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Optimal anti-surge control of gas pumping unit with gas turbine drive

***Abstract**. The article is devoted to the determination of the optimal transient process in the real system of anti-surge control of a gas pumping unit with a gas turbine drive. The literature review on this topic was carried out. The problem of optimal control of the controlled object was set on the basis of the Mayer problem for the corresponding higher-order function, the optimal transition process was constructed and the optimal trajectory of the transition process was determined taking into account the obstacles. The conclusions based on the results of the research were formulated.*

***Streszczenie.** Artykuł poświęcony jest wyznaczeniu optymalnego procesu przejściowego w rzeczywistym układzie regulacji przeciwzakłóceniowej gazowego zespołu pompowego z napędem turbiną gazową. Dokonano przeglądu literatury na ten temat. Postawiono problem optymalnego sterowania obiektem sterowanym na podstawie problemu Mayera dla odpowiedniej funkcji wyższego rzędu, skonstruowano optymalny proces przejściowy oraz wyznaczono optymalną trajektorię procesu przejściowego z uwzględnieniem przeszkód. Sformułowano wnioski oparte na wynikach badań. (**Optymalne sterowanie przeciwpompażowe gazowym zespołem pompowym z napędem turbinowym**).*

Keywords: optimal control, anti-surge system, moments of switching.
Słowa kluczowe: sterowanie optymalne, układ przeciwpompażowy, momenty przełączeń.

Introduction

One of the most important tasks of optimal control of gas pumping unit with gas turbine drive is to increase the speed of such a system, which requires scientific analysis and development of appropriate hardware and software. However, this problem has to be considered separately for each gas pumping unit, as the transmission functions of each of them have unique parameters. Thus, it is important to determine the optimal transient process in the real system of anti-surge control of gas pumping unit with gas turbine drive.

In many works of such foreign scientists as S. Mirsky, W. Jacobson, D. Tiscornia, J. McWhirter, M. Zaghloul [1], T.A. Johansen [2], A. Grancharova [3] the issue of improving the anti-surge control systems of gas pumping units and, in particular, the possibility of increasing the speed of the systems is analyzed. The scientific works of such national scientists as Sementsov G.N., Lahoida A.I. [4], S.H. Hirenko [5], M.V. Becker, R. Ya. Shymko, M.O. Petesh, O.V. Sukach, A.F, Repeta [6], V.P. Herasymenko [7], Y.E, Bliaut [8], M.I. Horbiichuk, M.I. Kohutiak [9,14,15] and others are devoted to the research of the method of anti-surge control and protection of compressors against surging [10,11,12].

However, the question of studying the influence of the dynamic properties of centrifugal superchargers on the possibility of increasing the speed of the anti-surge control system remains open [13,14]. To achieve this goal it is necessary to determine the moments of switching states $t_1 = f_1(t_2)$ and $t_1 = f_2(t_2)$, as well as to build the optimal transition process on the first and second intervals.

Main part

The problem of optimal Mayer control is analyzed under the condition when the criterion is the functionality of the form

(1) $$I = I_0\left[x(t_0), x(t_k), t_0, t_k\right],$$

which depends on the boundary conditions

(2) $$x = 0; \dot{x} = 0 \text{ when } t = 0,$$

$$x = x_n; \dot{x} = 0 \text{ when } t = t_k$$

And the problem of optimal speed according to the criterion is solved

(3) $$I = \int_0^{t_k} dk = t_k - t_0 = T \underset{u(t)\in U}{\rightarrow} \min,$$

t_0, t_k – fixed start and end time, $x(t)$ – controlled value, $u(t)$ – control action.

Technological restrictions are imposed on the control actions $u \in \{0, u_{\max}\}$ of the controlled object

(4) $$u_{i_{\min}} \le u_i \le u_{i_{\max}} .$$

Therefore, the problem of synthesis of the optimal speed control system is to determine the control algorithm $u(x_1, x_2, x_3, ..., x_n, t)$ which provides the transition control object from the initial state to the specified possible time with restrictions on action control.

