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doi:10.15199/48.2021.06.05

Nonlinear predictive control for trajectory tracking of underactuated mechanical systems

***Abstract**. The objective of this article is to present an automatic demonstrator of underactuated mechanical systems. It is the inertia wheel inverted pendulum, which has two degrees of freedom and a single actuator. Nonlinear predictive control is applied to the underactuated system allowing dynamic control for optimal tracking of periodic reference trajectories. The simulation results show the performance and efficiency of the proposed control.*

***Streszczenie.** Celem tego artykułu jest przedstawienie automatycznego demonstratora niedostatecznie uruchomionych układów mechanicznych. Jest to odwrócone wahadło stabilizowane kołem zamachowym, które ma dwa stopnie swobody i jeden siłownik. Nieliniowe sterowanie predykcyjne jest stosowane do niedostatecznie uruchomionego systemu, umożliwiając dynamiczne sterowanie w celu optymalnego śledzenia okresowych trajektorii odniesienia. Wyniki symulacji pokazują wydajność i skuteczność proponowanego sterowania. (**Nieliniowe predykcyjne sterowanie demonstratora niedostatecznie uruchomionych układów mechanicznych**)*

Keywords: underactuated system**,** Nonlinear predictive control, stability, trajectory tracking.
Słowa kluczowe: niedostatecznie uruchamiany system, nieliniowa kontrola predykcyjna, stabilność, śledzenie trajektorii.

Introduction

Robotic systems are mechanical systems with actuators to control the evolution of the system over time. Depending on the number of degrees of freedom and actuators of the system, 3 classes of mechanical systems result, namely: fully actuated mechanical systems, underactuated mechanical systems, and overactuated mechanical systems [1], [2], [3].

Underactuated mechanical systems are defined as systems where the number of actuators is less than the number of degrees of freedom. Thus, a subset of degrees of freedom of the system does not have a command entry [4], [5]. Therefore, conventional controls of fully actuated mechanical systems cannot be implemented for such systems [6]. Indeed, the command inputs only allow to completely control a part of the dynamics, the other part, called internal dynamics [7], depends on the evolution of the actuated coordinates (because of the coupling between the coordinates of the system). The system is said to be at minimum-phase if its internal dynamics are stable, otherwise the system is at non-minimum phase.

Underactuation is often introduced on purpose to reduce the number of actuators, and therefore the cost of building prototypes. This problem is of great academic interest since the classical techniques for controlling nonlinear systems are no longer valid for this kind of systems. Due to its complexity and richness, the task of designing control laws for this type of system is attracting more and more researchers who are interested in the development of new control techniques [8].

The inertia wheel inverted pendulum*,* is, because it has fewer degrees of freedom actuators, an underactuated mechanical system. Moreover, because its internal dynamic is unstable, it has non-minimum phase. Different control approaches have been developed for stabilizing and tracking desired trajectories of this system [9], [10]. The control of the systems under-actuated by the PID controller is not recommended to use it as it is a non-robust control [11], [12].

In this paper, a nonlinear predictive control strategy will be applied for tracking a sinusoidal reference path is developed.

Nonlinear predictive control

Model Predictive Control (MPC), also called optimal control over a sliding finite horizon, is a strategy well suited for the control of nonlinear processes subjected to constraints on the control variables and / or states. The pursuit objective is formulated as a constrained nonlinear optimization problem. It is then a matter of minimizing a performance criterion, a function of the difference between the reference trajectory and the output of the process, over a finite prediction horizon [13], [14].

One of the main advantages of this approach lies in the explicit taking into account of the constraints in the synthesis of the control law. The command structure considered here is the command structure with internal model to which an output return of the model has been added, figure 1. The process is written by a discrete nonlinear model [15]:

(1) $$\left(S_m\right)\begin{cases} x_m\left(k+1\right)=f_m\left(x_m\left(k\right),u\left(k\right)\right) \\ y_m\left(k\right)=h_m\left(x_m\left(k\right)\right)\end{cases}$$

where $x_m(k)\in\Re^n$, $u(k)\in\Re^m$, $y_m(k)\in\Re^p$ respectively the state, command and output vectors of the model. Functions $f_m:\Re^n\times\Re^m\to\Re^n$ and $h_m:\Re^n\to\Re^p$ are assumed to be continuously differentiable $\left(m\geq p\right)$.

