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Optimal selection of static capacitors battery and compensation factor depending on the network resistance

***Abstract**. The problem of optimal selection of a capacitor bank, taking into account the annual curve of reactive power in the network, is solved. The expressions for the cyclic duration factor of a capacitor bank and its own losses are determined. A method of reducing power losses in the network is shown. Expressions for determining the optimal capacity of the capacitor bank in the network, with the active network resistance taken into account, are obtained. On their basis practical recommendations are formulated.*

***Streszczenie.** Rozwiązano problem optymalnego wyboru akumulatora kondensatora, biorąc pod uwagę roczny harmonogram mocy biernej w sieci. Określono wyrażenia czasu włączenia akumulatora kondensatora i strat własnych w nim. Pokazano technikę zmniejszania strat energii elektrycznej w sieci. Uzyskano wyrażenia określające optymalną moc akumulatora kondensatora w sieci, biorąc pod uwagę aktywną rezystancję sieci, na podstawie których sformułowano praktyczne zalecenia. (**Optymalny dobór baterii kondensatorów statycznych i współczynnika kompensacji w zależności od rezystancji sieci**)*

Keywords: Reactive power, load curve, power losses, power factor compensation, network resistance.
Słowa kluczowe: Moc bierna, Wykres obciążenia, utrata energii elektrycznej, kompensacja mocy biernej, rezystancja sieci.

Introduction

A large number of solutions to the problem of selecting power factor compensation means in electric networks have been developed by now, from simple formulas to complex optimization programs [1-5]. The existing methods of selecting compensating devices, as a rule, take into account only one operational state of the electric network and that is the state of maximum loads [6]. This relates to the fact that information about other modes is always less reliable, especially at the design stage. In addition, the maximum load mode is the determinant for most technical constraints. In particular, this mode corresponds to the "construction" component of the energy costs resulting from the introduction of new generating capacity and network expansion.

At the same time, this approach does not provide for the power control of compensating devices. Therefore, some regularities of optimization calculations are lost, and there is no possibility of correct technical and economic comparison of regulated and unregulated devices. This paper discusses the selection of an unregulated static capacitor bank (SCB), considering the reactive power control by disconnecting the SCB at low loads. The emphasis is made on the derivation of analytical formulas, so the simplest case is considered and a number of appropriate assumptions are introduced.

Problem statement

A schematic of the electrical network for selecting the SCB is shown in Fig. 1. Let us accept the following assumptions:

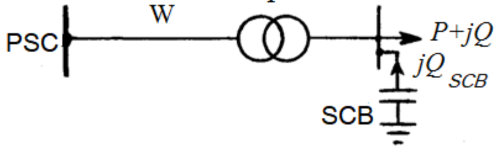


Fig. 1. Electrical network

1. The power loss in the network does not depend on the actual voltage level. Similarly, another assumption can be introduced: the voltage at the load node is maintained at a constant level by the available voltage regulation means.

2. The cost of the SCB (including installation costs) depends linearly on its capacity:

(1) $$K = K_{const} + k_{spec} Q_{SCB},$$

where K_{const} is the constant cost component; k_{spec} is the proportionality factor, which is numerically equal to the increase in the cost of SCB when its capacity increases by unity.

3. The power supply center (PSC) is the boundary of operational responsibilities of the network. Electricity is billed at a single-rate tariff.

4. The annual curve of load reactive power in terms of duration can be represented by a linear function of the form

(2) $$Q(t) = \begin{cases} Q_{\max}\left(1 - \dfrac{t}{t_0}\right), & t \le t_0 \text{ and } t \le 8760\text{ h}, \\ 0, & t > t_0 \text{ or } t > 8760\text{ h}, \end{cases}$$

where $Q_{\max}$ is the annual maximum of reactive power; t_0 is the coordinate of the line intersection, approximating the load curve with the abscissa axis. Various ways of approximating annual load curves, including linear approximation, are discussed in [7, 8].

5. Overcompensation is considered inadmissible, i.e. at any time the inequality constraint must be fulfilled:

(3) $$Q(t) \ge Q_{SCB}.$$

If this constraint is not met, the SCB is disconnected, that is, condition (3) is introduced as a criterion for controlling (disconnecting) the SCB. We can show that it is also a control condition according to the minimum energy loss criterion [9].