It should be noted that, in the general case, the time-optimal control is determined by the Pontriagin maximum assumption procedure [10]. In the general case, the solution of the problem of optimal control of a nonlinear object is reduced to the calculation of the moments of switching control effects to obtain the optimal control program $u^*(t)$.

Calculation of a closed anti-surge control system, optimal for speed. The control object is described by the following equation [11]:

(5) $$T_1 T_2 \frac{d^2 x}{dt^2} + (T_1 + T_2)\frac{dx}{dt} + x = kU$$

where $T_1 = 0.5$s and $T_2 = 0.3$s are the time constants, k is the transmission factor and

(6) $$|u| \le u_{\max} = 127$$

Table 1. Estimated and real values of coordinates

Coordinates	*t*,s					
x	0	0.2	0.4	0.6	0.8	1.0
x_p	0	20.98	74.02	122.62	163.5	196.2
x_f	0	23.08	81.42	134.89	178.9	215.8

The object operates under the influence of an obstacle. The calculated and actual values of the x coordinates are not shown in table 1.

Extrapolation of the obstacle of the polynomial $x_f = a_0 + a_1 t$ based on the data of tab.1., allowed to establish that

(7) $$x_f = -3,06 + 26,16t$$

from the time $t_{\Pi} = 0,4s$. This helps to determine the switching moments t_1 and t_2 taking into account the obstacle acting on the control object. It is necessary to determine the control algorithm that transfers the control object position, to the position $x = x_n; \dot{x} = 0$ in a minimum of time; the control action is limited to. $|u| \le u_{\max}$

To solve this problem, we write equation (5) as a system of two equations

(8) $$\begin{cases} T_1 \dfrac{dx_1}{dt} + x_1 = kU, \\ T_2 \dfrac{dx}{dt} + x = x_1 \end{cases}$$

From (8) we define

(9) $$\begin{cases} \dfrac{dx_1}{dt} = \dfrac{1}{T_1} + (ku - x_1) = f_1, \\ \dfrac{dx}{dt} = \dfrac{1}{T_2} + (x_1 - x) = f_2. \end{cases}$$

On the base of equation (9), using the principle of the maximum equation for conjugate variables ψ and we find

(10) $$\begin{cases} \psi_1 = -e^{\frac{1}{T_2}} = \left[\int \dfrac{1}{T_2} c_0 e^{\frac{t}{T_2}} e^{\frac{t}{T_1}} dt + c_2 \right] = \\ = c_0 e^{\frac{t}{T_2}} - c_2 e^{\frac{t}{T_1}}; \\ \psi_2 = c_0 e^{\frac{t}{T_2}} \end{cases}$$

Now we compose Hamilton's function [12]

(11) $$H^* = \psi_1 \frac{1}{T_1}(ku - x_1) - \psi_2 \frac{1}{T_2}(x_1 - x).$$

We take only the term that depends on u, and taking into account the values of ψ_1 and ψ_2, using (11) we obtain

(12) $$H^* = \psi_1 \frac{1}{T_1}(ku - x_1) = \left(c_1 e^{\frac{t}{T_2}} - c_2 e^{\frac{t}{T_1}} \right) \frac{k}{T_1} u.$$

For the function to have a maximum value, as required by the maximum principle, it is necessary to comply with the condition $u = u_{\max}$ and ensure that the sign $u_{\max}$ is changed as many times as it is changed by the function ψ_1.

So you can write that

(13) $$H^*_{\max} = \left(c_1 e^{\frac{t}{T_2}} - c_2 e^{\frac{t}{T_1}} \right) \frac{k}{T_1} u_{\max}.$$

Then we determine the switching moments t_1, t_2 and construct the optimal transition process.

To find the optimal control function, we choose the method of joining differential equations with the alternating right particle. [12,13].

So we write equation (5) on the first control interval

(14) $$x(t) = c_0' + c_1' e^{-\alpha_1 t} + c_2' e^{-\alpha_2 t}, \quad c_0' = ku$$

c_1', c_2' - constant integrations on the first control interval, $-\alpha_1 = -1/T_1; -\alpha_2 = -1/T_2$ - roots of the characteristic equation.

