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New multipoint algorithm for eliminating chaotic vibrations in complex non-linear systems

***Abstract**. The paper presents a very effective algorithm for stabilizing unstable periodic orbits, consisting of slight changes in selected parameters of a chaotic system at any time of sampling. Modification of the parameters leads to minimization of the distance of the phase trajectory from the fixed point on the cross-section of the generalized Poincaré map. By modifying several parameters, it is possible to effectively eliminate chaotic vibrations in complex non-linear dynamical systems in the presence of strong disturbances and noise.*

***Streszczenie.** W pracy przedstawiono efektywny algorytm stabilizacji niestabilnych orbit okresowych, polegający na niewielkich zmianach wybranych parametrów układu chaotycznego w każdej chwili próbkowania. Modyfikacja parametrów prowadzi do minimalizacji odległość trajektorii fazowej od punktu stałego na przekroju uogólnionego odwzorowania Poincarégo. Realizacja zagadnienia sterowania poprzez zmianę kilku parametrów umożliwia efektywną eliminację drgań chaotycznych w złożonych nieliniowych układach dynamicznych w obecności silnych zakłóceń i szumów. (**Nowy algorytm multipunktowy służący do eliminacji drgań chaotycznych w złożonych układach nieliniowych**).*

Keywords: non-linear systems, chaotic vibrations, strange attractors, multipoint stabilization algorithms.
Słowa kluczowe: układy nieliniowe, drgania chaotyczne, dziwne atraktory, multipunktowe algorytmy stabilizacji.

Introduction

Before chaotic systems were discovered, it was believed that the transient states of deterministic systems are predictable, i.e. knowing the mathematical description of a given system and initial conditions, its state can be determined at any time. However, it turned out that there are some systems with elements with specific non-linear characteristics which are characterized by strong sensitivity to initial conditions. Moreover, the courses of certain state variables of these systems resemble completely random [1-5]. A rapid increase in the interest of research centres in the subject of chaotic dynamics could be observed over the past 30 years. This fact has been caused by the extensive use of computers to solve non-linear problems. An important factor that motivates the development of research related to chaotic dynamics is a rapid development of power electronics, which has contributed to the widespread installation of non-linear receivers in the power system. The use of such highly non-linear loads can have a negative effect on the supply network. At present, it can be estimated that the level of voltage and current distortion in electrical systems has exceeded the alarm threshold. In this regard, various countermeasures are being taken, which are aimed at preventing further increase in deformation [2, 4].

Considering the fact that chaotic oscillations are usually unwanted, the question arises whether it is possible to eliminate these unfavourable vibrations and make a given chaotic system predictable. It turns out that the development of effective control strategies [4-15] is, apart from effective methods of predicting chaotic states [16-19], one of the most important problems that arises when analyzing chaotic phenomena. This problem is also revealed in plasma physics, in particular in research on effective control of an electric arc furnace [4, 5, 20-27]. In the initial stage of melting the load, the furnace is in a chaotic state, which is manifested by the continuity of the spectral characteristics of the currents drawn from the electrical grid. This is a particularly unfavourable phenomenon, because the occurrence of subharmonic vibrations may in many cases be dangerous for elements such as synchronous machines and transformers, as well as for the electrodes of the furnace due to the formation of mechanical resonances, which may cause serious failures in the supply system of the electric arc furnace [4]. Moreover, chaotic vibrations of low-frequency voltage and current waveforms, significantly exceeding the permissible standards in real systems of this type, have a negative impact on users connected in parallel.

The problems of controlling or elimination of chaos began to be dealt with as early as in the 1980s [1]. Due to the fact that chaos can appear in non-linear systems only, the control process is seriously hampered. Non-linear circuits are subject to neither the principle of proportionality nor the principle of superposition, and thus even relatively small changes in excitation can cause dramatic and unpredictable effects [2-5, 7, 12, 14, 28-35].

Among many methods, the OGY method [14], which was presented in 1990 by Ott, Grebogi and Yorke, turned out to be a very important concept of controlling chaotic systems. This method involves small modifications in one of the parameters of the chaotic system to stabilize the unstable periodic orbit immersed in a strange attractor. The advantage of this concept is that no high energy expenditure is required to eliminate unwanted vibrations. The OGY method and the minimum distance method [7], presented by Galias in 1995, inspired the author to develop new multipoint method of controlling complex chaotic systems [4].

