k=1}^{5}\left(P_{1,k}u_1^{(k)}\right) - \sum_{k=1}^{5}\left(F_{1,k}v_1^{(k)} + D_{1,k}u_1^{(k)}\right)\right)\right\},$$

where $\mathbf{P} = (\mathbf{LC})^{-1}$, $\mathbf{D} = \mathbf{Prg}$, $\mathbf{F} = \mathbf{P}(\mathbf{Lg} + \mathbf{rC})$, C_{11}, G_{11} – the elements of matrices $\mathbf{C}$ and $\mathbf{G}$, respectively.

After analysing (15), we see that it also shows the voltages of fictitious nodes at the beginning of the line for the wire $T2$ ($u_0^{(T2)}$) and phases of the line ($u_0^{(A)}$, $u_0^{(B)}$, $u_0^{(C)}$). We have already mentioned that the boundary conditions of the first kind are given here (voltages at the beginning of the line $u_1^{(T2)}$, $u_1^{(A)}$, $u_1^{(B)}$, $u_1^{(C)}$ have been specified), therefore, the unknown voltages for (15) can be easily found from the second equation in (6), if we formulate it for the first discrete node of the line.

Since voltages at the beginning of the line for the ground wire $T2$ and phase conductors are

$$(16)\qquad u_1^{(T2)} = 0, \qquad u_1^{(m)} = U_{\mathrm{m}}^{(m)}\sin\left(\omega t + \varphi^{(m)}\right), \; m = A, B, C,$$

their first and second derivatives in (15) will look like:

$$(17)\qquad v_1^{(m)} = \omega U_{\mathrm{m}}^{(m)}\cos\left(\omega t + \varphi^{(m)}\right), \quad \frac{dv_1^{(m)}}{dt} = -\omega^2 U_{\mathrm{m}}^{(m)}\sin\left(\omega t + \varphi^{(m)}\right).$$

To avoid overloading the paper with mathematical derivations, we will not search for fictitious voltages at the end of the power line. In [13], based on the boundary conditions of the second kind, a universal expression for finding the voltage $\mathbf{u}_{N+1}$ in fictitious nodes at the end of the line is obtained.

$$(18)\qquad \mathbf{u}_{N+1} = \mathbf{u}_{N-1} + 2\left(\mathbf{u}_{EL} - \mathbf{u}_N\right), \quad \mathbf{u}_{EL} = \left(u_{EL}^{(T1)}, u_{EL}^{(T2)}, u_{EL}^{(A)}, u_{EL}^{(B)}, u_{EL}^{(C)}\right)_t.$$

(18) makes it possible to use a line model based on the transmission line equation autonomously in any network configuration, regardless of the circuit of their connection. However, in (18) there is an actual voltage at the end of the transmission line $\mathbf{u}_{EL}$, which is to be determined. In our case, we must determine the voltages of the wires ($u_{EL}^{(T1)}$, $u_{EL}^{(T2)}$) which are connected to each other at the end of the line, since the phase voltages at the end of the line ($u_{EL}^{(A)}$, $u_{EL}^{(B)}$, $u_{EL}^{(C)}$) are set. As the ground wires at the end of the line are connected to each other, then $u_{EL}^{(T1)} = u_{EL}^{(T2)} = u_{EL}^{(T)}$. To find this voltage, let us consider the equivalent circuit of the last discrete sections of ground wires (Fig. 3). Here, we have traditionally used a straight equivalent *Γ*-circuit of the elementary section of the line.

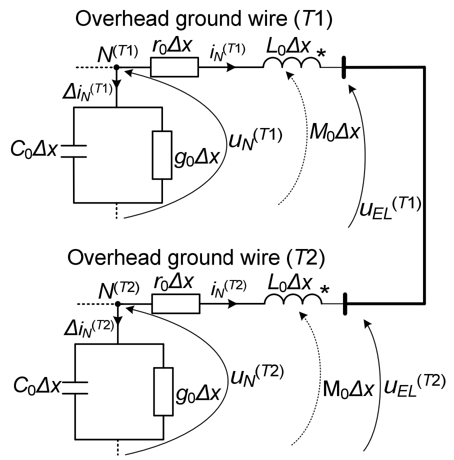

Fig. 3. The straight equivalent *Γ*-circuit of the last discretes of wires $T1$ and $T2$ at the end of the line