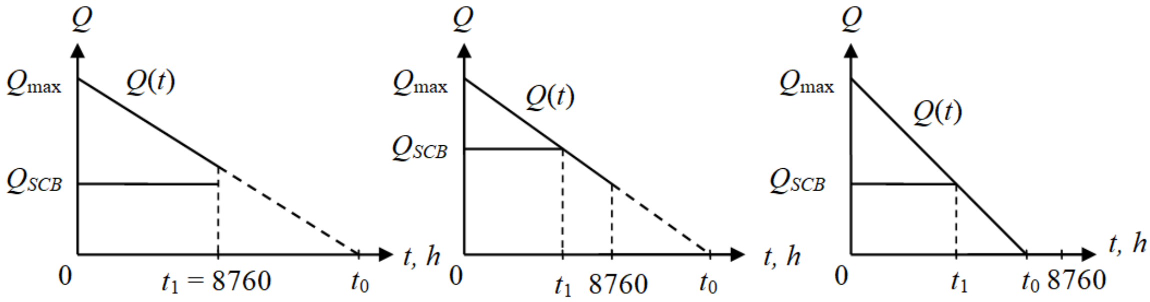
6. There are no other technical restrictions.

The installation of the SCB reduces the energy losses in the line and transformer by certain amount δW, but in the capacitor bank itself the losses ΔW_{SCB} will occur. The costs of installing the SCB given above and maintaining the network are used as the target function

(4) $$C(Q_{SCB}) = (E_n + a_r)C_i + C_e(\Delta W_{SCB} - \delta W),$$

where C_i is the capital investment; E_n is the coefficient of comparative efficiency of capital investments; a_r is the rate of deductions for repair and maintenance of SCB; C_e is the cost of electricity.

Dividing (4) by the cost of electricity we exclude the constant component of the SCB cost from the expenses,



a) $t_0 > 8760$ h, $Q_{SCB} \le Q_{min}$ b) $t_0 > 8760$ h, $Q_{SCB} > Q_{min}$ c) $t_0 \le 8760$ h

Fig. 2. Cyclic duration factor of SCB at different load curves

taking into account (1). As a result, the target function takes the form

(5) $$F\left(Q_{SCB}\right) = k_c k_{spec} Q_{SCB} + \Delta W_{SCB} - \delta W \Rightarrow \min,$$

where $k_c = \dfrac{E_n + a_r}{C_e}$.

Annual load cycle and energy losses in the SCB

The load cycle (cyclic duration factor) of the SCB for the year t_1 depends on the load curve and the capacity of the SCB. We introduce the notation $Q_{\min} = Q(8760)$. Three cases are possible (Fig. 2):

1. $t_0 > 8760$ h, $Q_{SCB} \le Q_{\min}$. In this case the SCB is not cut off, and $t_1 = 8760$ h.

2. $t_0 > 8760$ h, $Q_{SCB} > Q_{\min}$. Then, according to condition (3), the SCB must be cut off for part of the year, i.e., $t_1 <$ 8760 h.

3. $t_0 \le 8760$ h. Since $Q(t_0) = 0$, the SCB is also cut off, with $t_1 < t_0$.

For cases 2 and 3 the load cycle is determined by the formula

(6) $$t_1 = t_0\left(1 - \frac{Q_{SCB}}{Q_{\max}}\right).$$

This formula follows directly from the condition (3) and the dependence (2).

With the known specific power losses in the SCB p_{spec} per a unit of generated reactive power, the annual power losses in the SCB are determined by the expression

(7) $$\Delta W_{SCB} = t_1 p_{spec} Q_{SCB}$$

Or, with consideration of the cases

(8) $$\Delta W_{SCB} = \begin{cases} 8760\, p_{spec} Q_{SCB}, \quad Q_{SCB} \le Q_{\min}, \\ t_0 p_{spec} Q_{SCB}\left(1 - \dfrac{Q_{SCB}}{Q_{\max}}\right), \\ Q_{SCB} > Q_{\min} \text{ or } t_0 \le 8760 \text{ h.} \end{cases}$$

The formula (8) shows that, despite the linear relationship between the power losses in the SCB and the generated reactive power, in cases 2 and 3 the dependence of the energy losses in the SCB on the generated power is non-linear. At the power of the SCB $Q^*_{SCB}=Q_{max}/2$, the energy losses in the SCB pass through a maximum which is equal to

(9) $$\Delta W_{SCB,\max} = \frac{1}{2} t_0 p_{spec} Q^*_{SCB}$$

This means that if the reactive power of the load varies over the year by at least two times, with a degree of compensation exceeding 0.5 (relative to the maximum load) further increase in power of the unregulated SCB will lead not to an increase, but to a reduction of own losses in the SCB. This conclusion is rigorous only under the assumptions made and the adopted method of SCB control. However, the above regularity will also occur with other forms of the load curve, but with different quantitative ratios. The decrease of energy losses in the SCB with the increase of its power at the transition through the maximum is due to the reduction of the SCB load cycle.