Multipoint minimum distance method

Algorithms for stabilizing unstable periodic orbits in the single-point version [7, 14] are difficult to implement in real systems. This is because the system parameter is modified only once during the stabilized orbit period. Multipoint algorithms presented in [7, 14] can also be unreliable, especially when controlling complex chaotic systems in the presence of strong disturbances and noise. Another limitation of the discussed methods is the modification of one control parameter only. Such algorithms can be used to stabilize chaotic systems of higher order in case when a fixed-point on the Poincaré section, corresponding to a given periodic orbit, has one unstable eigenvalue only [14]. When dealing with periodic orbits that have more unstable eigenvalues, it is necessary to modify more than one parameter to stabilize a given orbit immersed in a strange attractor.

For these reasons, the author has developed a new effective multipoint algorithm [4], based on the minimum distance method developed by Galias [7], in which it is possible to modify several control parameters at any time of sampling. That is, depending on the implementation, the control parameters can be changed even several of thousands of times during the period of stabilized orbit. This approach allows controlling complex chaotic systems in the presence of strong disturbances and noise.

Let us consider an autonomous non-linear system, described by a system of ordinary differential equations of n-th order, which depends on k parameters $\mathbf{p}$:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{F}(\mathbf{x}(t), \mathbf{p}), \tag{1}$$

where $\mathbf{F}$ is a continuous vector field, $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{p} \in \mathbf{R}^k$, wherein $k \leq (n-1)$. Let's assume that for the values of $\mathbf{p} = \mathbf{p}_0$ the periodic orbit γ belongs to a strange attractor. We also assume that a small modification of parameters $\mathbf{\Delta}_{\mathbf{p}_{max}}$:

$$\mathbf{p}_0 - \Delta_{\mathbf{p}_{max}} \leq \mathbf{p} \leq \mathbf{p}_0 + \Delta_{\mathbf{p}_{max}} \tag{2}$$

does not lead to the disappearance of the attractor and the chosen periodic orbit.

In the case of the multipoint method, we do not permanently determine the position of the $\sum_i$ hyperplanes of general Poincaré map, but we dynamically determine the map at each sampling time which points from one hyperplane assigns to points from another hyperplane. Therefore, the problem of control consists in placing the hyperplane $\sum_i$ in such a way that the phase trajectory of the chaotic system intersects it transversally at the $\mathbf{x}_{\mathbf{P}i}$ at a specific sampling time. By $\mathbf{P}_i$, we denote the general Poincaré map, defined in a certain point $\mathbf{x}_{\mathbf{P}_{F_i}}$ environment, where the map depends on k parameters:

$$\mathbf{P}_i : \mathbf{R}^{(n-1)} \times \mathbf{R}^k \ni (\mathbf{x}_\mathbf{P}, \mathbf{p}) \to \mathbf{P}(\mathbf{x}_\mathbf{P}, \mathbf{p}) \in \mathbf{R}^{(n-1)}. \tag{3}$$

Let $\mathbf{x}_{\mathbf{P}_{F_i}}$ be the map's fixed-point (3) for parameter values $\mathbf{p} = \mathbf{p}_0$:

$$\mathbf{P}_i(\mathbf{x}_{\mathbf{P}F_i}, \mathbf{p}_0) = \mathbf{x}_{\mathbf{P}F_{i+1}}. \tag{4}$$

Linear approximation of the general Poincaré map around the point $(\mathbf{x}_{\mathbf{P}_{F_i}}, \mathbf{p}_0)$ can be presented in the following form:

$$\mathbf{P}_i(\mathbf{x}_\mathbf{P}, \mathbf{p}) \approx \mathbf{P}_i\left(\mathbf{x}_{\mathbf{P}_{F_i}}, \mathbf{p}_0\right) + \mathbf{J}_{\mathbf{P}_i}\left(\mathbf{x}_\mathbf{P} - \mathbf{x}_{\mathbf{P}_{F_i}}\right) + \mathbf{G}_i(\mathbf{p} - \mathbf{p}_0), \tag{5}$$