Taking into account the initial conditions at $t = 0, x = 0, \dot{x} = 0$, we determine the integration constants

(15) $$\begin{cases} x = c_0' + c_1' + c_2' = 0; \\ \dot{x} = -\alpha_1 c_1' + \alpha_2 c_2' = 0. \end{cases}$$

Now we write equation (15) for the second control interval

(16) $$x(t) = c_0'' + c_1'' e^{-\alpha_1 t} + c_2'' e^{-\alpha_2 t}, \quad c_0'' = ku$$

where $c_1''; c_2''$ - constant integration on the second control interval

Using the final conditions at $t = t_2, x = x_n, \dot{x} = 0$, we determine the integration constants

(17) $$\begin{cases} x_n = c_0'' + c_1'' e^{-\alpha_1 t_2} + c_2'' e^{-\alpha_2 t_2}; \\ \dot{x} = -\alpha_1 c_1'' e^{-\alpha_1 t_2} + \alpha_2 c_2'' e^{-\alpha_2 t_2} = 0. \end{cases}$$

Using the system of equations (17) we obtain

(18) $$\begin{cases} c_1'' = \dfrac{\alpha_2 (x_n + ku) e^{\alpha_1 t_2}}{\alpha_1 - \alpha_2}, \\ c_2'' = \dfrac{\alpha_1 (x_n + ku) e^{\alpha_2 t_2}}{\alpha_1 - \alpha_2}. \end{cases}$$

We join the equation at the time of switching t_1

(19) $$\begin{cases} c_0' + c_1' e^{-\alpha_1 t} + c_2' e^{-\alpha_2 t} = c_0'' + c_1'' e^{-\alpha_1 t_1} + c_2' e^{-\alpha_2 t_1} \\ -\alpha_1 c_1' e^{-\alpha_1 t_1} - \alpha_2 c_2' e^{-\alpha_2 t_1} = -\alpha_1 c_1'' e^{-\alpha_1 t_1} + \alpha_2 c_2'' e^{-\alpha_2 t_1} \end{cases}$$

After simplification, we obtain such a system of equations to determine the switching moments

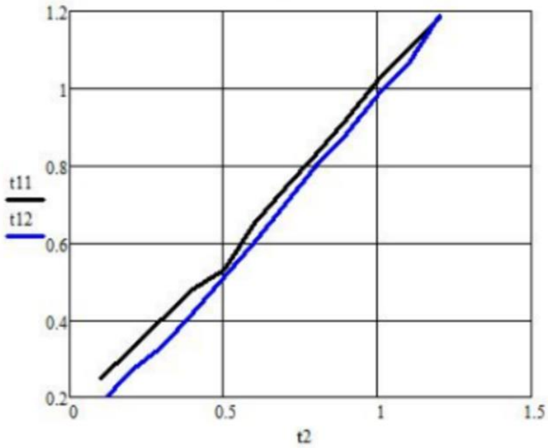
(20) $$\begin{cases} \left(1 + \dfrac{x_n}{ku}\right) e^{\alpha_1 t_2} - 2e^{\alpha_1 t_1} + 1 = 0 \\ \left(1 + \dfrac{x_n}{ku}\right) e^{\alpha_2 t_2} - 2e^{\alpha_2 t_1} + 1 = 0 \end{cases}$$

After substituting the numerical values of the coefficients into the equation of system (20) we obtain

(21) $$\begin{cases} 1,86e^{2t_2} - 2e^{2t_1} + 1 = 0 \\ 1,86e^{3,3t_2} - 2e^{3,3t_1} + 1 = 0 \end{cases}$$

The system of equations which can only be solved approximately, using, for example, the graphic method.

