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Analysis of Linear Periodically Time-Varying Circuits by the Frequency Symbolic Method with Applying the D-Trees Method

***Abstract**. The D-trees method ensures near optimal factoring in the formed algebraic equations, which results in a significant cut-down of the time needed for their formation. The analysis of the example linear circuits with fixed and variable parameters presented in the paper revealed that application of the D-trees methods ensures from 10-to-100-fold time saving as compared to the standard MATLAB tools. Such reduction of time allows a considerable rise in the efficiency of the FS method in problems of statistical studies or optimization of electronic devices modelled by linear periodically time-varying circuits*

***Streszczenie.** Analiza przykładowych obwodów liniowych o stałych i zmiennych parametrach przedstawionych w artykule wykazała, że zastosowanie metody D-trees zapewnia od 10 do 100-krotną oszczędność czasu w porównaniu ze standardowymi narzędziami MATLAB. Takie skrócenie czasu pozwala na znaczny wzrost efektywności metody FS w problemach badań statystycznych lub optymalizacji urządzeń elektronicznych modelowanych przez liniowe obwody zmieniające się okresowo w czasie. (**Analiza obwodów liniowych okresowo zmieniających się w czasie z zastosowaniem metody D-trees**)*

Keywords: circuit analysis computing, frequency symbolic models, frequency symbolic method, linear periodically time-varying circuits.
Słowa kluczowe: metodas D-trees, analiza obwodów liniowych

Introduction

A number of multivariate problems that require multiple computing of the characteristics of electronic circuits at the change of their elements' parameters can be conveniently solved by symbolic methods. These methods provide for the formation in the PC memory of the formula for computing the needed circuit characteristic, in which the parameters of the circuit elements are presented by symbols. Further, on, the formula is multiple-computed for various sets of numerical values of these parameters. This technique is used, for instance, for problems of statistical analysis and optimization of electronic devices.

The proposed research focuses on the analysis of electronic devices modelled by linear electric circuits with time-varying parameters. The speed of parameter variation is commensurate with the speed of variation of the circuit signals [1,2,3,4,5]. Such circuits are linear periodically time-varying (LPTV) circuits. The study used the symbolic analysis of LPTV circuits, which relies on the solution of symbolic systems of linear algebraic equations (SSLAE) in the form of rational fraction algebraic symbolic equations. Importantly, the time of formation of these expressions and the number of further substitutions of numerical values in them determine the speed of obtaining the results of designing the device as a whole. The required number of substitutions in said problems can reach hundreds or thousands, which is why the speed becomes a defining factor of effective solution of the problem within reasonable time limits.

The proposed symbolic method is a frequency method as it models a LPTV circuit in the frequency domain. Therefore, the paper discusses the frequency symbolic method (FS method) [2,3] of analysis of the stationary mode of LPTV circuits in the frequency domain. The method bases on the approximation of the parametric transfer functions $W(s,t)$ of the circuit in the form of the Fourier polynomials, where s is the complex variable and t is the time variable. In transfer functions, the user-selected parameters of the circuit elements and variables s and t are presented by symbols, therefore such operations as factoring, differentiation or integration with respect to these parameters and variables can be applied to them. This increases the self-descriptiveness and, consequently, the value of such functions. According to the FS method, transfer functions, or, to be exact, the coefficients of the corresponding Fourier polynomials, are the solutions of SSLAE, which can be fairly high-order. The drawback of solving such SSLAE using conventional methods and tools is that the process is tedious, requires a significantly longer computing time for higher-order SSLAE or there can even be no solution [3]. In our research, we propose eliminating this drawback or diminishing its effect by applying a sub-circuits method, namely the D-trees method [3]. The method is software implemented in the form of the D_trees() function in the system UDF MAOPCs [5,6]. The efficiency of the D_trees() function is comparable with the features offered by the symbolic solution of SSLAE in the MATLAB environment.

The rc-circuits were selected as test circuits. They consist of resistors r and capacities c with lumped parameters and can model long two-wire lines which contain both fixed parameters and variable ones [7].

The principles of the D-trees method

The D-trees method is a method of analysis of complex linear electronic circuits with fixed parameters. It is based on the division of a certain circuit into parts (sub-circuits), analysis of these sub-circuits and synthesis of the results in order to determine the characteristics of the whole circuit. At the nodes, the circuit is divided into sub-circuits (further referred to as initial sub-circuits). For the initial sub-circuits, as it will be demonstrated below, the sets of certain algebraic degree polynomials are calculated as functions of the complex variable s, which fully characterizes these circuits. Then these sub-circuits with shared nodes (neighbouring sub-circuits) are paired to form higher-order sub-circuits. Due to the pairing of two sub-circuits, based on the sets of polynomials of these two sub-circuits, a similar set of polynomials of the resulting sub-circuit can be computed. All the sub-circuits (both initial and combined) are described by similar polynomial sets, which standardizes the combination of two neighbouring circuits into one regardless of the number of initial sub-circuits that these two sub-circuits comprise. After (n-1) pairings (where n is the number of initial sub-circuits in the circuit), such pairing of sub-circuits results in a polynomial set of the whole circuit. This set expressly defines all possible transfer functions of the circuit between its inputs and outputs.

It is shown in [3] that several neighbouring elements or even separate elements of the circuit can be selected as the initial sub-circuits. The developed algorithm for determining the sequence of combining sub-circuits into pairs ensures minimal or near minimal duration of the analysis of the whole circuit.

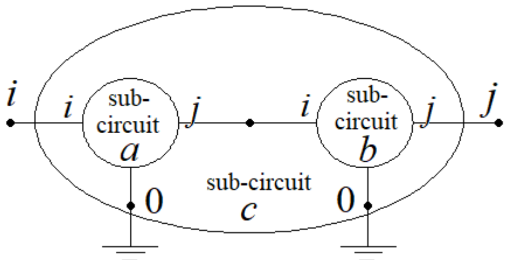

Fig.1. Combining the sub-circuits a and b into the sub-circuit c

Combining two neighbouring sub-circuits

Let us assume that a certain circuit is dividable into three-pole sub-circuits, and two neighbouring sub-circuits a and b form a combined sub-circuit c, as shown in Fig.1. The D-trees method states that for each such pair of sub-circuits a, b and the combined sub-circuit c (since they are three-pole circuits), the polynomial sets describing them are identical and are presented by the determinant of the conductivity matrix of a respective sub-circuit and its algebraic complements [3]:

(1) $$\Delta, \Delta_{ij}, \Delta_{ii}, \Delta_{ji}, \Delta_{jj}, \Delta_{ii,jj}$$

It is known [3] that this determinant and algebraic complements are equal to the weights of certain d-trees (or the sum of the weights of the d-trees) of the conductivity graph of such sub-circuit; they are degree polynomials of the complex variable s. In our research, we opted for the matrix form as it is more widespread. Then, according to the D-trees method, the correlations between the determinants and algebraic complements of the sub-circuits a, b and the combined sub-circuit c will appear as [3]:

(2) $$\begin{aligned} &\Delta^c = \Delta^a \cdot \Delta^b_{ii} + \Delta^a_{jj} \cdot \Delta^b, \Delta^c_{ij} = \Delta^a_{ij} \cdot \Delta^b_{ij}, \\ &\Delta^c_{ii} = \Delta^a_{ii,jj} \cdot \Delta^b + \Delta^a_{ii} \cdot \Delta^b_{ii}, \Delta^c_{ji} = \Delta^a_{ji} \cdot \Delta^b_{ji}, \\ &\Delta^c_{jj} = \Delta^a \cdot \Delta^b_{ii,jj} + \Delta^a_{jj} \cdot \Delta^b_{jj}, \Delta^c_{ii,jj} = \Delta^a_{ii} \cdot \Delta^b_{ii,jj} + \Delta^a_{ii,jj} \cdot \Delta^b_{jj}. \end{aligned}$$

In (2) the superscripts signify the sub-circuit a, b or c that a determinant or algebraic complement belongs to. It should be noted that the sub-circuit c is described by the same number of expressions (1) as the sub-circuits a and b that the sub-circuit c is formed of. For pairing the sub-circuits a and b we assume that each sub-circuit (initial sub-circuits and all those obtained by their combination) is described by six expressions (1), the formation of which for initial sub-circuits poses no difficulties. Then combining two sub-circuits into one resulting sub-circuit consists in computing the expressions of the form (1) for the combined sub-circuit based on (2), using the known expressions (1) for these two sub-circuits. The expressions (1) obtained for the whole circuit determine all the transfer functions of this circuit with respect to its inputs and outputs. For better understanding of the similarities between the matrix representation of the sub-circuits and their representation using a set of D-trees (1), the following explanation can be made. According to [8], the matrix equation characterizing the multi-pole sub-circuits a or b in Fig. 1 in relation to their external nodes i, j is written using the algebraic complements of the conductivity matrix of the corresponding sub-circuit as:

(3) $$\frac{1}{\Delta_{ii,jj}} \cdot \begin{array}{c} \begin{array}{cc} i & j \end{array} \\ \begin{bmatrix} \Delta_{jj} & -\Delta_{ji} \\ -\Delta_{ij} & \Delta_{ii} \end{bmatrix} \end{array} \cdot \begin{bmatrix} U_i \\ U_j \end{bmatrix} = \begin{bmatrix} I_i \\ I_j \end{bmatrix}$$

where $U_i, U_j -$ denote node voltages between the nodes $i,0$ and $j,0$; $I_i, I_j -$ are currents in the nodes i and j ; $\Delta_{ii}, \Delta_{ij}, \Delta_{jj}, \Delta_{ji}, \Delta_{ii,jj} -$ are algebraic complements of the conductivity matrix of the sub-circuit. The matrix equation (3) completely describes the sub-circuit and therefore can be a basis for its combination with other sub-circuits. This can be done even using the expressions (2). For this to be done, five algebraic complements from (3) suffice, as they define the determinant Δ of the sub-circuit matrix and together form its set (1). For instance, according to Jacobi's theorem [8], Δ is determined from (3) as:

(4) $$\Delta = \frac{1}{\Delta_{ii,jj}} \cdot \begin{vmatrix} \Delta_{jj} & -\Delta_{ji} \\ -\Delta_{ij} & \Delta_{ii} \end{vmatrix}.$$

However, as a result of making substitutions with (4) for the corresponding sub-circuits into the expressions (2), they come to have denominators. As the combination process proceeds, the number of such algebraic expressions with denominators will increase drastically and they will become complex fractions. This will result in a significant extension of the computing time required for combining the sub-circuits between themselves. The formation of symbolic transfer functions in such a form becomes tedious and most probably unnecessary.

On the other hand, the determinant Δ of the initial sub-circuit a or b in Fig. 1, as well as its other algebraic complements from (3), can be found from the conductivity matrix of this sub-circuit using the known rules [8] in the form of degree polynomials of *s* with no denominators. The sets (1) of the sub-circuits a and b formed in this way according to the expressions (2) determine the set (1) of the combined sub-circuit c with no denominators either. It is obvious that the result of using the expressions (1) and (2) for combining the previously combined sub-circuits will not comprise denominators. This is the main advantage of the D-trees method. It is hoped that these explanations make the application of the D-trees method easy-to-understand and practicable.

The D-three method presented herein is designed for arbitrary possible combinations of multi-pole sub-circuits between themselves by shared nodes [3]; an algorithm for a time-optimal sequence of their combination was developed. Although the D-trees method appeared in the 1970-ies [3], it has not lost its importance. This is due to the fact that the method ensures optimal or near optimal factoring in the formed algebraic expressions, this ensuring that the number of required arithmetic operations is close to minimal. We affirm that owing to that, the D-trees method has long demonstrated its efficacy as compared to other symbolic methods, in particular the methods implemented in the MATLAB det() function [9]. This is confirmed by the experimental outcomes presented below. Therefore, we believe that the application of the D-trees method in the FS method [3] in problems of analysis of linear parametric circuits is a logical way to improve the efficiency of the system UDF MAOPCs developed by the authors [6]. This is true both from the standpoint of increasing the admissible complexity of circuits and from the point of view of increasing the speed of the system in the parametric analysis of parametric circuits in the frequency domain.

Comparison of the D_trees() and det() functions in test circuits

Three types of linear rc-circuits were taken as test circuits, each formed of in-series rc-elements shown in Fig. 2a, Fig. 2b or Fig. 2c. At the driving point of each circuit there is a power supply source *i(t).* Each element of the circuit for the D-trees method is considered an initial sub-circuit. The number of elements n in the test circuit is selected depending on the needs of a specific computer experiment. The first type of linear circuit is the circuit with fixed parameters. The second type has variable parameters, and the parameter of all parametric capacities varies synchronously in time according to the expression:

(5) $$c(t) = c_0(1 + m \cdot \cos(\Omega t)),$$

where c_0, m, Ω are the mean value of the capacity, its modulation depth and pumping frequency, respectively. The third type also contains time-varying, though different $c_p(t)$, capacities:

(6) $$c_p(t) = c_0(1 + m \cdot \cos(\Omega t + \varphi_p)), \quad p = 1, 2, \ldots, n,$$

in which the initial phase φ_p of the change of the capacity parameter in each element has a steady increment with respect to the initial phase φ_{p-1} of the change of the capacity of previous element. This increment is such that regardless of the number of the elements n from the first element to the last (numbering them from left to right) φ varies steadily from 0 to π radians ($\varphi_0 = 0, \varphi_n = \pi$).

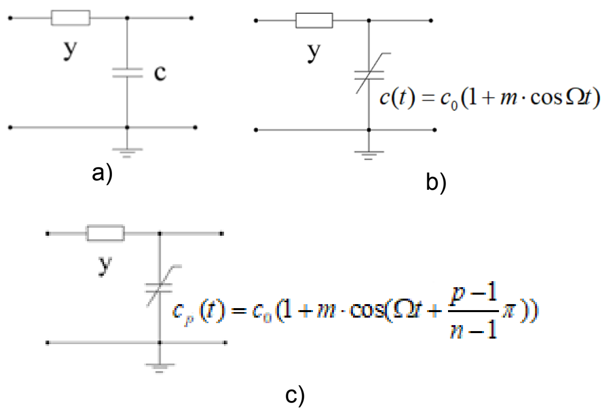

Fig. 2. Elements rc $(r = 1/y)$ by in-series connection of which test circuits are formed: a) circuits with fixed parameters; b) circuits with synchronously time-varying capacity parameters; c) circuits with time-varying capacity parameters with the delay of the phase of their variation ($p = 1, 2, ..., n$ is the ordinal number of the element)

The selection of such test circuits is associated with a number of features. All combinations of sub-circuits occur according to the non-complex expressions (2) presented above; each initial sub-circuit of the parametric circuit (each element of the circuit) contains a parametric element; all the variants of the circuits can be physically implemented. In particular, they can model two-wire long lines with fixed or variable parameters, as, for instance, in [7].