where $\mathbf{x}_\mathbf{P} \in \mathbf{R}^{(n-1)}$, $\mathbf{p} \in \mathbf{R}^k$, $\mathbf{J}_{\mathbf{P}_i} \in \mathbf{R}^{(n-1)\times(n-1)}$, $\mathbf{G}_i \in \mathbf{R}^{(n-1)\times k}$. The coefficients of the linear approximation (5) can be determined with high accuracy using numerical integration procedures [4]. We have obtained very good results using Runge-Kutta second-order algorithms. In the case under consideration $\mathbf{J}_{\mathbf{P}_i} = \frac{\partial \mathbf{P}_i}{\partial \mathbf{x}_\mathbf{P}}(\mathbf{x}_{\mathbf{P}_{F_i}}, \mathbf{p}_0)$ is a Jacobi's matrix of the general Poincaré map in point $(\mathbf{x}_{\mathbf{P}_{F_i}}, \mathbf{p}_0)$, while $\mathbf{G}_i = \frac{\partial \mathbf{P}_i}{\partial \mathbf{p}}(\mathbf{x}_{\mathbf{P}_{F_i}}, \mathbf{p}_0)$ is a matrix of partial derivatives of this map in relation to the set of $\mathbf{p}$ parameters.

By using the Poincaré map, we reduce the size of the state space by 1, while remembering that the individual hyperplanes of the generalized map (3) are located along the x_i direction. In accordance with the idea of the minimum distance method, we look for a value of the $\mathbf{p} = [p_1 \;\; p_2 \;\; \dots \;\; p_k]^T$ parameters to minimize the distance of the trajectory from the fixed-point in each iteration of the general Poincaré map:

$$d(\mathbf{P}_i(\mathbf{x}_\mathbf{P}, \mathbf{p}), \mathbf{x}_{\mathbf{P}_{F_{i+1}}}) = \min. \tag{6}$$

Using linearization (5) and the definition of pseudo-inversion of the matrix [36] the value of the control parameters $\mathbf{p}$ for which:

$$\left\| \mathbf{J}_{\mathbf{P}_i}(\mathbf{x}_\mathbf{P} - \mathbf{x}_{\mathbf{P}_{F_i}}) + \mathbf{G}_i(\mathbf{p} - \mathbf{p}_0) \right\| \tag{7}$$

reaches a minimum value, is:

$$\mathbf{p} = \mathbf{p}_0 - (\mathbf{G}_i{}^T\mathbf{G}_i)^{-1}\mathbf{G}_i{}^T\mathbf{J}_{\mathbf{p}i}(\mathbf{x}_\mathbf{P} - \mathbf{x}_{\mathbf{P}Fi}), \tag{8}$$

where $\mathbf{x}_\mathbf{P}, \mathbf{x}_{\mathbf{P}Fi} \in \mathbf{R}^{(n-1)}$, $\mathbf{p}, \mathbf{p}_0 \in \mathbf{R}^k$, $\mathbf{J}_{\mathbf{P}_i} \in \mathbf{R}^{(n-1)\times(n-1)}$, $\mathbf{G}_i \in \mathbf{R}^{(n-1)\times k}$. We assume that the rank of the $\mathbf{G}_i$ matrix is k. When we have (n - 1) parameters that are involved in the control process, i.e. k = (n - 1), formula (8) can be simplified to form:

$$\mathbf{p} = \mathbf{p}_0 - \mathbf{G}_i{}^{-1}\mathbf{J}_{\mathbf{p}i}(\mathbf{x}_\mathbf{P} - \mathbf{x}_{\mathbf{P}Fi}). \tag{9}$$

In the presented multipoint method, we modify the parameters at any time in accordance with (8) in the event that the distance of the trajectory from the point $\mathbf{x}_{\mathbf{P}_{F_i}}$ on the Poincaré cross-section is less than the assumed value of $d_{\max}$. Otherwise we set the control parameters to the nominal value of $\mathbf{p}_0$ and wait with the modification of parameters until the trajectory passes again close to one of the fixed points on the $\sum_i$ hyperplane, located in any area of the phase space within the chaotic attractor. On the other hand, if one of the control parameters calculated from (8) differs from the nominal value more than the acceptable change of a particular parameter $\mathbf{\Delta}_{\mathbf{p}_{jmax}}$, then this parameter is set to the value $p_j = p_{0j} + \Delta_{p_{jmax}}$ in the situation when $(p_j - p_{0j}) > \Delta_{p_{jmax}}$ or to the value $p_j = p_{0j} - \Delta_{p_{jmax}}$ in the case when $(p_j - p_{0j}) < (-\Delta_{p_{jmax}})$, while the rest of the control parameters is set according to the relation (8). The elimination of chaotic vibrations with the use of the presented multipoint algorithm is possible when we have an appropriate analytical description of the position of an unstable periodic orbit in the phase space. This description is necessary to determine the location of the fixed points $\mathbf{x}_{\mathbf{P}_{F_i}}$ on Poincaré cross-sections in the whole attractor space. Interested readers are invited to study the thesis [4].

