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doi:10.15199/48.2022.10.06

TP Transformation Based Controller and Observer Design of the Inverted Pendulum

Abstract. *The paper discusses that applying of CNO and IRNO type weighting functions results different TP models, including controller and observer. Selection of weighting functions has influence on the TP model transformation and the LMI based design. Present paper introduces the LMI based observer and controller design of the inverted pendulum via the TP model transformation based method using CNO and IRNO type weighting functions. The TP transformation is accomplished by qLPV modeling and HOSVD method. The paper gives a conclusion and a comparison of the original and estimated state variables and the impact of applying two types of weighting functions. The aim of present paper is to prove that using different type of weighting functions influences the TP models and LMI methods, but LMIs still result feasible solution for controller and observer design. Furthermore the modification allows to choose the better controller/observer.*

Streszczenie. *W artykule wykazano, że zastosowanie funkcji wagowych typu CNO i IRNO skutkuje różnymi modelami TP, w tym kontrolerem i obserwatorem. Wybór funkcji wagowej ma wpływ na transformację modelu TP i konstrukcję w oparciu o LMI. W niniejszym artykule przedstawiono projekt obserwatora i kontrolera w oparciu o LMI odwróconego wahadła za pomocą metody opartej na transformacji modelu TP z wykorzystaniem funkcji wagowych typu CNO i IRNO. Transformacja TP realizowana jest za pomocą modelowania qLPV i metody HOSVD. Artykuł zawiera wnioski i porównanie pierwotnych i oszacowanych zmiennych stanu oraz wpływu zastosowania dwóch typów funkcji wag. Celem niniejszego artykułu jest wykazanie, że zastosowanie różnego rodzaju funkcji wagowych wpływa na modele TP i metody LMI, ale LMI nadal dają wykonalne rozwiązanie dla konstrukcji kontrolera i obserwatora. Ponadto modyfikacja pozwala wybrać lepszego kontrolera/obserwatora.* (***Transformatorowy sterownik TP i konstrukcja obserwatora odwróconego wahadła***) ***()***

Keywords: TP transformation, qLPV model, LMI, observer design, controller design
Słowa kluczowe: funkcja wagowa typu CNO, odwrócone wahadło, modele TP

Introduction

This paper introduces the Tensor Product (TP) transformation based controller and observer design via Linear Matrix Inequality (LMI) methods through Higher Order Singular Value Decomposition (HOSVD) with Close to NOrmal (CNO) and Inverted and Relaxed NOrmal (IRNO) type weighting functions, where the Linear Parameter Varying (LPV) model can be described by parameter varying combination of Linear Time Invariant (LTI) systems. Due to this control design approach, the LMI results globally asymptotically stable observer and controller, in this instance for nonlinear inverted pendulum.

The TP model transformation was introduced in paper [1]. Novel researches on the subject are focused on varying the input space of the TP model [2], and extracting LPV and qLPV structures from state space representations [3].

The goal of current paper is to examine that TP transformation based globally and asymptotically stable controller and observer are feasible in LMI with different type of weighting functions. Furthermore designing of observer and controller can be separate, however present paper shows the non-separate approach. Generally, not all state variables are available, so these need to be estimated in the case of state-feedback control method. The mathematical modeling of the nonlinear system is based on Euler-Lagrange equation and for the numerical methods we use MATLAB and TPtool Toolbox [4].

Our previous papers [5] [6] investigate the HOSVD [7] [8] [9] [10] based TP transformation [11] [12] [13] [14] [15] [16] [17] [18] [19] and qLPV model representation of the inverted pendulum [20] [21] [22] [23] [24] [25] [26] [27] [28] and the design of LMI based controller [29] [30] with applying decay rate.

Inverted pendulum is a heavily unstable system and nonlinear, respectively. The pendulum mounted on cart is shown on Fig 1., where $2L$ is the length of the rod on the car, M is the mass of the rod, m is the mass of the cart in a coherent unit system SI and the generalized coordinates are horizontal position of the cart x and angular position φ. F is the actuator force. Parameters are the same as in paper [5]. For detailed mathematical description and Euler-Lagrange equation see paper [5].