From Fig. 3 we can write [11]:

$$(19)\qquad i_N^{(T1)} = -i_N^{(T2)} \;\Rightarrow\; \frac{di_N^{(T1)}}{dt} = -\frac{di_N^{(T2)}}{dt}.$$

Given that $i_N^{(Z)} = i_N^{(A)} + i_N^{(B)} + i_N^{(C)}$, ($i_N^{(Z)}$ is the current in the ground of the last discrete branch), based on Kirchhoff's second law for the circuit (Fig. 3), we will write the equation for finding currents in the last discrete branches of ground wires.

$$(20)\qquad \frac{di_N^{(T1)}}{dt} = \frac{1}{\Delta x L_0}\left(u_N^{(T1)} - \Delta x\left(\sum_{m=T2}^{C}\left(M_{T1,m}\frac{di_N^{(m)}}{dt}\right) + \right.\right.$$

$$+\left(r_0+r_Z\right)i_N^{(T1)}+r_Z\sum_{m=T2}^{C}\left(i_N^{(m)}\right)\Bigg)-u_{EL}^{(T)}\Bigg);$$

$$\text{(21)}\quad \frac{di_N^{(T2)}}{dt}=\frac{1}{\Delta x L_0}\Bigg(u_N^{(T2)}-\Delta x\Bigg(\sum_{n=T1}^{C}\left(M_{T2,n}\frac{di_N^{(n)}}{dt}\right)+$$

$$+\left(r_0+r_Z\right)i_N^{(T2)}+r_Z\sum_{n=T1}^{C}\left(i_N^{(n)}\right)\Bigg)-u_{EL}^{(T)}\Bigg),\ n=T1,\ A,\ B,\ C,$$

where $m,\ n$ – the names of ground wires or phase conductors.

If we apply (20) and (21) it on the second expression in (19) and express the voltage of ground wires at the end of the line ($u_{EL}^{(T)}$), we produce:

$$\text{(22)}\quad u_{EL}^{(T)}=\frac{1}{2}\Bigg\{u_N^{(T2)}+u_N^{(T1)}-\Delta x\Bigg(\sum_{m=T2,n=T1}^{C,C}\Bigg(M_{T1,m}\frac{di_N^{(m)}}{dt}+$$

$$+M_{T2,n}\frac{di_N^{(n)}}{dt}\Bigg)+2r_Z\left(i_N^{(A)}+i_N^{(B)}+i_N^{(C)}\right)\Bigg)\Bigg\}.$$

The currents in all the discrete branches of ground wires and phase conductors of the line can be found from the second equation (6).

The following system of differential equations is subject to joint integration: (5), (6) including (1), (2), (15) – (18), (22).

Computer simulation results

Computer simulation is performed to study transient electromagnetic processes that take place in a symmetrical three-phase transmission line with two overhead ground wires (Fig. 1) during short circuits. The studies serve testing purposes in order to represent the adequate behaviour of the developed model in theoretical terms and to lay a real 750 kV transmission line with a length of 476 km with the following specific parameters: $r_{0F}=1.9\cdot10^{-5}$ Om/m, $r_{0T}=$ $=4.28\cdot10^{-4}$ Om/m, $r_{0Z}=5\cdot10^{-5}$ Om/m, $L_{0F}=1.647\cdot10^{-6}$ H/m, $L_{0T}=2.4049\cdot10^{-6}$ H/m, $M_{0FF}=7.41\cdot10^{-7}$ H/m, $M_{0FT}=7.4\cdot10^{-7}$ H/m, $M_{0TT}=7.05\cdot10^{-7}$ H/m, $g_{0F}=3.253\cdot10^{-11}$ Sm/m, $g_{0FF}=g_{0FT}=$ $=3.253\cdot10^{-13}$ Sm/m, $g_{0T}\approx0$, $g_{0TT}\approx0$, $C_{0F}=0.8647\cdot10^{-11}$ F/m, $C_{0FF}=0.103\cdot10^{-11}$ F/m, $C_{0FT}=0.0723\cdot10^{-11}$ F/m, $C_{0T}=$ $=0.3501\cdot10^{-11}$ F/m, $C_{0TT}=0.04162\cdot10^{-11}$ F/m.