Due to the very small distances between the individual hyperplanes of the $\mathbf{P}_i$ map, the linear approximation (5) is very accurate in a long trajectory distance from a given fixed- point $x_{P_{F_i}}$. This is an extremely advantageous property of the presented multipoint algorithm, because it is possible to significantly increase the $d_{\max}$ parameter, which affects the moment of starting the stabilization process. Moreover, the possibility of increasing the $d_{\max}$ parameter is of great importance in the case of stabilization of a selected unstable orbit in the presence of strong disturbances and noise. It is also worth emphasizing that the possibility of changing several parameters of the system allows the use of such control for any complex chaotic systems.

Chaotic vibrations in Chua's circuit

The considered multipoint algorithm will be tested using the Chua's circuit [2, 4, 5], which, due to its properties, is an extremely popular subject of research at scientific centres dealing with the problems of chaos theory. The vast majority of tests of the Chua circuit is carried out using the piecewise-linear characteristic of a non-linear element, and the research has shown that it is representative of other non-linear characteristics that may be smooth, and thus differentiable over the entire voltage range [2]. It is worth emphasizing that the results obtained during the simulation of this circuit are very similar to the results obtained on the basis of a physically realized experimental system. On the one hand, the not very complicated structure of the circuit, and on the other hand, the enormity of specific phenomena that can be identified in it, mean that the interest in this circuit is currently increasing and there are often descriptions that reveal many of unknown issues in the field of non-linear systems. With the appropriate selection of parameters, a full range of behaviours can be observed in this circuit that are characteristic of a wide class of non-linear dynamical systems, namely: asymptotic stability of the equilibrium state at a point, periodic oscillations, bifurcations and chaotic vibrations. Chaotic phenomena, which are generated in the considered circuit, can be described by the following system of differential equations:

$$\frac{du}{dt} = \frac{1}{C_2}\left(\frac{u_1 - u}{R} - i(u)\right)$$
(10)
$$\frac{du_1}{dt} = \frac{1}{C_1}\left(\frac{u - u_1}{R} + i_L\right) ,$$
$$\frac{di_L}{dt} = -\frac{1}{L}\left(u_1 + R_L \cdot i_L\right)$$

where the current-voltage characteristics of a non-linear resistor can be described by the following relationship:

(11) $i(u) = G_1 u + \frac{1}{2}(G_2 - G_1)[|u + v| - |u - v|],\ |G_1| < |G_2|.$

In order to verify the effectiveness of the developed algorithm for stabilizing unstable periodic orbits, we have realized a series of numerical experiments using the MATLAB package. Fig. 1 shows the Chua's system bifurcation diagram obtained for the following set of parameters: C_1=1000mF, C_2=64.103mF, G_1=-0.7143S, G_2= -1.3429S, R=1Ω, R_L=0, v=1V. Solution with the following initial conditions: $u(0)$=2V, $u_1(0)$=0.26V, $i_L(0)$=1.9A, was obtained using the *Ode23* numerical integration procedure.

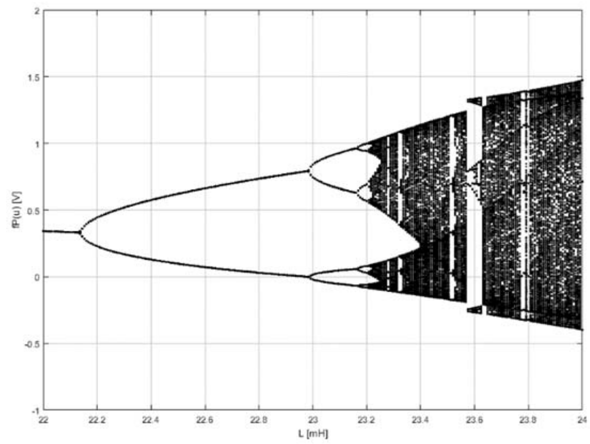

Fig.1. Chua's system bifurcation diagram

The above diagram shows diversity of Chua's circuit. As a result of changes in L parameter, a qualitative change in the system dynamics occurs. Initially, periodic oscillations are generated in the circuit. For L = 22mH, the phase trajectory attractor consists of a single periodic orbit. By increasing the bifurcation parameter, we reach the critical value L = 22.14mH, for which the oscillation period is doubled. At this point, as a result of bifurcation, the original stable orbit of period 1 is transformed into an unstable orbit of the same period and a stable orbit of period 2. Further increase this parameter leads to the transition of the system into a chaotic state as a result of bifurcation cascade of doubling the period [1, 2, 4]. Eventually, for L = 23.23mH, chaos arises in considered system.