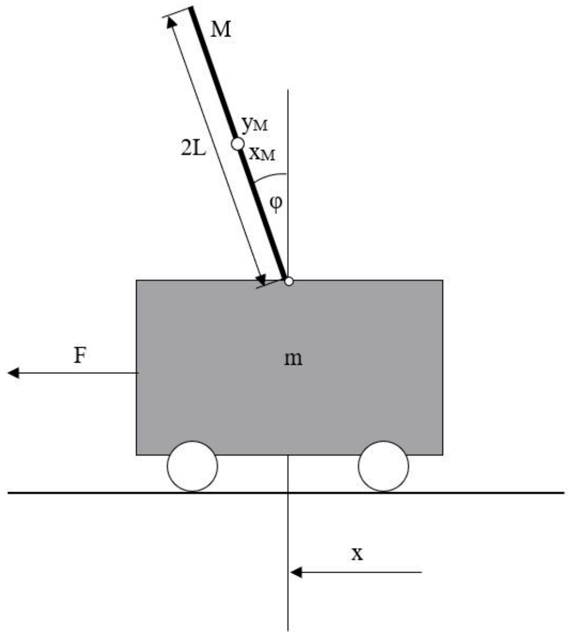

Fig. 1. Inverted Pendulum

The paper is structured as follows: Section I defines the statements and proves. Section II presents the TP model transformation of the inverted pendulum. Section III presents the design of LMI based controller and observer.

Statements and proves

Present paper investigates that selection of weighting function has influence on the TP model and LMI based controller and observer design.

Statement 1. Varying and selecting the weighting functions influence the feasibility of the LMI method and the TP model.

Statement 2. The selection of weighting functions shows how the LMI based controller and observer change, then we can choose the better controller/observer.

Proves. The proves are based on the inverted pendulum control design example. The examinations follow these key points:

1. Mathematical modeling of the inverted pendulum via Euler-Lagrange method.
2. qLPV modeling of the state space representation.
3. Applying HOSVD based method in numerical way, that generates TP models.
4. TP model transformation generates the LTI vertex systems.
5. LMI based controller and observer design.

TP model transformation

This section describes the TP model transformation of the inverted pendulum for controller and observer design.

First of all, we need to define the qLPV representation for the system:

$$\begin{bmatrix}\dot{\mathbf{x}}(t)\\ \mathbf{y}(t)\end{bmatrix} = \mathbf{S}(\mathbf{p}(t))\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{u}(t)\end{bmatrix} \tag{1}$$

where $\mathbf{x}(t)$, $\mathbf{u}(t)$ and $\mathbf{y}(t)$ are the state, input and output vectors, $\mathbf{S}(\mathbf{p}(t))$ is the system matrix respectively and parameter vector $\mathbf{p} = \mathbf{p}(t) \in \Omega$. The following state-space representation is adapted to design the controller and observer:

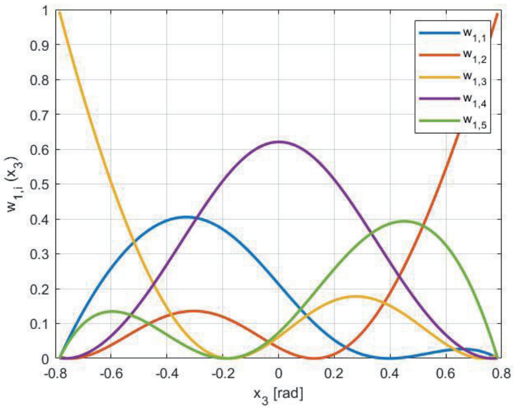

Fig. 2. CNO type weighting functions for x_3

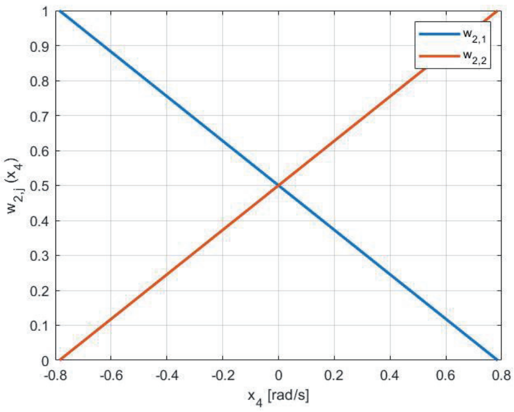

Fig. 3. CNO type weighting functions for x_4

$$\dot{\hat{x}}(t) = \mathbf{A}(\mathbf{p}(t))\hat{\mathbf{x}}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t) + \mathbf{K}(\mathbf{p}(t))(\mathbf{y}(t) - \hat{\mathbf{y}}(t)), \tag{2}$$