The adopted values of the amplitudes and arguments of the phase voltages of the beginning and end of the line correspond to the normal steady-state mode of transmission of active power P in the range (0.55 – 0.65) of its natural value P_C, namely: $u_1^{(A)}=615\ \sin(\omega t+20^{\circ})$ kV, $u_1^{(B)}=615\ \sin(\omega t-100^{\circ})$ kV, $u_1^{(C)}=615\ \sin(\omega t+140^{\circ})$ kV, $u_{EL}^{(A)}=598\ \sin(\omega t+4^{\circ})$ kV, $u_{EL}^{(B)}=598\ \sin(\omega t-116^{\circ})$ kV, $u_{EL}^{(C)}=598\ \sin(\omega t+124^{\circ})$ kV, $\omega=314.15\ s^{-1}$.

To approximate real conditions, the simulation of line activation begins from the time $t=0$ s, taking into account the possible phase-by-phase simultaneous switching on of switches at the beginning and at the end of the line. After entering the steady-state mode, at time $t=0.14$ s, a single-phase short circuit of phase A to ground s simulated at the end of the line.

Fig. 4, 5 present the change in time of phase voltages at a distance of 23 km to the end of the line and phase currents at the end of the line (short-circuit currents), respectively. We can see that due to the simulation of controlled switching of circuit breakers, overvoltages during line activation are practically absent (Fig. 4). In normal steady-state mode, the amplitude values of the voltages are 600 kV and in the steady-state short-circuit mode the voltage amplitude of phase A is 30 kV, while other phases remain virtually unchanged.

When we analyse the transients of currents (Fig. 5), we see that after entering the steady-state mode, the currents acquire amplitude values of 1.36 kA. After a single-phase short circuit, the maximum values of current modules are: in phase A – 6.62 kA, in phase B – 2.12 kA, and in phase C – 3.32 kA. In the steady-state short-circuit mode, the currents have the following amplitude values: phase A – 3.65 kA, phase B – 1 kA, phase C – 2.29 kA.

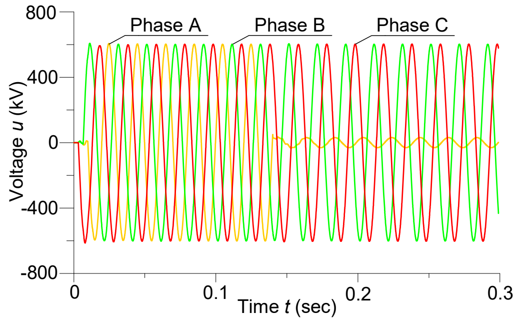

Fig. 4. Phase voltages 23 km to the end of the line

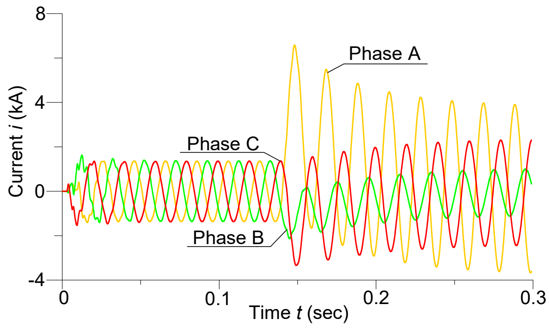

Fig. 5. Phase currents at the end of the line

Fig. 6 shows the change over time of the voltages of wire $T1$ at the beginning and end of the line, and Fig. 7 – the currents of wire $T2$ at the beginning and end of the line, too.

If we analyse the voltage of the ground wire $T1$ (Fig. 6), we see that when the line is activated, the maximum value of the voltage at the beginning of the line reaches -110 kV and at the end of the line -200 kV. As it enters the normal steady-state mode, the voltages of ground wires reduce to zero. After a short circuit, the maximum voltage at the beginning of the line reaches -230 kV and at the end of the line -300 kV. The amplitude values of the $T1$ wire voltages in the steady-state short-circuit mode are 173 kV and 230 kV at the beginning and end of the line, respectively.