Reduction of load energy losses in the network

The value by which the annual load energy losses in the line and transformer will be reduced is equal to the following integral:

(10) $$\delta W = \int_0^{t_1} \left(\frac{Q^2(t)}{U^2} R - \frac{\left(Q(t) - Q_{SCB}\right)^2}{U^2} R \right) dt,$$

where R is the total resistance of the line and transformer; U is the network voltage.

By performing integration with (6), we obtain the following calculated expressions for energy loss reduction:

(11) $$\delta W = \begin{cases} 8760 \dfrac{R}{U^2} Q_{SCB}\left(Q_{\max} + Q_{\min} - Q_{SCB}\right), \\ Q_{SCB} \le Q_{\min}, \\ t_0 \dfrac{R}{U^2} Q_{SCB}\left(Q_{\max} - Q_{SCB}\right), \\ Q_{SCB} > Q_{\min} \text{ or } t_0 \le 8760 \text{ h.} \end{cases}$$

According to (11), the dependence δW (Q_{SCB}) always has a maximum. If $Q_{min} < Q_{max}/2$ or $t_0 \le 8760$ h, the maximum reduction of losses in the network occurs at $Q^*_{SCB}=Q_{max}/2$ and is equal to

(12) $$\delta W_{\max 1} = t_0 \frac{R}{U^2} \frac{Q^2_{\max}}{4}$$

This means that if the reactive power of the load varies over the year by at least two times, with a degree of compensation exceeding 0.5 (relative to the maximum load) further increase in power of the unregulated SCB will lead not to a reduction, but to an increase of energy losses in the SCB. Obviously, such a solution would be economically inefficient, even without taking into account the cost of the SCB. This conclusion is rigorous only under the assumptions made and the adopted method of SCB control. However, we should note that changing the cut-off condition (control method) of the SCB could give additional loss reduction only if part of the time the network operates in the overcompensation mode. The decrease of energy losses in the network with the increase of SCB power at the transition through the maximum is due to the reduction of the SCB load cycle.

If $Q_{min} \geq Q_{max}/2$, then the maximum loss reduction will be observed at the break point of the dependence $\delta W(Q_{SCB})$ at $Q_{SCB}=Q_{min}$ (the break point is caused by the transition from the year-round operation mode of the SCB to the cut-off mode):

(13) $$\delta W_{\max 2} = 8760 \frac{R}{U^2} Q_{\max} Q_{\min}$$

Under the maximum at the break point, the solutions $Q_{SCB} \leq Q_{min}$ that correspond to the round-the-year operation mode of the SCB can be cost-effective.

Optimal SCB power

Let us substitute (8) and (11) in the expression for the target function (5):

(14) $$F = \begin{cases} k_c k_{spec} Q_{SCB} - 8760 \dfrac{R}{U^2} Q_{SCB} \bullet \\ \bullet \left(Q_{\max} + Q_{\min} - Q_{SCB} \right) + \\ +8760 p_{spec} Q_{SCB}, \quad Q_{SCB} \leq Q_{\min}, \\ k_c k_{spec} Q_{SCB} - t_0 \dfrac{R}{U^2} Q_{SCB} \left(Q_{\max} - Q_{SCB} \right) + \\ +t_0 p_{spec} Q_{SCB} \left(1 - \dfrac{Q_{SCB}}{Q_{\max}} \right), \\ Q_{SCB} > Q_{\min} \text{ or } t_0 \leq 8760 \text{ h.} \end{cases}$$

By differentiating (14) by the SCB power and equating the derivatives to zero, we obtain the following expressions for possible points of minimum:

(15) $$Q_{SCB1} = \frac{Q_{\max} + Q_{\min}}{2} - \frac{U^2}{2R} \left(\frac{k_c k_{spec}}{8760} + p_{spec} \right), \quad Q_{SCB1} \leq Q_{\min},$$

(16) $$Q_{SCB2} = \frac{\dfrac{Q_{\max}}{2} - \dfrac{U^2}{2R} \left(\dfrac{k_c k_{spec}}{t_0} + p_{spec} \right)}{1 - \dfrac{p_{spec} U^2}{Q_{\max} R}}, \quad Q_{SCB2} > Q_{\min} \text{ or } t_0 \leq 8760 \text{ h.}$$

A primary analysis of formulas (15) and (16) suggests the following conclusions:

1. At a sufficiently small resistance $R_{\min}$, the optimum powers Q_{SCB1} and Q_{SCB2} turn to zero. This indicates that reactive power compensation is not cost-effective in this case. From formulas (15) and (16), it follows that the value of $R_{\min}$ is