$$\hat{\mathbf{y}}(t) = \mathbf{C}(\mathbf{p}(t))\hat{\mathbf{x}}(t), \tag{3}$$

which can be rewritten the following state-space form:

$$\dot{\hat{x}}(t) = \mathbf{A}(\mathbf{x}(t))\hat{\mathbf{x}} + \mathbf{B}(\mathbf{x}(t))u + \mathbf{K}(\mathbf{x}(t))(\mathbf{y}(t) - \hat{\mathbf{y}}(t)), \tag{4}$$

$$\hat{\mathbf{y}}(t) = \mathbf{C}(\mathbf{x}(t))\hat{\mathbf{x}}(t). \tag{5}$$

Therefore, structure of the TP transformation is

$$\dot{\hat{x}} \cong \sum_{i=1}^{r} w_i(x_3, x_4)((\mathbf{A}_i\hat{\mathbf{x}} + \mathbf{B}_i u) + \mathbf{K}(\mathbf{y} - \hat{\mathbf{y}})). \tag{6}$$

Elements of matrix $\mathbf{A}(\boldsymbol{x}(t))$ (15) and vector $\mathbf{B}(\mathbf{x}(t))$ (16) have been determined in a numerically way. The TP model transformation is required a transformation space $\Omega = [(-45/180)\pi, (45/180)\pi] \times [(-45/180)\pi, (45/180)\pi]$, which is defined and discretized by $M_1 \times M_2 = 136 \times 136$ grid points. Hence, the elements are stored in a four dimensional tensor $S \in R^{136\times136\times4\times5}$. Mathematical modeling [5] of the nonlinear system is defined by Euler-Lagrange method:

$$(M+m)\ddot{x} + MLC_\varphi\ddot{\varphi} - MLS_\varphi\dot{\varphi}^2 = F, \tag{7}$$

$$MLC_\varphi\ddot{x} + (\Theta + ML^2)\ddot{\varphi} - MgLS_\varphi = 0. \tag{8}$$

After some mathematical manipulations of (7) and (8), the equations are as follows:

$$\ddot{x} = \frac{F - MLC_\varphi\ddot{\varphi} + MLS_\varphi\dot{\varphi}^2}{m+M}, \tag{9}$$

$$\ddot{\varphi} = \frac{gS_\varphi - \frac{C_\varphi}{m+M}(F + MLS_\varphi\dot{\varphi}^2)}{L\left(\frac{4}{3} - \frac{MC_\varphi^2}{m+M}\right)}. \tag{10}$$

Then the state-space representation of the model is

$$\dot{x}_1 = x_2, \tag{11}$$

$$\dot{x}_2 = \frac{\frac{4}{3}MLsin(x_3)x_4^2 - Mgcos(x_3)sin(x_3) + \frac{4}{3}F}{\frac{4}{3}(m+M) - Mcos^2(x_3)}, \tag{12}$$

$$\dot{x}_3 = x_4, \tag{13}$$

$$\dot{x}_4 = \frac{MLsin(x_3)cos(x_3)x_4^2 + cos(x_3)F - (m+M)gsin(x_3)}{MLsin^2(x_3) - \frac{4}{3}(m+M)L}, \tag{14}$$

where the state-space variables are $x_1 = x$, $x_2 = \dot{x}$, $x_3 = \varphi$, $x_4 = \dot{\varphi}$. The elements of matrix $\mathbf{A}(\boldsymbol{x})$ and vector $\mathbf{B}(\mathbf{x})$ only depend on the state variables x_3 and x_4.