Graphs of the dependence $t_1 = f_1(t_2)$ and $t_1 = f_2(t_2)$ are shown in Figure 1. The point of their intersection gives the value $t_1 = 1.14\text{s}, t_2 = 1.16\text{s}$. The difference of switching moments. $\Delta t = t_2 - t_1 = 0,02s$



$$t11 := \begin{pmatrix} 0.25 \\ 0.32 \\ 0.4 \\ 0.48 \\ 0.53 \\ 0.65 \\ 0.74 \\ 0.83 \\ 0.92 \\ 1.02 \\ 1.1 \\ 1.18 \end{pmatrix} \quad t12 := \begin{pmatrix} 0.18 \\ 0.27 \\ 0.33 \\ 0.42 \\ 0.51 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.88 \\ 0.98 \\ 1.06 \\ 1.19 \end{pmatrix} \quad t2 := \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1 \\ 1.1 \\ 1.2 \end{pmatrix}$$

Fig. 1. Graphical solution of two equations

Construction of the optimal transition process

To construct the optimal transient process on the first interval in expression (14) it is necessary to substitute the values of the integrations from the system (16). Then

$$x(t)=\left(1+\frac{\alpha_2}{\alpha_1-\alpha_2}e^{-\alpha_1 t}-\frac{\alpha_1}{\alpha_1-\alpha_2}e^{-\alpha_2 t}\right)ku .$$

The optimal transition process on the second interval $0 \le t \le t_2 - t_1$ is described by equation (17). If we conditionally take the switching moment for the reference $t=0$, we can write

(22) $$x(0)=x_1; \frac{dx}{dt}(0)=x_1, \quad x_1=c_0''+c_1''+c_2''; x_1=-\alpha_1 c_1''-\alpha_2 c_2''.$$

From these relations we define constant integrations:

(23) $$\begin{cases} c_2''=\dfrac{\alpha_1}{\alpha_1-\alpha_2}(x_1-ku)+\dfrac{\dot{x}_1}{\alpha_1-\alpha_2}, \\ c_1''=-\dfrac{\alpha_2}{\alpha_1-\alpha_2}(x_1+ku)-\dfrac{\dot{x}_1}{\alpha_1-\alpha_2}, \end{cases}$$

After substituting c_1'' i c_2'' in equation (17) we have

(24) $$x(t)=-ku-\left[\frac{\alpha_2}{\alpha_1-\alpha_2}(x_1+ku)\frac{\alpha_1}{\alpha_1-\alpha_2}\right]e^{-\alpha_2 t}+ \left[\frac{\alpha_1}{\alpha_1-\alpha_2}(x_1+ku)+\frac{\alpha_1}{\alpha_1-\alpha_2}\right]e^{-\alpha_2 t}= -ku+\left[\left(\alpha_1 e^{-\alpha_2 t}-\alpha_2 e^{-\alpha_1 t}\right)(x_1+ku)+x_1\left(e^{-\alpha_2 t}-e^{-\alpha_1 t}\right)\right]\frac{1}{\alpha_1-\alpha_2}$$

Here $x_1=223$ units – from the solution on the first interval. Then

(25) $$\dot{x}_1=\left(2.0\cdot 2.54\cdot e^{-2\cdot 1.14}-1.54\cdot 3.3\cdot e^{-3.3\cdot 1.14}\right)272=115$$

After substituting the numerical values in equation (24) we determine $x(t)$ for $t=0,02$ as well the second interval is very small. Then

(26) $$x(t)=-272- -\left[\left(2\cdot 0.93-3.3\cdot 0.96\right)\cdot 495+115\left(0.93-0.96\right)\right]\cdot 0.77= =-272+650\cdot 0.77=227.5$$

The next step is to determine the switching moments t_1 and t_2, taking into account the obstacle acting on the OK and the calculation of the optimal trajectory.

Obstacle $x_f=a_0+a_1 t$ acts on the control object from time $t_{\Pi}=0,4s, a_0=-3,06; a_1=26,16$.

To determine the switching states t_1, and t_2 we again use the method of joining the solutions of differential equations with the alternating right-hand side.

From the decision (27)

(27) $$x(t)=c_0+c_1 e^{-\alpha_1 t}+c_2 e^{-\alpha_2 t}+a_0+a_1, \quad \dot{x}(t)=-\alpha_1 c_1 e^{-\alpha_1 t}-\alpha_2 c_2 e^{-\alpha_2 t}+a_0 .$$

on the first and second intervals, we define constant integrations.