To simplify computer experiments, the solution of SSLAE will be understood as determination not of all the transfer functions but only of those that co-relate the circuit inputs with its outputs, which, in their turn, are determined by the determinant and algebraic complements of the corresponding conductivity matrix of the circuit. For further simplification of the computer experiments, only the determinant of the SSLAE matrix of the circuit is calculated, as computation of the algebraic complements can be performed in the similar way using the preliminary modified matrix by Cramer's rule [10]. At that, we should remember that the D-trees method by definition provides for the determination of not only the determinant, but also of all its algebraic complements needed for the formation of the transfer functions.

For the test circuits the determinants of the matrices of the respective SSLAE are calculated using both functions, D_trees() and det(), on condition that all the parameters of the circuit elements are designated by symbols. The values of the computing time spent by the two functions are presented in the tables. As the symbolic expressions formed by both functions were rather bulky, their identity was every time checked by verification that their difference was equal to zero.

Computer experiments

Computer experiment 1. Problem. Assess the time of formation of the determinant Δ of the conductivity matrix for a circuit formed by in-series connection of n rc-elements from Fig. 2a, using the D_trees() and det() functions for all the parameters of the circuit elements designated by symbols. Each time the number of elements n is to be increased from $n = 16$ until the system message «Out of memory».

Solution. The system of equations written by the node voltage method has a dimension $(n+1)$ for each value of n. For instance, for $n = 2$ it will be as follows:

(7) $$\begin{bmatrix} y & -y & 0 \\ -y & 2y+sc & -y \\ 0 & -y & y+sc \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ 0 \end{bmatrix} \quad or \quad Y(s) \cdot U(s) = I(s)$$

For the convenience of description of further experiments, we write (7) as:

(8) $$\begin{bmatrix} y & -y & 0 \\ -y & 2y+sc & -y \\ 0 & -y & y+sc \end{bmatrix} \cdot \begin{bmatrix} W_{11} \\ W_{21} \\ W_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad or \quad Y(s) \cdot W(s) = 1,$$

where $W_{11}(s) = U_1/I_1$, $W_{21}(s) = U_2/I_1$, $W_{31}(s) = U_3/I_1$.

We can build SSLAE and conductivity matrix Y of the circuit for $n > 2$. However, this is unnecessary. According to the D-trees method, each rc-element of the circuit is considered to be an initial sub-circuit, for which the conductivity matrix is

(9) $$\begin{bmatrix} y & -y \\ -y & y+sc \end{bmatrix},$$

and the polynomial set for it will appear as

$$\Delta = ysc,\ \Delta_{ii} = y + sc,\ \Delta_{ij} = y,\ \Delta_{jj} = y,\ \Delta_{ji} = y, \Delta_{ii,jj} = 1.$$

The sequence of sub-circuits combination for $n = 4$ is shown in Fig. 3. For instance, for $n = 4$ the result of combining two initial sub-circuits 1 and 2 into a sub-circuit 5 or sub-circuits 3 and 4 into a sub-circuit 6 from Fig. 3 according to the expressions (2) is a set of polynomials

$$\Delta = c \cdot s \cdot y^2 + c \cdot s \cdot y \cdot (y + c \cdot s);\ \Delta_{ii} = (y + s \cdot c)^2 + y \cdot s \cdot c;$$

$$\Delta_{ij} = y^2;\ \Delta_{jj} = (y + s \cdot c) \cdot y;\ \Delta_{ji} = y^2;\ \Delta_{ii,jj} = 2y + c \cdot s.$$

The next combination of the two neighbouring sub-circuits 5 and 6 from Fig. 3, according to those same expressions (2) results in the following set of polynomials:

$$\Delta = ((y + cs)^2 + cs \cdot y) \cdot (cs \cdot y^2 + cs \cdot y \cdot (y + cs)) +$$
$$+(y^2 + cs \cdot y) \cdot (cs \cdot y^2 + cs \cdot y \cdot (y + cs));\ \ \Delta_{ii} = (2y + cs) \cdot$$

$\cdot(cs\cdot y^2+cs\cdot y\cdot(y+cs))+((y+cs)^2+cs\cdot y)^2;\ \Delta_{ij}=y^4;$

$$\Delta_{jj}=(y^2+cs\cdot y)^2+(2y+cs)\cdot(cs\cdot y^2+cs\cdot y\cdot(y+cs));$$

$$\Delta_{ji}=y^4;\ \Delta_{ii,jj}=(2y+cs)\cdot(y^2+cs\cdot y)+((y+cs)^2+$$
$$+cs\cdot y)\cdot(2y+cs).$$

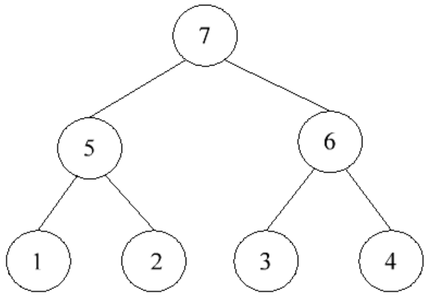

Fig. 3. The order of combining four initial sub-circuits 1, 2, 3 and 4 into combined sub-circuits 5, 6 and 7

For other values of n , the sequence of combination is selected in a similar way. It is clear that for a specific value of n , (n -1)[th] combination results in the set of polynomials (1), which is the determinant and algebraic complements of the conductivity matrix of the whole circuit. It should be noted that this set determines all possible transfer functions between the external nodes of this circuit, which is a positive feature of the method.

Table I presents the time* of determining Δ of the conductivity matrix of the circuit using both D_trees() and det() functions for n varying from 16 to 8192. As seen from Table I, the function D_trees() is hundreds or thousands times more time-efficient than the function det(), and the «Out of memory». situation for the function D_trees() did not occur till $n=8192$, which is a significantly better result than for the function det(), in which case the «Out of memory». situation was obtained at $n=1024$.

Table 1. The time of formation of Δ of the symbolic SLAE of the circuit with fixed parameters using the standard Matlab tools and D-trees method

n	16	512	1024	2048	4096	8192
Function det(), t,s	0.36	2363.07	OutOfM	OutOfM	OutOfM	OutOfM
Function D_trees(), t,s	0.21	7.22	15.46	44.13	60.36	329.09

- *n* denotes the number of elements in the circuit from Fig.2a;

- *Function det(), t,s* is the time taken by the *MATLAB* function *det()* to form the determinant of the SSLAE matrix of the circuit;

- *Function D_trees(), t,s* is the time taken by the function *D_trees()* in the system UDF MAOPCs in the MATLAB environment to form the determinant of the SSLAE matrix of the circuit.