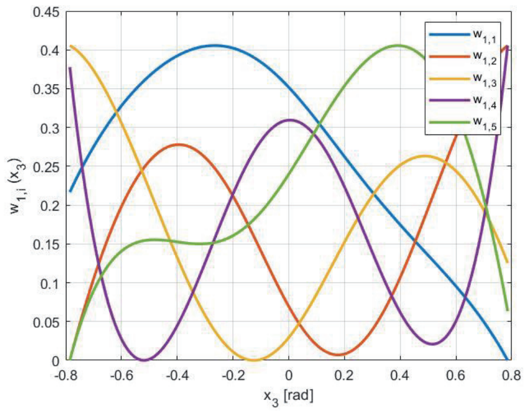

Fig. 4. IRNO type weighting functions for x_3

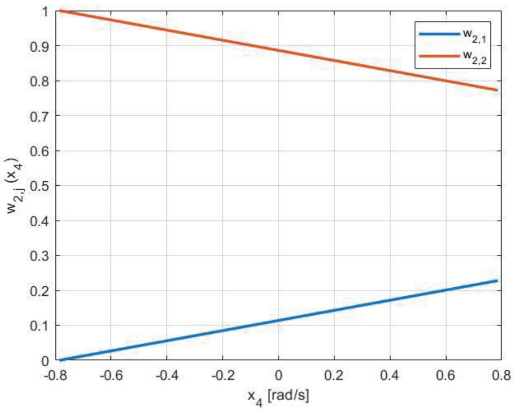

Fig. 5. IRNO type weighting functions for x_4

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-Mgcos(x_3)}{\frac{4}{3}(m+M)-Mcos^2(x_3)}\frac{sin(x_3)}{x_3} & \frac{\frac{4}{3}MLsin(x_3)}{\frac{4}{3}(m+M)-Mcos^2(x_3)}x_4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{(m+M)g}{MLsin^2(x_3)-\frac{4}{3}(m+M)L}\frac{sin(x_3)}{x_3} & \frac{MLsin(x_3)cos(x_3)}{MLsin^2(x_3)-\frac{4}{3}(m+M)L}x_4 \end{bmatrix}, \quad (15)$$

$$\mathbf{B}(\mathbf{x}) = \begin{bmatrix} 0 \\ \frac{\frac{4}{3}}{\frac{4}{3}(m+M)-Mcos^2(x_3)} \\ 0 \\ \frac{cos(x_3)}{MLsin^2(x_3)-\frac{4}{3}(m+M)L} \end{bmatrix}, \quad (16)$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad (17)$$

then the system matrix is as follow:

$$\mathbf{S}(\mathbf{x}) = \begin{bmatrix} \mathbf{A}(\mathbf{x}) & \mathbf{B}(\mathbf{x}) \\ \mathbf{C}(\mathbf{x}) & \mathbf{D}(\mathbf{x}) \end{bmatrix}. \quad (18)$$

After defining the system matrix, we use HOSVD method where the weighting functions for TP transformation are defined:

- NO: the TP function is Normal (NO), if the weighting functions are normal; the sum of the weighting functions for all $\mathbf{p} \in \mathbf{\Omega} = 1$, and the values of the weighting functions is non-negative.
- CNO: the TP function is CNO, if the weighting functions are NO and the largest value of the weighting funtcion is 1 or close to 1.
- RNO: the TP function is Relaxed Normal type, if the weighting functions are NO, and the largest values of all weighting functions are between 0 and 1.
- INO: the TP function is Inverted Normal, if the smallest value of all weighting function is 0.
- IRNO: the TP function is Inverted and Relaxed Normal, if the smallest values of all weighting functions are 0, and the largest values are the same.

After using HOSVD on $\mathbf{S}(\mathbf{x})$ (18) with CNO, IRNO type weighting functions, the following TP model transformation

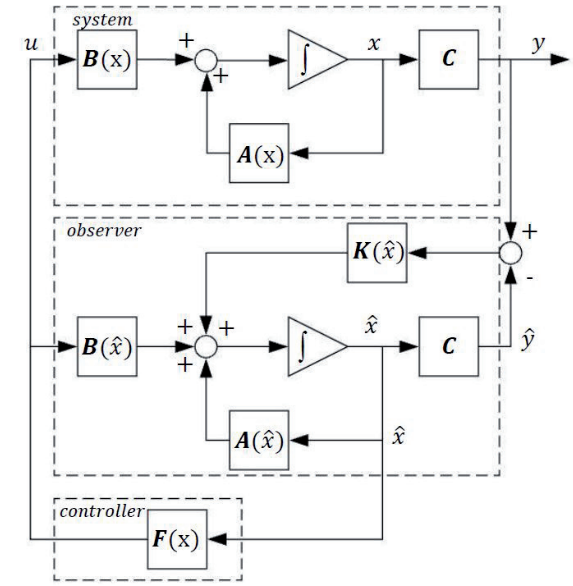

Fig. 6. Block diagram of observer and controller

describes the inverted pendulum system:

$$\dot{\mathbf{x}} \cong \sum_{i_1=1}^{r_1}\sum_{i_2=1}^{r_2} w_{i_1}(x_3)w_{i_2}(x_4)(\mathbf{A}_{i_1,i_2}\hat{\mathbf{x}} + \mathbf{B}_{i_1,i_2}u + \mathbf{K}_{i_1,i_2}(\mathbf{y}-\hat{\mathbf{y}})), \quad (19)$$

where $w_{i_1}(x_3)$ and $w_{i_2}(x_4)$ are the weighting functions, $\mathbf{A}_{i_1,i_2}$ and $\mathbf{B}_{i_1,i_2}$ are state independent system matrix and vector. $\mathbf{K}_{i_1,i_2}$ is the gain. The nonlinear system can be approximated by the combinations of $r_1 r_2$ LTI systems. It can be written as follows:

$$\dot{\mathbf{x}} \cong \sum_{i_1=1}^{5}\sum_{i_2=1}^{2} w_{i_1}(x_3)w_{i_2}(x_4)(\mathbf{A}_{i_1,i_2}\mathbf{x} + \mathbf{B}_{i_1,i_2}u + \mathbf{K}_{i_1,i_2}(\mathbf{y}-\hat{\mathbf{y}})), \quad (20)$$

$$\dot{\mathbf{x}} \cong \sum_{i=1}^{r} w_i(x_3,x_4)(\mathbf{A}_i\mathbf{x} + \mathbf{B}_i u + \mathbf{K}_{i_1,i_2}(\mathbf{y}-\hat{\mathbf{y}})). \quad (21)$$

where $r = r_1 r_2$ is the number of LTI systems with minimum $5 \times 2 = 10$ LTI vertex models. The CNO and IRNO type weighting functions are shown in Fig 2-3. and Fig 4-5. It means, that the system is decomposed in angular velocity $\dot{\varphi}$ into 2 weighting functions, and in angular position φ into 5 weighting functions. It can be seen that using various weighting function has influence on the LMI based controller and observer design. The feasibility test shows that LMIs are still feasible in both case of weighting functions.

Controller and Observer design

In this section, designing of the globally asymptotically stable controller and observer is represented investigating CNO and IRNO type weighting functions.

The observer based output-feedback design (see in Fig 6.) is determined by LMI based method, which is guaranteed that the observer is asymptotically stable [31]. The observer and the controller can be designed separately, but the TP model transformation makes it advisable to design them simultaneously.

The controller guarantees the stability of the nonlinear system and the observer satisfies the following condition:

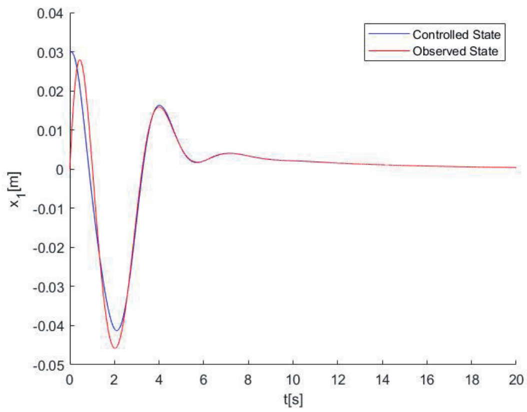

Fig. 7. State variable x_1 with CNO type weighting functions

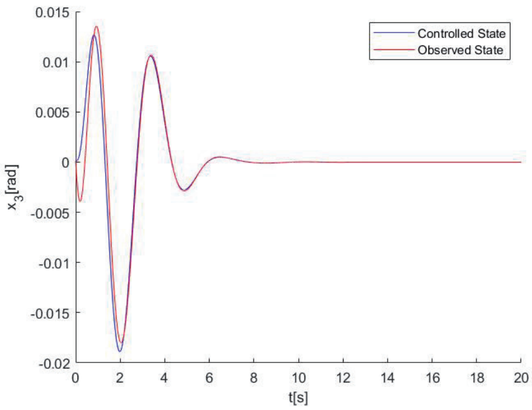

Fig. 8. State variable x_3 with CNO type weighting functions

$\mathbf{x}(t) - \hat{\mathbf{x}}(t) \to 0$, where $\hat{\mathbf{x}}(t)$ denotes the estimated state variables generated by the TP model transformation based observer. This means that the error between $\mathbf{x}(t)$ and $\hat{\mathbf{x}}(t)$ has to converge to 0. Thus, the TP model transformation based observer structure is written as follow:

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}(\mathbf{p}(t))\hat{\mathbf{x}}(t)+\mathbf{B}(\mathbf{p}(t))\mathbf{u}(t)+\mathbf{K}(\mathbf{p}(t))(\mathbf{y}(t)-\hat{\mathbf{y}}(t)), \tag{22}$$

$$\hat{\mathbf{y}}(t) = \mathbf{C}(\mathbf{p}(t))\hat{\mathbf{x}}(t). \tag{23}$$

The angular position x_3 and angular speed x_4 are observable and $p(t)$ does not contain elements from $\hat{x}(t)$.

The aim is to determine $\mathbf{K}$ gain. The observer and controller is globally asymptotically stable if there exist a common positive definite matrix $\mathbf{P}$ where $\mathbf{P}_1, \mathbf{P}_2 > 0$. Matrices $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{M}$ and $\mathbf{N}$ can be found by convex optimization methods

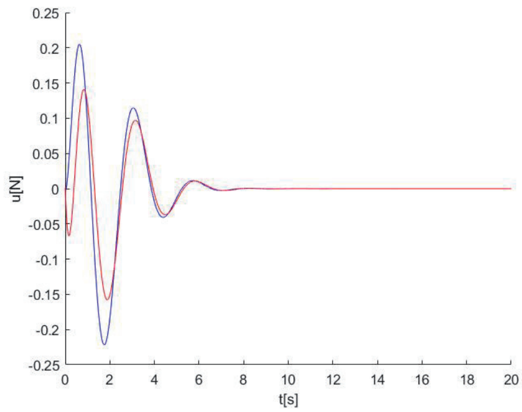

Fig. 9. Control signal u with CNO type weighting functions

involving LMIs. Therefore the LMI conditions [31] are:

$$\mathbf{P}_1\mathbf{A}_i^T - \mathbf{M}_i^T\mathbf{B}_i^T + \mathbf{A}_i\mathbf{P}_1 - \mathbf{B}_i\mathbf{M}_i \prec 0, \tag{24}$$

$$\mathbf{A}_i^T\mathbf{P}_2 - \mathbf{C}_i^T\mathbf{N}_i^T + \mathbf{P}_2\mathbf{A}_i - \mathbf{N}_i\mathbf{C}_i \prec 0, \tag{25}$$

$$\begin{aligned}\mathbf{P}_1\mathbf{A}_i^T - \mathbf{M}_j^T\mathbf{B}_i^T + \mathbf{A}_i\mathbf{P}_1 - \mathbf{B}_i\mathbf{M}_j + \mathbf{P}_1\mathbf{A}_j^T \\ -\mathbf{M}_i^T\mathbf{B}_j^T + \mathbf{A}_j\mathbf{P}_1 - \mathbf{B}_j\mathbf{M}_i \prec 0,\end{aligned} \tag{26}$$

$$\begin{aligned}\mathbf{A}_i^T\mathbf{P}_2 - \mathbf{C}_j^T\mathbf{N}_i^T + \mathbf{P}_2\mathbf{A}_i - \mathbf{N}_i\mathbf{C}_j + \mathbf{A}_j^T\mathbf{P}_2 \\ -\mathbf{C}_i^T\mathbf{N}_j^T + \mathbf{P}_2\mathbf{A}_j - \mathbf{N}_j\mathbf{C}_i \prec 0,\end{aligned} \tag{27}$$

where $\mathbf{F}_i = \mathbf{M}_i\mathbf{P}_1^{-1}$, $\mathbf{F}_j = \mathbf{M}_j\mathbf{P}_1^{-1}$, $\mathbf{K}_i = \mathbf{P}_2^{-1}\mathbf{N}_i$, $\mathbf{K}_j = \mathbf{P}_2^{-1}\mathbf{N}_j$, and $i = 1, \cdots, r$, $j = i+1, \cdots, r$, where r is the number of the total LTI vertex systems.