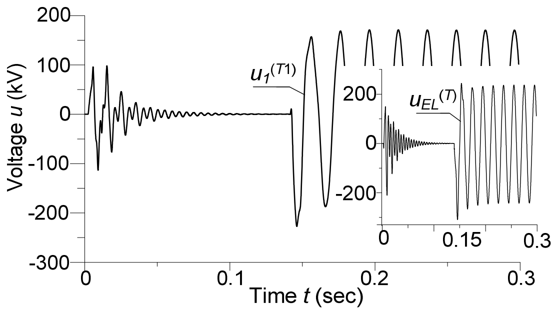

Fig. 6. Voltages of the ground wire $T1$ at the beginning and end of the line

If we analyse the currents of the grounded wire $T2$ at the beginning of the line (Fig. 7), we see that when the line is switched on, the highest value of current at the beginning of the line is -326 A and at the end of the line -120 A. Both the voltages and the currents across the wires tend to zero as they enter the steady-state mode. Following a short circuit, the current in the wire $T2$ at the beginning of the line reaches a maximum of 450 A and at its end, 370 A. In the steady-state short-circuit mode, these currents have amplitudes of 340 A and 290 A, respectively.

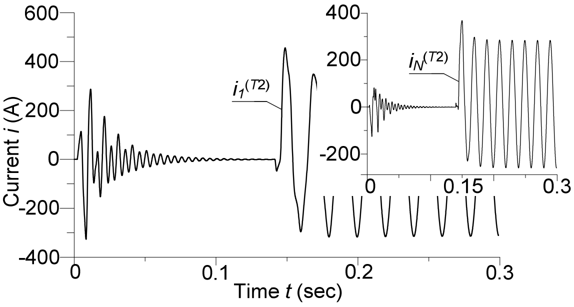

Fig. 7. Currents of the ground wire $T2$ at the beginning and end of the line

Fig. 8 shows the voltage distribution of wires $T1$ and $T2$ along the line during a single-phase short circuit at its end, where: $U^*_{m,\text{r.u.}}$ – maximum values of wire voltage modules during the short-circuit transient process; $U^*_{a,\text{r.u.}}$– amplitude values of wire voltages in the steady-state short-circuit mode. Here, the voltage values are given in relative units (r.u.) relative to the corresponding nominal line voltage.

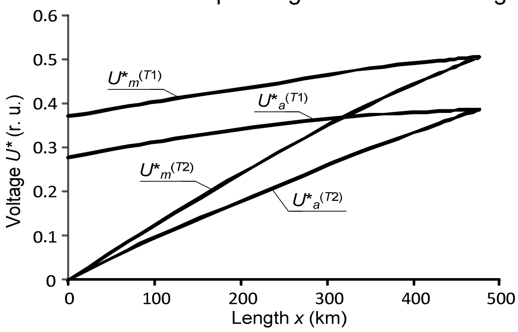

Fig. 8. Voltage distribution of wires $T1$ and $T2$ along the line during a single-phase short circuit at its end

Fig. 8 is a significant addition to the picture of the course of physical processes in the line with ground wires and the possibility of their analysis presented in Fig. 4 – 7.

In our further research, we plan to use the mathematical model of a long line developed in the present paper for an analysis of transients in transmission lines during a lightning strike.

Conclusion

The application of boundary conditions of the second kind (Neumann's conditions) to solving the differential equation of an ultrahigh voltage transmission line with ground wires enables searching for the unknown functions of voltages of ground wires at the beginning and end of the line. This makes it possible to solve the equation of the electromagnetic state not only for the phase conductors of the line, but also for its ground wires. This approach makes it possible to fully take into account overhead ground wires in the line model and to study transients at any point of the line, and if necessary to reproduce the spatial and temporal-spatial distributions of voltages and currents of ground wires and phase conductors of the line.

The results of computer simulation confirm theoretical studies, in particular, the absence of voltages and currents in ground wires in symmetrical line modes and their presence during asymmetric switching and emergency modes, which gives grounds for asserting the adequacy of the developed mathematical model of the line with overhead ground wires.

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