Computer experiment 2. Problem. Assess the time of formation of the determinant Δ of the SSLAE matrix formed with respect to unknown transfer functions by the FS method for a parametric circuit formed by in-series connection of n rc-elements from Fig. 2b, using the D_trees() and det() functions for all the parameters of the circuit elements designated by symbols. Each time the number of elements n is to be increased from $n=8$ until the system message «Out of memory».

Solution. Based on the system of differential equations of the parametric circuit written by the node voltage method, using the FS method, we write SSLAE having the form of [3]:

(10) $$F\cdot W=D,$$

where W denotes the vector of unknown coefficients of the Fourier polynomials of all the transfer functions from the signal source *i(t)* to the node voltages of the circuit. The transfer functions are approximated by the Fourier polynomials containing *k* harmonic components.

For example, let $n=2$. Then the circuit contains three nodes (in addition to the zero node), and by approximating $\hat{W}_{11},\hat{W}_{21},\hat{W}_{31}$ of the transfer functions W_{11},W_{21},W_{31} by the Fourier trigonometric polynomial with one harmonic component ($k=1$) we obtain:

(11) $\hat{W}_{11}(s,t)=W_{0,11}(s)+W_{c1,11}(s)\cos(\Omega t)+W_{s1,11}(s)\sin(\Omega t)$,

(12) $\hat{W}_{21}(s,t)=W_{0,21}(s)+W_{c1,21}(s)\cos(\Omega t)+W_{s1,21}(s)\sin(\Omega t)$,

(13) $\hat{W}_{31}(s,t)=W_{0,31}(s)+W_{c1,31}(s)\cos(\Omega t)+W_{s1,31}(s)\sin(\Omega t)$.

At that, SSLAE (10) by the FS methods assumes the form:

(14) $$\begin{bmatrix} y & & & -y & & & & & \\ & y & & & -y & & & & \\ & & y & & & -y & & & \\ -y & & & Y_{44} & Y_{45} & 0 & -y & & \\ & -y & & Y_{54} & Y_{55} & Y_{56} & & -y & \\ & & -y & Y_{64} & Y_{65} & Y_{66} & & & -y \\ & & & -y & & & Y_{77} & Y_{78} & 0 \\ & & & & -y & & Y_{87} & Y_{88} & Y_{89} \\ & & & & & -y & Y_{97} & Y_{98} & Y_{99} \end{bmatrix}\cdot\begin{bmatrix} W_{0,11} \\ W_{c1,11} \\ W_{s1,11} \\ W_{0,21} \\ W_{c1,21} \\ W_{s1,21} \\ W_{0,31} \\ W_{c1,31} \\ W_{s1,31} \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

where $Y_{44}=Y_{55}=Y_{66}=2y+c_0 s,\ \ Y_{77}=Y_{88}=Y_{99}=y+c_0 s,$ $Y_{54}=Y_{87}=c_0 sm,\ \ Y_{45}=Y_{78}=c_0 sm/2,\ \ Y_{64}=Y_{97}=-c_0\Omega m,$ $Y_{65}=Y_{98}=-c_0\Omega,\ Y_{56}=Y_{89}=c_0\Omega$.

The system (14) will be solved by applying the Cramer's rule, and for the formation of the determinants required for this, we will use the D-trees method.

When comparing (8) and (14), the focus will be on the following.

1. Each unknown transfer function from (8), which is a rational fraction expression at fixed circuit parameters, according to (11)-(13), is represented by three rational fraction expressions in (14) for variable parameters. Generally, the number of algebraic expressions (coefficients in the Fourier polynomial) in the transfer functions of the parametric circuit is determined by the selected number of harmonic components k in them and by analogy with (11)-(13) equals $(2k+1)$.

2. The matrices in the expressions (8) and (14) have the dimension of the conductivity.

3. If in the expression (14) we put $m=0$, then $W_{c1,11}=W_{s1,11}=W_{c1,21}=W_{s1,21}=W_{c1,31}=W_{s1,31}=0$, and for $c_0=c$ and $W_{0,11}=W_{11},W_{0,21}=W_{21},W_{0,31}=W_{31}$ it is transformed into the expression (8).

The comparison of the expressions (8) and (14) prompts the following suggestions. The expression (14) formed for the parametric circuit with variable parameters is the transformed expression (8) formed for the circuit with fixed parameters, if we start to change these fixed parameters. Considering this, the expression (14) conveniently appears as

(15) $$\begin{bmatrix} \boldsymbol{Y} & -\boldsymbol{Y} & 0 \\ -\boldsymbol{Y} & 2\boldsymbol{Y}+s\boldsymbol{C} & -\boldsymbol{Y} \\ 0 & -\boldsymbol{Y} & \boldsymbol{Y}+s\boldsymbol{C} \end{bmatrix}\cdot\begin{bmatrix} \hat{\boldsymbol{W}}_{11} \\ \hat{\boldsymbol{W}}_{21} \\ \hat{\boldsymbol{W}}_{31} \end{bmatrix}=\begin{bmatrix} \boldsymbol{1}_1 \\ 0 \\ 0 \end{bmatrix},$$

where $\boldsymbol{Y}=\begin{bmatrix} y & & \\ & y & \\ & & y \end{bmatrix}$, $\boldsymbol{C}=\begin{bmatrix} c_0 & \frac{1}{2}c_0 m & 0 \\ c_0 m & c_0 & c_0\Omega/s \\ -c_0 m\Omega/s & -c_0\Omega/s & c_0 \end{bmatrix}$,

$\hat{\boldsymbol{W}}_{\boldsymbol{11}}=\begin{bmatrix} W_{0,11} \\ W_{c1,11} \\ W_{s1,11} \end{bmatrix}$, $\hat{\boldsymbol{W}}_{\boldsymbol{21}}=\begin{bmatrix} W_{0,21} \\ W_{c1,21} \\ W_{s1,21} \end{bmatrix}$, $\hat{\boldsymbol{W}}_{\boldsymbol{31}}=\begin{bmatrix} W_{0,31} \\ W_{c1,31} \\ W_{s1,31} \end{bmatrix}$, $\boldsymbol{1_1}=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

As we see, the expressions (15) and (8) are similar in form and dimension. We should also keep in mind that the Y-matrix is the sum of the Y-matrices of each element. Therefore, we conclude that the conductivity matrix of the element with fixed parameters (9) can be placed in correspondence with the conductivity matrix of a parametric element with variable parameters

(16) $$\begin{bmatrix} \boldsymbol{Y} & -\boldsymbol{Y} \\ -\boldsymbol{Y} & \boldsymbol{Y}+s\boldsymbol{C} \end{bmatrix},$$

in which the role of the conductivity y and capacity c from (9) is performed by the conductivity matrix $\boldsymbol{Y}$ and the capacity matrix $\boldsymbol{C}$, respectively. In general, the order of the matrices $\boldsymbol{Y}$ and $\boldsymbol{C}$ in (16) is $(2k+1)$. Then, everything bases on the analogy between the matrices (9) and (16). For instance, by analogy with the experiment 1, we can build SSLAE and its matrix of the parametric circuit for $n>2$. However, as in the computer experiment 1, this is not obligatory. According to the D-trees method, each $rc(t)$-element of the parametric circuit is considered an initial sub-circuit, for which the matrix (16) is considered to be the conductivity matrix, and therefore the set (1) for it is formed using the same rules as for the matrix (9). However, if for the matrix (9) this led to the formation of algebraic expressions, for the matrix (16) the set (1) is formed as matrix expressions:

(17) $\Delta=\boldsymbol{Y}s\boldsymbol{C};\Delta_{ii}=\boldsymbol{Y}+s\boldsymbol{C};\Delta_{jj}=\boldsymbol{Y};\Delta_{jj}=\boldsymbol{Y};\ \Delta_{ji}=\boldsymbol{Y};\ \Delta_{ii,jj}=1$.