The output variables x_1 and x_3 and control signal u with CNO type weighting functions are shown in Fig. 7-9. Consequently, x_1,x_3 and u using the IRNO type weighting functions are indicated in Fig. 10-12. The initial condition is $\mathbf{x}(0) = [0.03, 0, 0, 0]$ in both case.

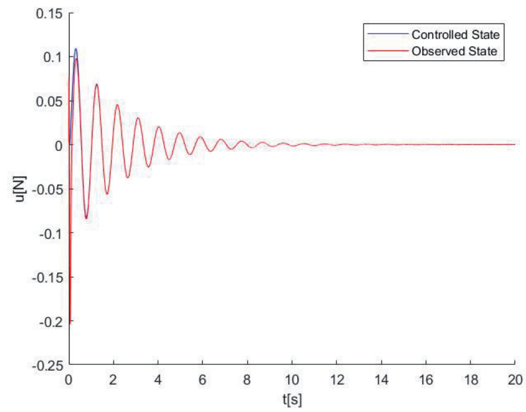

Fig. 10. Control signal u with IRNO type weighting functions

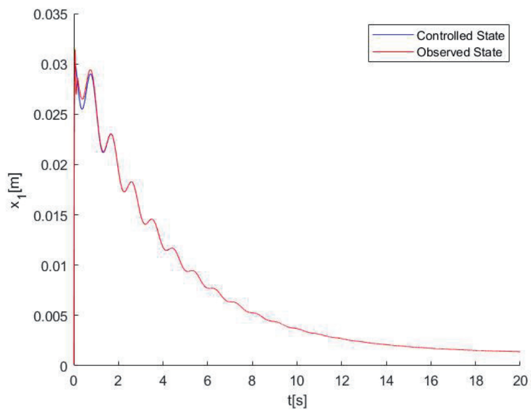

Fig. 11. State variable x_1 with IRNO type weighting functions

Applying CNO and IRNO type weighting functions has influence on performance of LMI, but still gives feasible solution for LMI. However, it is clear that the rate of the oscillations is different, but it can be seen that the system gets into stable position about 18 seconds with IRNO type weighting

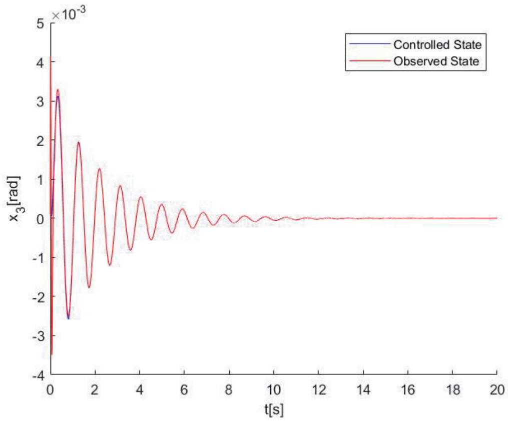

Fig. 12. State variable x_3 with IRNO type weighting functions

functions. Using CNO type weighting functions results better controller and observer, because the system reach stable state quicker, about 8 seconds.

The error between $\mathbf{x}(t)$ and $\hat{\mathbf{x}}(t)$ converges to 0, hence this statement is in accordance with the observer design criteria.

Conclusion

Current paper describes the HOSVD based TP model transformation of inverted pendulum with CNO and IRNO type weighting functions via LMI based controller and observer design method. The paper presents that selection of weighting function influences the TP model and LMI feasibility regions.

In conclusion, applying CNO and IRNO type of weighting functions, the LMI is still feasible and the resulted controller and observer is globally asymptotically stable. Using the CNO type weighting functions results better LMI based controller and observer, because the system gets into stable position about 8 seconds and controller results less oscillations.

The statements are proven through an academical example. The different types of weighting functions has influence on the feasibility regions of LMI and the TP models. This shows how the controller and observer are changed whilst we can choose the better controller and observer.

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