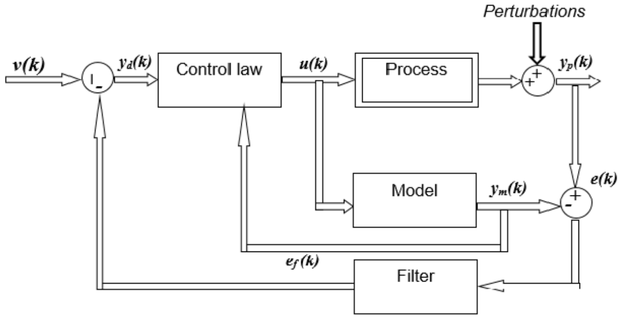


Fig.1. the internal model control structure

In a predictive control strategy associated with an internal model control structure, the objective of pursuing the reference trajectory v by the output of the method y_p amounts to determining a sequence of commands $\tilde{u}=\left\{u\left(k\right),u\left(k+1\right),\ldots,u\left(k+Nc-1\right)\right\}$ over a control horizon N_c such that the difference between the desired

trajectory y_d and the output predicted by the model y_m is minimal over a prediction horizon N_p.
Indeed, according to the control diagram, figure 1, one can write:

(2) $$y_d(k) = v(k) - e_f(k)$$

For k sufficiently large compared to the dynamics of the filter:

(3) $$\lim_{k\to\infty} y_d(k) = \lim_{k\to\infty}\left(v(k) - e(k)\right)$$

(4) $$\lim_{k\to\infty} y_d(k) = \lim_{k\to\infty}\left(v(k) - \left(y_p(k) - y_m(k)\right)\right)$$

(5) $$\lim_{k\to\infty}\left(y_d(k) - y_m(k)\right) = \lim_{k\to\infty}\left(v(k) - y_p(k)\right)$$

In the multivariable case, the mathematical formulation of this problem is written [14]:

(6) $$\min_{\tilde{u}}\left\{ J(\tilde{u}) = \sum_{j=k+1}^{k+N_p} \left(y_d(j) - y_m(j)\right)^T Q\left(y_d(j) - y_m(j)\right)\right\}$$

(7) $$y_d(j) = v(j) - e_f(j)$$

(8) $$(S_m)\begin{cases} x_m(j+1) = f_m\left(x_m(j), u(j)\right) \\ y_m(j) = h_m\left(x_m(j)\right) \\ x_m(k) \text{ is the initial condition} \end{cases}$$

subject to the constraints on the control variables (of threshold type and / or speed) and states:

(9) $$(S_m)\begin{cases} x_m(j+1) = f_m\left(x_m(j), u(j)\right) \\ y_m(j) = h_m\left(x_m(j)\right) \\ x_m(k) \text{ is the initial condition} \end{cases}$$

(10) $$\begin{cases} u_{min} \le u(j) \le u_{max} \\ \Delta u_{min} \le u(j) - u(j-1) \le \Delta u_{max} \\ x_{min} \le x_m(j+1) \le x_{max} \end{cases}$$

$y_d(j)$: desired output at the moment $j.T_e$
$y_m(j)$: model output at the instant j.Te
N_p : finite prediction horizon
N_c : order horizon ($N_c \le N_p$)
Q : symmetric positive definite matrix
T_e : sampling period

Dynamic modelling of the plant

the underactuated system studied in this paper is an inertia wheel inverted pendulum, figure1, which consists of an inverted pendulum equiped with a rotating wheel. Figure 2, illstrates its mechanical structure.

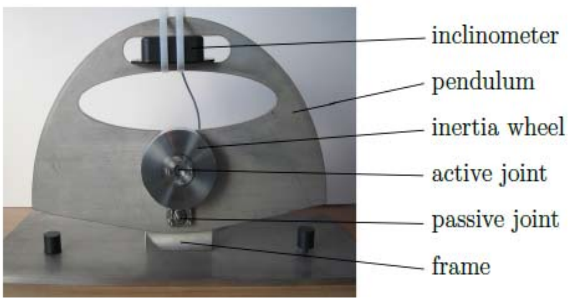


Fig.2. The inertia wheel inverted pendulum

In order to develop the dynamic model of the Inertia wheel inverted pendulum, the following assumptions are considered:
• Hypothesis 1: The masses of the pendulum and the inertia wheel are considered to be point masses located at their centers of gravity.
• Hypothesis 2: The study of the dynamics of the inverted pendulum is carried out by neglecting the mechanical phenomena related to friction.
• Hypothesis 2: The dynamics of the actuator motor associated with the inertia wheel is not taken into account in the modeling of the system.