Each matrix expression in (17) determines matrix operations, the performance of which for $k=1$ results in the formation of the $(2k+1)=3$ order matrix. As the $\boldsymbol{Y}$ matrix is diagonal and consists of identical elements, then the matrices $\boldsymbol{Y}$ and $\boldsymbol{C}$ are commuting matrices, and the order of their multiplication is not important: $\boldsymbol{Y}\cdot\boldsymbol{C}=\boldsymbol{C}\cdot\boldsymbol{Y}$. Therefore, **in contrast to circuits from the computer experiment 1 (fixed parameters), the sub-circuits of which are described by a set of polynomials, the sub-circuits in the computer experiment 2 (variable parameters) are characterized by a set of matrices.** By analogy, **if the combination of two sub-circuits with fixed parameters results in polynomials, then the combination of two sub-circuits with variable parameters results in matrices.**

Let us consider the combination of the two initial sub-circuits 1 and 2 into the sub-circuit 5 (Fig. 3) in more detail. From (14)-(16) appears that the matrices of the sub-circuits 1 and 2 are block matrices [10] and have the following form:

(18) $$\boldsymbol{M}^1=\begin{bmatrix} \boldsymbol{Y} & -\boldsymbol{Y} \\ -\boldsymbol{Y} & \boldsymbol{Y}+s\boldsymbol{C} \end{bmatrix},$$

(19) $$\boldsymbol{M}^2=\begin{bmatrix} \boldsymbol{Y} & -\boldsymbol{Y} \\ -\boldsymbol{Y} & \boldsymbol{Y}+s\boldsymbol{C} \end{bmatrix},$$

where the superscript signifies that the matrix belongs to a respective sub-circuit, 1 or 2. Since the matrix $\boldsymbol{Y}$ is diagonal and its elements are identical, the determinants and algebraic complements of the block matrices $\boldsymbol{M}^1$ and $\boldsymbol{M}^2$ are determined using the conventional rules [10]. Thus, for the sub-circuit 1,

(20) $$\boldsymbol{\Delta}^1=\boldsymbol{Y}(\boldsymbol{Y}+s\boldsymbol{C})-\boldsymbol{YY}=\boldsymbol{Y}s;\quad \boldsymbol{\Delta}^1_{ii}=(\boldsymbol{Y}+s\boldsymbol{C});$$
$$\boldsymbol{\Delta}^1_{ij}=\boldsymbol{Y};\quad \boldsymbol{\Delta}^1_{jj}=\boldsymbol{Y};\quad \boldsymbol{\Delta}^1_{ji}=\boldsymbol{Y};\quad \boldsymbol{\Delta}^1_{ii,jj}=\boldsymbol{1},$$

and for the sub-circuit 2,

(21) $$\boldsymbol{\Delta}^2=\boldsymbol{Y}(\boldsymbol{Y}+s\boldsymbol{C})-\boldsymbol{YY}=\boldsymbol{Y}s\boldsymbol{C};\quad \boldsymbol{\Delta}^2_{ii}=(\boldsymbol{Y}+s\boldsymbol{C});$$
$$\boldsymbol{\Delta}^2_{ij}=\boldsymbol{Y};\quad \boldsymbol{\Delta}^2_{jj}=\boldsymbol{Y};\quad \boldsymbol{\Delta}^2_{ji}=\boldsymbol{Y};\quad \boldsymbol{\Delta}^2_{ii,jj}=\boldsymbol{1}.$$

For the combination of the sub-circuits 1 and 2, to the sets of the matrices (20) and (21) we apply the expressions which are correct for this parametric circuit and which directly result from (2) for $a=1$, $b=2$ and $c=5$. We obtain the set of six matrices of the combined sub-circuit 5:

(22) $$\boldsymbol{\Delta}^5=\boldsymbol{\Delta}^1\cdot\boldsymbol{\Delta}^2_{ii}+\boldsymbol{\Delta}^1_{jj}\cdot\boldsymbol{\Delta}^2=\boldsymbol{Y}s\boldsymbol{C}\cdot(\boldsymbol{Y}+s\boldsymbol{C})+\boldsymbol{Y}\cdot\boldsymbol{Y}s\boldsymbol{C}=$$
$$=\boldsymbol{Y}s\boldsymbol{C}\cdot\boldsymbol{Y}+\boldsymbol{Y}s\boldsymbol{C}s\boldsymbol{C})+\boldsymbol{Y}\cdot\boldsymbol{Y}s\boldsymbol{C}=\boldsymbol{Y}(2\boldsymbol{Y}+\boldsymbol{Y}s\boldsymbol{C})s\boldsymbol{C};$$
$$\boldsymbol{\Delta}^5_{ii}=\boldsymbol{\Delta}^1_{ii,jj}\cdot\boldsymbol{\Delta}^2+\boldsymbol{\Delta}^1_{ii}\cdot\boldsymbol{\Delta}^2_{jj};\qquad \boldsymbol{\Delta}^5_{ij}=\boldsymbol{\Delta}^1_{ij}\cdot\boldsymbol{\Delta}^2_{ij}=\boldsymbol{Y}\cdot\boldsymbol{Y};$$
$$\boldsymbol{\Delta}^5_{jj}=\boldsymbol{\Delta}^1\cdot\boldsymbol{\Delta}^2_{ii,jj}+\boldsymbol{\Delta}^1_{jj}\cdot\boldsymbol{\Delta}^2_{jj};\qquad \boldsymbol{\Delta}^5_{ji}=\boldsymbol{\Delta}^1_{ji}\cdot\boldsymbol{\Delta}^2_{ji}=\boldsymbol{Y}\cdot\boldsymbol{Y};$$
$$\boldsymbol{\Delta}^5_{ii,jj}=\boldsymbol{\Delta}^1_{ii}\cdot\boldsymbol{\Delta}^2_{ii,jj}+\boldsymbol{\Delta}^1_{ii,jj}\cdot\boldsymbol{\Delta}^2_{jj}.$$

By multiplication and addition of the matrices, from (22) we obtain the resulting six matrices $\boldsymbol{\Delta}^5,\boldsymbol{\Delta}^5_{ii},\boldsymbol{\Delta}^5_{ij},\boldsymbol{\Delta}^5_{jj},\boldsymbol{\Delta}^5_{ji},\boldsymbol{\Delta}^5_{ii,jj}$ of the combined sub-circuit 5:

(23) $$\Delta^5=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix},$$

$a=c_0\cdot s\cdot y^2+(c_0{}^2\cdot m^2\cdot s^2\cdot y)/2+c_0\cdot s\cdot y\cdot(y+c_0\cdot s);$
$b=(c_0\cdot m\cdot s\cdot y^2)/2+(c_0{}^2s^2m\cdot y)/2+(c_0ms\cdot y\cdot(y+c_0s))/2;$
$c=(\Omega\cdot c_0{}^2\cdot m\cdot s\cdot y)/2;d=c_0\cdot m\cdot s\cdot y^2-\Omega^2\cdot c_0{}^2\cdot m\cdot y+$
$+c_0{}^2\cdot m\cdot s^2\cdot y+c_0\cdot m\cdot s\cdot y\cdot(y+c_0\cdot s);\quad e=c_0\cdot s\cdot y^2-$
$-\Omega^2\cdot c_0{}^2\cdot y+(c_0{}^2\cdot m^2\cdot s^2\cdot y)/2+c_0\cdot s\cdot y\cdot(y+c_0\cdot s);$
$f=\Omega\cdot c_0\cdot y^2+\Omega\cdot c_0{}^2\cdot s\cdot y+\Omega\cdot c_0\cdot y\cdot(y+c_0\cdot s);$
$g=-\Omega\cdot c_0\cdot m\cdot y^2-\Omega\cdot c_0\cdot m\cdot y\cdot(y+c_0\cdot s)-$
$-2\cdot\Omega\cdot c_0{}^2\cdot m\cdot s\cdot y;\quad h=-\Omega\cdot c_0\cdot y^2-\Omega\cdot c_0{}^2\cdot s\cdot y-$
$-\Omega\cdot c_0\cdot y\cdot(y+c_0\cdot s)-(\Omega\cdot c_0{}^2\cdot m^2\cdot s\cdot y)/2;$
$k=c_0\cdot s\cdot y^2-\Omega^2\cdot c_0{}^2\cdot y+c_0\cdot s\cdot y\cdot(y+c_0\cdot s);$

$$\Delta^5{}_{ii}=\begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & e_1 & f_1 \\ g_1 & h_1 & k_1 \end{bmatrix};\quad \Delta^5{}_{ij}=\begin{bmatrix} y^2 & 0 & 0 \\ 0 & y^2 & 0 \\ 0 & 0 & y^2 \end{bmatrix},$$

$a_1=(y+c_0s)^2+c_0s\cdot y+(c_0{}^2\cdot m^2\cdot s^2)/2;$
$b_1=(c_0\cdot m\cdot s\cdot y)/2+c_0\cdot m\cdot s\cdot(y+c_0s);c_1=(\Omega\cdot c_0{}^2ms)/2;$
$d_1=c_0\cdot m\cdot s\cdot y-\Omega^2\cdot c_0{}^2\cdot m+2\cdot c_0\cdot m\cdot s\cdot(y+c_0s);$
$e_1=(y+c_0s)^2-\Omega^2\cdot c_0{}^2+c_0\cdot s\cdot y+(c_0{}^2m^2s^2)/2;$
$f_1=2\Omega\cdot c_0\cdot(y+c_0s)+\Omega\cdot c_0\cdot y;\ g_1=-\Omega\cdot c_0{}^2ms-$
$-2\cdot\Omega\cdot c_0\cdot m\cdot(y+c_0\cdot s)-\Omega\cdot c_0\cdot m\cdot y;h_1=-2\cdot\Omega\cdot c_0\cdot(y+$
$+c_0s)-\Omega c_0y-(\Omega c_0{}^2m^2s)/2;k_1=(y+c_0s)^2-\Omega^2c_0{}^2+c_0sy;$

$$\mathbf{\Delta}^{5}{}_{jj}=\begin{bmatrix} y^2+c_0\cdot s\cdot y & (c_0\cdot s\cdot m)/2 & 0 \\ c_0\cdot s\cdot m\cdot y & y^2+c_0\cdot s\cdot y & c_0\cdot\Omega\cdot y \\ -c_0\cdot\Omega\cdot m\cdot y & -c_0\cdot\Omega\cdot y & y^2+c_0\cdot s\cdot y \end{bmatrix};$$

(24) $$\mathbf{\Delta}^{5}{}_{ji}=\begin{bmatrix} y^2 & 0 & 0 \\ 0 & y^2 & 0 \\ 0 & 0 & y^2 \end{bmatrix};$$

$$\mathbf{\Delta}^{5}{}_{ii,jj}=\begin{bmatrix} 2\cdot y+c_0\cdot s & (c_0\cdot s\cdot m)/2 & 0 \\ c_0\cdot s\cdot m & 2\cdot y+c_0\cdot s & c_0\cdot\Omega \\ -c_0\cdot\Omega\cdot m & -c_0\cdot\Omega & 2\cdot y+c_0\cdot s \end{bmatrix}.$$

Similarly, we obtain the matrices of the set (1) for the sub-circuit 6:

(25) $$\mathbf{\Delta}^{6},\mathbf{\Delta}^{6}{}_{ii},\mathbf{\Delta}^{6}{}_{ij},\mathbf{\Delta}^{6}{}_{jj},\mathbf{\Delta}^{6}{}_{ji},\mathbf{\Delta}^{6}{}_{ii,jj},$$

which in our case are equal to the corresponding matrices from (24), as the sub-circuits 1,2 and 3,4 are identical in terms of structure and parameters.

The sub-circuits 5 and 6 are combined into the sub-circuit 7 in a similar way. Using the formed sets of matrices (24) and (25) based on the expressions (2), which are correct also for this parametric circuit, we determine the set of matrices (1) for the sub-circuit 7:

(26) $$\mathbf{\Delta}^{7},\mathbf{\Delta}^{7}{}_{ii},\mathbf{\Delta}^{7}{}_{ij},\mathbf{\Delta}^{7}{}_{jj},\mathbf{\Delta}^{7}{}_{ji},\mathbf{\Delta}^{7}{}_{ii,jj}.$$

The adequacy of the operations mentioned in (20)-(26) can be demonstrated using Fig. 4, 5, 6 and their descriptions presented below. Fig. 4 offers the graphic imaging of the matrix F from the expression (10), which is the 15-order matrix obtained (by the FS method by analogy to (14)) for the circuit for $n=4$. Fig. 5 shows the graphic imaging of that same matrix for the initial sub-circuit. In these figures the elements of the diagonal matrices $\boldsymbol{Y}$ are marked by the symbol '○', and the other by the symbol '●', because for further explanation this is the fact of their presence that is important, not their value.

From the expressions (24) and the form of the matrix equation (3), which is correct for the case of parametric circuit, we can build the graphic image of the matrix F, which corresponds to the combined sub-circuits 5 (see Fig. 6). The matrices F obtained for the combined sub-circuits 6 and 7 will have similar graphic representation. Therefore, we can assume that the graphic image in Fig. 6 is common for all the sub-circuits 1-7, irrespective of the fact if these sub-circuits are initial (Fig. 5) or they already comprise other initial sub-circuits (Fig. 6). The only difference is that in case of initial sub-circuits the elements of the diagonal matrices *B*, *C* from Fig. 6 equal y, while in the other cases, these elements equal y^i. The degree i here shows that the sub-circuit contains the number of i of the initial sub-circuits. This is well illustrated by the third and fifth matrices from the set of matrices (24) for the sub-circuit 5, the diagonal elements of which are equal to y^2 for $i=2$. This is also the case for an arbitrary value of the number of initial sub-circuits *n* in the test circuit. Thus, from Fig. 6 and explanations thereto, taking into account the rules of operations with block matrices [10], we can conclude that the determinant of the block matrix from Fig. 6 is found using the same rules as for ordinary matrices. For instance, for the symbols used in Fig. 6 the determinant of such a block matrix appears as