(17) $$R_{\min} = \begin{cases} \dfrac{U^2}{Q_{\max} + Q_{\min}} \left(\dfrac{k_c k_{spec}}{8760} + p_{spec} \right), & t_0 > 8760 \text{ h,} \\ \dfrac{U^2}{Q_{\max}} \left(\dfrac{k_c k_{spec}}{t_0} + p_{spec} \right), & t_0 \leq 8760 \text{ h.} \end{cases}$$

2. As the resistance increases, the power Q_{SCB1} increases monotonically up to the value of $Q_{\min}$, followed by the transition to the dependence (16).

3. If $Q_{min} < Q_{max}/2$, then a further increase in resistance R leads to a continuation of the growth of the SCB optimum power, which asymptotically approaches to $Q_{max}/2$. This again leads to the earlier conclusion that if the reactive power of the load (under the accepted assumptions) varies over a year by more than two times, then the optimal degree of compensation (in relation to the maximum of the loads) cannot be more than 50%.

4. If $Q_{min} \geq Q_{max}/2$, then formula (16) cannot give a result that satisfies the given power range. This means that the target function defined by the second formula (14) does not have an extremum. In this case, the target function defined by the first formula (14) may or may not have an extremum (depending on the network resistance).

5. If the target function has an extremum, that is, there is one of the values of Q_{SCB1} or Q_{SCB2}, then this value is the optimal power of SCB Q^*_{SCB}. If the obtained value is negative, then reactive power compensation is not cost-effective.

6. If the target function has no extremum (no values of Q_{SCB1} and Q_{SCB2} exist) the solution is the break point: $Q^*_{SCB} = Q_{min}$.

Quantitative evaluation of the results

The cost of electricity at CH-2 voltage is approximately 3-4 rubles/ (kWh), the cost of SCB is from 200 to 400 rubles per square meter. We assume C_e = 3.2 rubles/ (kWh), k_{spec} = 300 rubles/kvar. Then when E_n = 0.14 and a_r= 0.059 [10]

$$k_c k_{spec} = \frac{E_n + a_r}{C_e} k_{spec} = \frac{0.14 + 0.059}{3.2} 300 = 18.7 \text{ h.}$$

The specific power losses in the SCB are assumed to be p_{spec} = 0.001 kW / kvar.

Let the number of hours with maximum reactive power load $T_{\max Q}$ = 6000 hours. Then from the condition of equality of areas under the load curves we obtain t_0 = 13900 hours.

Consider the network voltage U = 10 kV (on the high-voltage side). Let us take $Q_{\max}$ = 1000 kvar. Then $Q_{\min}$ = 370 kvar. Under these conditions, the power range of the SCB without cut-off is quite large, and at the same time $Q_{min} < Q_{max}/2$. Thus, the optimal solution can be in both the year-round operation of the SCB, and when the SCB is off for part of the year.

The network resistance (Fig. 1) is the sum of the line resistance W and transformer resistance T and depends on the cross-section and length of the line, the material of the conductors and the transformer capacity. The most probable transformer capacity in this case would be 2500 kVA (a less powerful transformer would probably be overloaded, while more powerful transformer would be severely underloaded). The active resistance of the TM-2500/10 transformer is R_t = 0.42 ohms [11]. The maximum operating current of the line is approximately 100 A, and with aluminum conductors this current corresponds to a cross section of 35 mm^2 [12]. The resistance per unit length is approximately 0.8 ohm/km. The lengths of 10 kV distribution networks do not exceed several kilometers. If we take the maximum line length of approximately 10 km, considering the transformer and rounding the result, we obtain the maximum resistance of the network of 10 ohms. In addition, let us consider the case of installing a high-voltage SCB at the end of the line. In this case, the transformer is outside the compensation area and its resistance is not included in the calculation formulas. Then we finally obtain the network resistance variation range R = 0...10 Ohm.