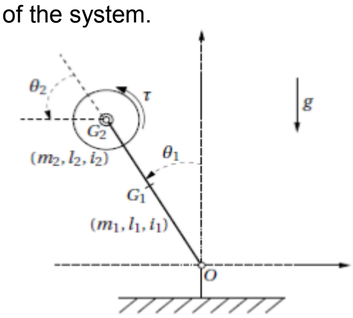


Fig.3. Synoptic of the system's mechanical structure

The nonlinear dynamic model of the inertia wheel inverted pendulum is obtained by applying the Lagrange formalism [16]. This approach requires the calculation of the Lagrangian according to the kinetic and potential energies of the various components of the system according to the generalized coordinates. Lagrange Formulation is based on the Lagrange equation:

(11) $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i\,;\ i = 1,2$$

where L=T – V is the Lagrangian, with T the kinetic energy, and V the potential energy. Knowing that q_i and $\dot{q}_i$ are the vectors of generalized positions and velocities. Q_i is the vector of generalized forces.
The application of such a formalism to the case of the inverted pendulum led to the nonlinear dynamic model [17]:

(12) $$\ddot{\theta}_1 = \frac{1}{I}\left[\tau_1 - \tau_2 + \overline{ml}\, g \sin\theta_1\right]$$

(13) $$\ddot{\theta}_2 = \frac{1}{I i_2}\left[-i_2\tau_1 + \left(i_2 + I\right)\tau_2 - i_2\overline{ml}\, g\sin\theta_1\right]$$

where $I = m_1 l_1^2 + m_2 l_2^2 + i_1$ $\overline{ml} = m_1 l_1 + m_2 l_2$

The different variables used in the modeling are grouped together in Table 1.

Table 1. Summary of the variables used in the modeling

Description	Parameter
θ_1	pendulum position (rad)
$\dot{\theta}_1$	Pendulum velocity (rad/s)
$\ddot{\theta}_1$	Pendulum acceleration (rad/s^2)
θ_2	Inertia wheel position (rad)
$\dot{\theta}_2$	Inertia wheel velocity (rad/s)
$\ddot{\theta}_2$	Inertia wheel acceleration (rad/s^2)
τ_1	External disturbing torque applied to the pendulum (N.m)
τ_2	torque exerted by the actuator (N.m)

Table 2 summarizes all the geometric and dynamic parameters of the inertia wheel inverted pendulum:

Table 2. Description of dynamical parameters of the inverted pendulum

Parameter value	Description
$m_1 = 3.30810\,Kg$	Body mass
$m_2 = 0.33081\,Kg$	Wheel mass
$l_1 = 0.06\,m$	Body center of mass position
$l_2 = 0.044\,m$	wheel center of mass position
$i_1 = 0.03146\,Kgm^2$	Body inertia
$i_2 = 0.00041\,Kgm^2$	Wheel inertia
$g = 9.81\,ms^{-2}$	Gravity acceleration

The external disturbing torque applied to the pendulum τ_1 is assumed to be zero, the dynamics of the pendulum can be rewritten:

$$(I + i_2)\ddot{\theta}_1 + i_2\ddot{\theta}_2 - \overline{m\,l}\,g\sin\theta_1 = 0 \quad (14)$$

$$i_2(\ddot{\theta}_1 + \ddot{\theta}_2) = \tau_2 \quad (15)$$

The discretization of the inverted pendulum using Euler's approximation. This Euler transformation uses the state vectors $x(t)$ and $\dot{x}(t)$ at time t approximated respectively as follows [18], [19]:

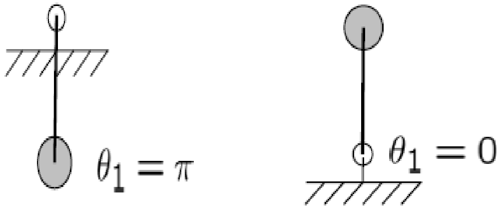
$$\begin{cases} x(t) = x(k) \\ \dot{x}(t) = \dfrac{x(k) - x(k-1)}{T_e} \end{cases} \quad (16)$$

where *x* at time *kT* is denoted by $x(k)$, Te being the sampling period.
It follows the approximate discrete model, corresponding to the inverted pendulum, as follows:

$$\begin{cases} x_1(k) = T_e x_2(k-1) + x_1(k-1) \\ x_2(k) = \dfrac{T_e}{I}\left(\tau_2(k-1) + \overline{m\,l}\,g\sin x_1(k-1)\right) \\ \qquad + x_2(k-1) \\ x_3(k) = T_e x_4(k-1) + x_3(k-1) \\ x_4(k) = \dfrac{T_e}{I i_2}\left((i_2 + I)\tau_2(k-1) - i_2\overline{m\,l}\,g\sin x_1(k-1)\right) \\ \qquad + x_4(k-1) \end{cases} \quad (17)$$

The inverted pendulum stabilized by an inertia wheel has two equilibrium points. The first is a point of unstable equilibrium, it corresponds to the state in which the pendulum is pointed upwards. This point of equilibrium is said to be unstable because in the absence of control torque, the pendulum, under the effect of the slightest disturbance, is unable to maintain this position indefinitely.