(27) $$\Delta=\begin{vmatrix} A & B \\ C & D \end{vmatrix}=A\cdot D-C\cdot B=D\cdot A-B\cdot C$$

and does not depend on the order of multiplications. This also means that all other methods, including the trees method and D-trees method [3] for calculating determinants in this case are correct and applicable. Regarding the algebraic complements from the set (1) for the matrix from Fig.6, formed in our case by deleting three rows and three columns (1,2,3 or 4,5,6) simultaneously, the results will also be a priori correct, as they are formed of separate matrices A,B,C,D or unit diagonal matrix. Now we can make another conclusion: the above-presented transfer from the operations with algebraic expressions in case of circuits with fixed parameters (computer experiment 1) to the operations with matrices for circuits with variable parameters (computer experiment 2) is correct. However, it should be understood that the correctness of such transfer in our case is ensured by the fact that the selected test circuit between the neighbouring nodes contains solely the fixed parameter resistor. This provides for the diagonal form of the matrices with identical elements marked in Fig.6 by the symbol '○'. For the other linear circuit structures, we also managed to reduce the problem to the above-considered form.

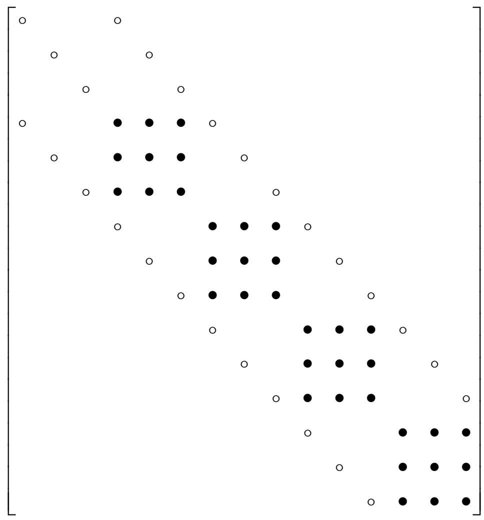

Fig. 4. Graphic imaging of the 15-order matrix F obtained by the FS method for the four sub-circuits 1-4

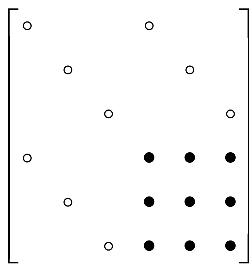

Fig. 5. Graphic imaging of the matrix of the initial sub-circuits ($rc(t)$-elements)

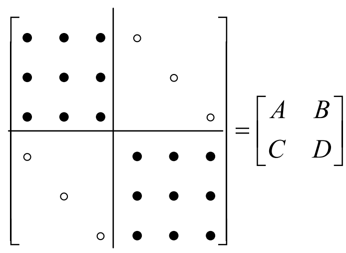

Fig. 6. Graphic imaging of the matrix obtained by the combination of two neighbouring sub-circuits

Thus, it is clear that by combining all the sub-circuits of the parametric circuit between themselves we obtain not polynomials but a set of six matrices of the form (1). The order of combining sub-circuits is similar to the computer experiment 1. The determinant Δ of the obtained matrix of the circuit from the set (1) is one of the required determinants that form the SSLAE solution by Cramer's rule. The other determinants are formed in a similar way using the Cramer's rule. This results in the formation of the required transfer functions between the input and outputs of the parametric circuit. However, the further experiments showed that the formation of determinants of such matrices, the degree of which is not very high and equals $(2k+1)$, can also take lots of computing time. This is due to the fact that the elements of such matrices are also bulky expressions. This problem can be solved both by using the MATLAB det () function and by the system UDF MAOPCs D_trees() function. As the experiments showed, in this case the D_trees() function is also much more effective that det(), which is why it was used in all the further experiments.

Summing up the above-presented information, we will make the following points. There are three ways of computing the determinant of the SSLAE of the LPTV circuit and its transfer functions: a) by applying the det() function to the SSLAE matrix of the circuit; b) by applying the D_trees() function to the SSLAE matrix of the circuit and the det() function to the final $(2k+1)$-degree matrix; c) by applying the D_trees() function both to the SSLAE matrix of the circuit and to the final $(2k+1)$-degree matrix.

Table 2 presents the time of forming the determinant of the SSLAE matrix of the circuit using the three methods (a), (b) and (c) for n varying from 8 to 256. From Table 2 it follows that:

- the det() function in the variant (a) formed the determinant for circuits for the cases when the number of sub-circuits does not exceed 32;
- for the case of 32 sub-circuits in the circuit, the variant (b) is 4.5 times faster than the variant (a), and the variant (c) is 500 times faster than the variant (a);
- the variant (c) produced results even for the case of 256 sub-circuits in the circuit;
- for the case of 32 sub-circuits in the circuit, the variant (c) was 110 times faster than the variant (b);
- the variant (c) turned out to be much more effective than the variants (a) and (b).

Table 2. The time of formation of Δ of the SSLAE of the circuit with variable parameters using the standard Matlab tools and D-trees method

n	8	32	64	128	256
The variant a, t,s	51.68	3592.98	OutOfM	OutOfM	OutOfM
The variant b, t,s	0.71	791.23	OutOfM	OutOfM	OutOfM
The variant c, t,s	0.09	7.13	56.03	772.26	10777.07

- *n* denotes the number of elements in the circuit from Fig.2b;
- The variant a —the time of formation the determinant of the SSLAE matrix of the circuit using the MATLAB det() function;
- The variant b – the time of formation the determinant of the SSLAE matrix of the circuit using the D_trees() function to the SSLAE matrix of the circuit and the MATLAB det() function to the final -degree matrix ;
- The variant c – the time of formation the determinant of the SSLAE matrix of the circuit using double application of the D_trees() function in the system UDF MAOPCs of the MATLAB environment.

Computer experiment 3. Problem. Assess the time of formation of the determinant Δ of the SSLAE matrix formed by the FS method for a parametric circuit formed of n $rc(t)$-elements (Fig. 2c), the parametric capacities of which vary with different initial phases $\varphi_p = \frac{(p-1)\pi}{n-1}$, where p is the ordinal number of the element (the circuit elements are numbered starting from the power supply source) for all the symbolic parameters of the circuit elements using the D_trees() and det() functions. Each time the number of n $rc(t)$-elements is to be increased from $n=8$ until the system message «Out of memory»..

The experiment is practically similar to the experiment 2 with the following slight differences.