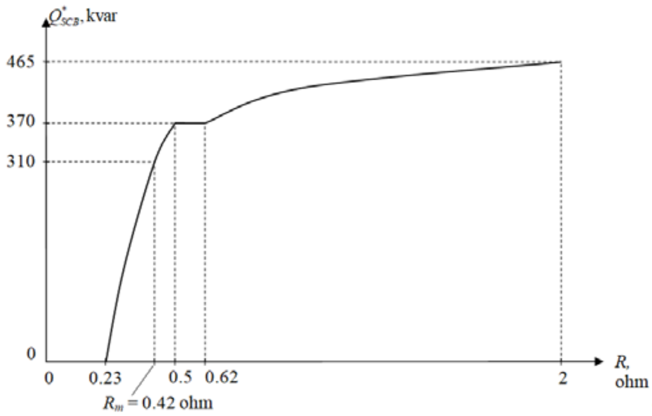


Fig. 3. Dependence of optimal SCB power on the network resistance

The dependence of the optimal power of the SCB on the network resistance, plotted according to the given data within the range from 0 to 2 Ohm, is shown in Fig. 3. The calculations showed the following:

1. Power factor compensation becomes appropriate when the resistance is greater than $R_{\min}$ = 0.23 ohm. This corresponds to a line length of approximately 250 meters in the absence of a transformer.

2. When increasing the resistance from $R_{\min}$ the optimum power first increases rapidly, and then at 0.5 ohm it reaches the minimum value of the load reactive power $Q_{\min}$. This corresponds to the presence of a transformer and a line length of approximately 100 m.

There is a horizontal section on the curve of the SCB optimum power dependence on the resistance, where $Q^{*}_{SCB} = Q_{min}$..

4. After the horizontal section, the optimal power increases again, but much slower than it was initially asymptotically, and it approaches $Q_{max}/2=500$kvar. At R = 10 ohm the optimal power of the SCB is 493 kvar.

Taking into account the controllability of the SCB, the construction component of the energy loss cost and the active power of the load

Many capacitor banks currently used are regulated devices that allow discrete changes in reactive power. The regulated SCB can also be selected by the condition of the minimum for the target function (4), but the reduction of energy losses in the network and own losses in the SCB in this case are determined in a more complex way.

The advantages of regulated SCB over unregulated SCB will be most apparent if the minimum reactive power of the load does not exceed twice the power of one section of the SCB (double control step). In this case, the SCB control range becomes operational. When the load diagram approximates to a uniform load curve, the economic effect of the SCB regulation decreases and becomes equal to zero under the condition of $Q^{*}_{SCB} \leq Q_{min}$., since no cut-off will be performed in this case.

The upper limit for the optimum power of the regulated SCB is no longer limited to 0.5 $Q_{\max}$. This limit depends on the number of sections, and as its number increases, this limit approaches $Q_{\max}$ (but always remains less than this value).

If the payment for electricity is made at a two-rate tariff, then the target function should include an additional summand, which is the cost of real-power losses in the mode of maximum loads. This component corresponds to the so-called construction component of the energy cost due to the expansion of the network and the increase in generating capacity with the growth of electric loads.

Let us consider the limiting case when only active power is paid for at maximum loads (i.e. free primary energy sources are used). When selecting the SCB, only the maximum load mode is considered relevant. The optimal power is limited to the value of $Q_{\max}$, but control of the SCB to reduce energy losses does not make sense. It follows that in real conditions with a two-rate tariff for electricity, the optimal capacity of the SCB is higher than with a single-rate tariff, and the economic effect of using regulated SCB, on the contrary, is lower.

Most of the existing procedures for selecting compensating devices are based on the use of load power factors that take into account the active power of consumers. However, actually in distribution networks, the modes of active and reactive power can be considered mutually independent, so the above formulas do not include the active power of consumers. At the same time, the greater the active power of the load is, the lower the network resistance is on average (as the rated capacity of the transformers and wire or cable cross-sections are increased). This in turn leads to a decrease in the optimal power of the SCB. Thus, the higher the initial power factor is, the lower the optimal reactive power compensation factor is on average. However, this pattern is merely statistical, since the resistances also depend on the lengths of the lines.

Conclusion

Accounting for low-load cut-offs leads to a significant change in the patterns of optimal selection of capacitor banks as compared to selection by a given power, which usually corresponds to the maximum of the loads. In particular, the optimal power of an unregulated SCB in many cases is determined by the mode of minimum loads rather than maximum loads. The advantage of the proposed approach is in keeping track of the actual operating time of the SCB for the year and the degree to which the annual load reactive power curve is filled.

The optimal compensation factor increases as the filling degree of the load curve and the network resistance increase. However, there may be a resistance range at which the optimal power of the SCB remains constant. This is one of the fundamental differences of the considered approach from the standard procedures for selecting SCB based on the given power. The solution in this case conforms to the break point of the target function.

Besides the use of analytical formulas (15) and (16), the problem of optimal selection of unregulated SCB in the network node can be solved by selecting options according to the criterion for the minimum of given costs (4). This provides an opportunity to take into account the real cost scale of capacitor batteries. Energy losses in the SCB and reduction of losses in the network are determined by formulas (8) and (11).

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