To do this, we note the time $t_{\Pi}=0.4$s. A new countdown $t=0$. Therefore, control starts from the point $x_{1p}=74.2; \dot{x}_{1p}=261.6$ (Table 1). Then the decision to start management will look like this

(28) $$c_0'+c_1'+c_2'+a_0=x_{1p} \quad -\alpha_1 c_1'-\alpha_2 c_2'+a_1=\dot{x}_{1p}$$

We find the values of the integration constants from the finite conditions by writing at the end of the second interval at $t=t_2, x=x_n, \dot{x}=0$.

(29) $$x_n=c_0''+c_1'' e^{-\alpha_1 t_2}+c_2'' e^{-\alpha_2 t_2}+a_0+a_1 t_2 \quad \dot{x}=-\infty_1 c_1'' e^{-\alpha_1 t_2}-\alpha_2 c_1'' e^{-\alpha_2 t_2}+a_1=0$$

Join the equation at the time of switching t_1

(30) $$c_0'+c_1' e^{-\alpha_1 t_1}+c_2' e^{-\alpha_2 t_1}+a_0+a_1 t= =c_0''+c_1'' e^{-\alpha_1 t_2}+c_2'' e^{-\alpha_2 t_2}+a_0+a_1 t_1; -\alpha_1 c_1' e^{-\alpha_1 t_1}-\alpha_2 c_1' e^{-\alpha_2 t_1}+a_1= =-\alpha_1 c_1'' e^{-\alpha_1 t_1}-\alpha_2 c_1'' e^{-\alpha_2 t_1}+a_1.$$

Subtracting the integration steel from these equations, we obtain the following system of equations to determine the switching moments:

(31) $$\left(1+\frac{x_n-a_0}{ku}-\frac{a_1}{\alpha_2 ku}-\frac{a_1 t_2}{ku}\right)e^{\alpha_1 t_2}- -2e^{\alpha_1 t_1}\left(1+\frac{a_0-x_{1p}}{ku}-\frac{a_1-\dot{x}_{1p}}{\alpha_2 ku}\right)=0; \left(1+\frac{x_n-a_0}{ku}-\frac{a_1}{\alpha_1 ku}-\frac{a_1 t_2}{ku}\right)e^{\alpha_2 t_2}- -2e^{\alpha_2 t_1}\left(1+\frac{a_0-x_{1p}}{ku}-\frac{a_1-\dot{x}_{1p}}{\alpha_1 ku}\right)=0.$$

After substituting numerical values, we obtain two transcendental equations

$$(32)\quad \begin{aligned}&\left(1.85\text{-}0.096\mathrm{t}_2\right)e^{2t_2}-2e^{2t_1}+0.4=0;\\&\left(1.83\text{-}0.096\mathrm{t}_2\right)e^{3.3t_2}-2e^{3.3t_1}+0.22=0.\end{aligned}$$

A graph of the solution of equations (26), from which we determine

$$t_1=0.64\mathrm{s}, t_2=0.67\mathrm{s}$$

$$\Delta t=t_2-t_1=0.67-0.64=0.03\mathrm{s}$$

Conclusions

Being based on the research, the optimal transient process in the system of anti-surge control of gas pumping unit with gas turbine drive was obtained.

The time of the transient process shows that they differ significantly from the transient process in the system for the case when the input of the controlled object is served with not optimal control action. In the course of the research, the following items were determined: - switching states of the different switches $t_1=f_1(t_2)$ and $t_1=f_2(t_2)$ at the point (1,14; 1,16), the difference of switching states $\Delta t=t_2-t_1=0.02\mathrm{s}$. The switching states t_1 and t_2 were determined taking into account the obstacle, acting on the controlled object. The direction of further research is to develop such control laws that would ensure the optimal duration of the transition process both with the using the methods of the investigated processes mathematical simulation. It can be realized in two ways – the new model design and the considering more difficult boundary and initial conditions and process parameters with the numerical methods using for the modelling realization – both with calculation schemes convergence and stability investigation.

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