The second point, on the other hand, corresponds to the state in which the pendulum is pointing downwards. In the presence of a disturbance acting on the pendulum, if the state of the system is in a neighborhood of this point, it naturally remains there in this state. These two equilibrium points are illustrated in figure 4. The control objective that we will discuss concerns particularly the monitoring of the system instructions.



a) Stable equilibrium point b) Unstable equilibrium point

Fig.4. Illustration of the equilibrium points of the system

Simulations Results

In this section simulation results are presented. They attest the feasibility of the proposed predictive control.

Consider the dynamic model of system inertia wheel inverted pendulum described by equations (14) and (15) with dynamical parameters described in Table 2. The initial conditions of simulations are as follows $(\theta_1 = \dot{\theta}_1 = \dot{\theta}_2 = 0)$.

The choice of the initial value of the position of the pendulum is not arbitrary, for reasons of the mechanical structure of the pendulum.

Figures 5 and 6 respectively represent the angular position and the angular velocity of the articulation of the body of the pendulum as a function of time in solid lines, while the dotted lines represent their corresponding position and reference velocity. It is clear that the nonlinear predictive control ensures a good convergence of the position and the velocity of the pendulum towards their reference trajectories.

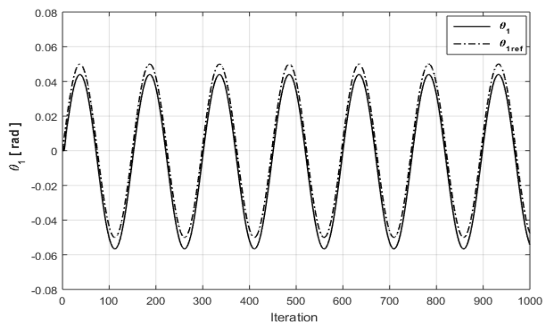


Fig.5. Evolution of angular position of the inverted pendulum

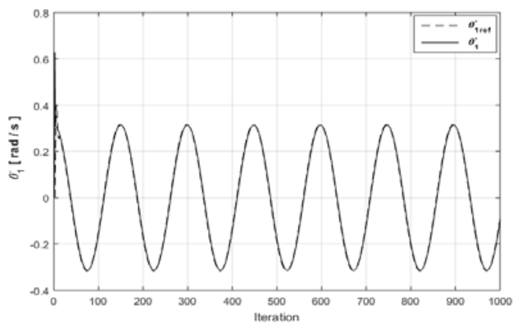


Fig.6. Evolution of angular velocity of the inverted pendulum

Figure 7 represents the control input which is made up of the voltage of the motor driver (proportional to the motor torque), where it can be seen that it remains within the admissible limits (± 10 V).

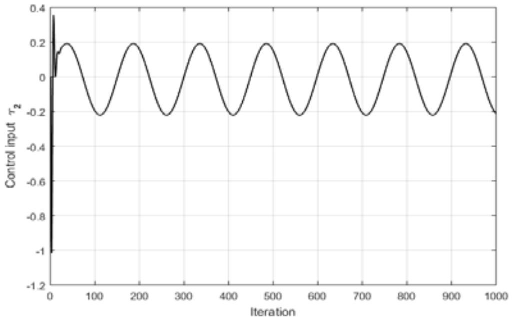


Fig.7. Control input τ_2

Figure 8 shows the evolution of the plan portrait $(\theta_1, \dot{\theta}_1)$. We notice the convergence of the initial condition towards a stable limit cycle. We can deduce that this predictive control technique meets the desired performance.

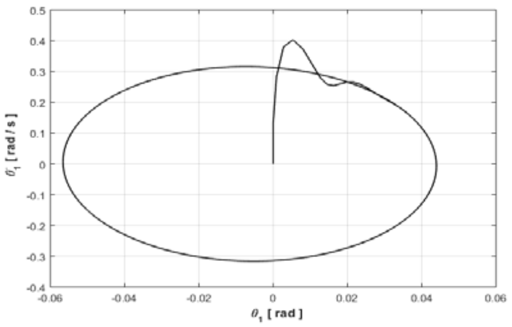


Fig.8. The inertia wheel inverted pendulum

Conclusion

In this paper, a nonlinear predictive control approach is proposed for reference trajectories tracking for underactuated mechanical systems. The control scheme is designed in the special case of the inertia wheel inverted pendulum. The Lagrangian dynamic model of the system is nonlinear and its internal dynamics is unstable. As a result, the system is at non-minimum-phase.

The simulation results obtained show the effectiveness of the proposed approach. however, a nonlinear predictive control, it can be easily applied to the more general case of underactuated mechanical systems.

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