For the case of $n=2$ SSLAE (10) by FS method will appear in the form (14) except for the elimination of signs in the expressions:

$$Y_{78} = -\frac{c_0 sm}{2},\ Y_{87} = -c_0 sm,\ Y_{97} = c_0\Omega m. \tag{28}$$

This means that the expression (15) is correct for both initial sub-circuits, but, as it follows from the FS method, the matrices $\boldsymbol{C}$ for them will be different. For the sub-circuit 1, this matrix (let us designate it $\boldsymbol{C}_1$) is identical to that in (15), and for the sub-circuit 2 this matrix $\boldsymbol{C}_2$ is different:

$$\boldsymbol{C}_1 = \begin{bmatrix} c_0 & 0.5\cdot c_0 m & 0 \\ c_0 m & c_0 & c_0\Omega/s \\ -c_0 m\Omega/s & -c_0\Omega/s & c_0 \end{bmatrix}, \tag{29}$$
$$\mathbf{C}_2 = \begin{bmatrix} c_0 & -0.5\cdot c_0 m & 0 \\ -c_0 m & c_0 & c_0\Omega/s \\ c_0 m\Omega/s & -c_0\Omega/s & c_0 \end{bmatrix}.$$

For $n>2$, since the parametric capacity of each element has a shift in the initial phase, the matrices $\boldsymbol{C}_p$ of the initial sub-circuit will also be different. Therefore, the conductivity matrix for different initial elements will be different. For instance, for the p^{th} initial element, the conductivity matrix is

$$\begin{bmatrix} \boldsymbol{Y} & -\boldsymbol{Y} \\ -\boldsymbol{Y} & \boldsymbol{Y}+s\boldsymbol{C}_p \end{bmatrix}. \tag{30}$$

At that, the matrix expressions (17) for the p^{th} initial element will appear as

$$\Delta = \boldsymbol{Y}s\boldsymbol{C}_p;\quad \Delta_{ii} = \boldsymbol{Y}+s\boldsymbol{C}_p;\quad \Delta_{ij} = \boldsymbol{Y}; \\ \Delta_{jj} = \boldsymbol{Y};\quad \Delta_{ji} = \boldsymbol{Y};\quad \Delta_{ii,jj} = 1. \tag{31}$$

The expressions (18) and (19) for the initial sub-circuits p and $p+1$ will be

$$\boldsymbol{M}^p = \begin{bmatrix} \boldsymbol{Y} & -\boldsymbol{Y} \\ -\boldsymbol{Y} & \boldsymbol{Y}+s\boldsymbol{C}_p \end{bmatrix}, \tag{32}$$

$$\boldsymbol{M}^{p+1} = \begin{bmatrix} \boldsymbol{Y} & -\boldsymbol{Y} \\ -\boldsymbol{Y} & \boldsymbol{Y}+s\boldsymbol{C}_{p+1} \end{bmatrix}. \tag{33}$$

As the matrix $\boldsymbol{Y}$ in (32) and (33) did not change and remained diagonal with identical elements, then the determinants and algebraic complements of the block matrices $\boldsymbol{M}^p$ and $\boldsymbol{M}^{p+1}$ are determined using expressions similar to (20) and (21):

$$\boldsymbol{\Delta^p} = \boldsymbol{Y}(\boldsymbol{Y}+\boldsymbol{sC_p}) - \boldsymbol{YY} = \boldsymbol{YsC_p};\quad \boldsymbol{\Delta^p_{ii}} = (\boldsymbol{Y}+\boldsymbol{sC_p}); \\ \boldsymbol{\Delta^p_{ij}} = \boldsymbol{Y};\quad \boldsymbol{\Delta^p_{jj}} = \boldsymbol{Y};\quad \boldsymbol{\Delta^p_{ji}} = \boldsymbol{Y};\quad \boldsymbol{\Delta^p_{ii,jj}} = \boldsymbol{1} \tag{34}$$
,

$$\Delta^{p+1} = Y(Y + sC_{p+1}) - YY = YsC_{p+1}; \ \Delta_{ii}^{p+1} = (Y + sC_{p+1}); \tag{35}$$
$$\Delta_{ij}^{p+1} = Y; \ \Delta_{jj}^{p+1} = Y; \ \Delta_{ji}^{p+1} = Y; \ \Delta_{ii,jj}^{p+1} = 1 .$$

However, the fact that the matrices C_p are not equal has no effect whatsoever on combining two neighbouring sub-circuits, as the D-trees method does not require such equality. Therefore, the combination occurs according to the expressions similar to (22).

Table 3, as Table 2, presents the time of determining Δ of the matrix of the circuit SSLAE using three variants (a), (b) and (c) for n varying from 8 to 256.

From Table 3 it follows that

- the det() function in the variant (a) formed the determinant for circuits for the cases when the number of sub-circuits does not exceed 10;
- for the case of 10 sub-circuits in the circuit, the variant (b) is 10.3 times faster than the variant (a), and the variant (c) is 1391 times faster than the variant (a);
- the variant (c) produced results even for the case of 256 sub-circuits in the circuit;
- for the case of 16 sub-circuits in the circuit, the variant (c) was 134 times faster than the variant (b);
- the variant (c) turned out to be much more effective than the variants (a) and (b).

Table 3. The time of formation of Δ of the SSLAE of the circuit with variable parameters using the standard Matlab tools and D-trees method

n	8	10	16	64	128	256
The variant a, t,s	64.96	1099.07	OutOfM	OutOfM	OutOfM	OutOfM
The variant b, t,s	4.96	106.54	2660.49	OutOfM	OutOfM	OutOfM
The variant c, t,s	0.09	0.49	0.66	87.97	867.96	15600

- *n* denotes the number of elements in the circuit from Fig.2b;
- The variant a —the time of formation the determinant of the SSLAE matrix of the circuit using the MATLAB det() function;
- The variant b – the time of formation the determinant of the SSLAE matrix of the circuit using the D_trees() function to the SSLAE matrix of the circuit and the MATLAB det() function to the final -degree matrix ;
- The variant c – the time of formation the determinant of the SSLAE of the circuit using double application of the D_trees() function in the system UDF MAOPCs of the MATLAB environment.

Conclusion

1. The FS method for the analysis of LPTV circuits is a development of the frequency symbolic methods of the analysis of circuits with fixed parameters and their extension to parametric circuits.
2. The application of the FS method to linear circuits with fixed or variable parameters described by differential equations using the node voltage method made it possible to obtain SSLAE with respect to the transfer functions that were solved by correctly applying the d-trees method.
3. The implementation of the D-trees method in the form of D_trees() function in the system UDF MAOPCs enabled an objective comparison of the D-trees method with the methods implemented in the MATLAB det() function. This comparison was done on the SSLAE describing linear circuits with fixed and variable parameters by the FS method.
4. Linear circuits were selected as test circuits, as they allow for the use of a significant number of parametric elements in the circuit, can effectively model long lines with variable parameters and simplify the representation by the D-trees method.
5. The results of the computer experiments presented in the paper convincingly demonstrate that the application of the D-trees method for the solution of symbolic SLAE when parametric circuits are analysed by the FS method provides a significant reduction of the time and increases the allowable complexity of the circuits being analysed. The presented computer experiments were hundreds and thousands times more time-efficient as compared to the symbolic methods used in the MATLAB det() function. The admissible complexity of the LPTV circuits in terms of the number of elements increased from 32 sub-circuits for the standard det() function to 256 sub-circuits for the D-trees method.

It should also be noted that:

1. The D-trees method was developed both for linear circuits and for circuits that have an arbitrary structure.
2. The presence of variable resistors, capacities and inductances do not change the character of the obtained results.
3. We believe that the presented material opens up the opportunities for applying the system UDF MAOPCs in multivariate problems of analysis and design, in particular of electronic devices modelled by linear parametric circuits.

*Hereinafter, each value of the time was obtained by averaging the time values obtained in ten corresponding computations. For computer experiments, MATLAB R2014a and Dell/Intel(R) Core(TM) i-5-3317U CPU, 1.70 GHz, RAM:8.00 GB were used.

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