Two types of strange attractors are characteristic for the Chua system, the Rössler-type attractor and the double-scroll attractor [2, 4]. A characteristic feature of strange attractors is a fractal structure. They express the natural ability of dynamical systems to self-organize in such a way that their phase trajectories fill the phase space more or less evenly. Fig. 2a shows a strange Rössler-type attractor obtained for L = 23.81mH. It should be emphasized that such an attractor consists of an infinitely many unstable periodic orbits. Fig. 2b shows a phase portrait in order to better identify the location of the periodic orbit of interest, while Fig. 2c shows an unstable periodic orbit, immersed in the concerned attractor. The location of this orbit was determined on the basis of the location of pseudo-periodic orbits within the attractor [4, 7].

Based on the bifurcation diagram, it is possible to select parameters of a given system in such a way as to obtain the desired behaviour, but this approach is not always possible. In such a case, an interesting alternative may be the use of developed multipoint algorithms of stabilizing unstable periodic orbits, consisting of small modification in selected parameters of the system, which leads to the elimination of chaotic oscillations.

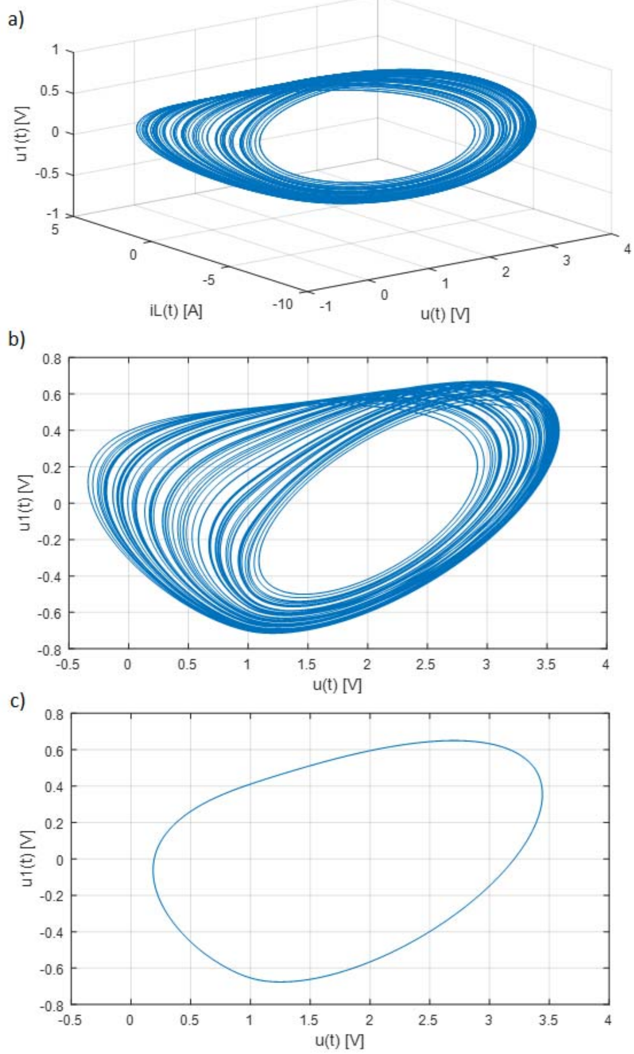

Fig.2. Chaotic phenomena of the Chua's system: a) Rössler-type attractor, b) phase portrait of the Chua's system, c) unstable periodic orbit immersed in the attractor

Stabilization of short periodic orbit

Elimination of chaotic vibrations in a given non-linear system, using the considered multipoint algorithm, consists in stabilization of a selected periodic orbit by small modifications in selected control parameters at each sampling time. The control process can be started when the trajectory passes close enough to the stabilized orbit. Fig. 3a shows a trajectory of voltage $u(t)$ obtained for the sampling period h = 0.001s and $d_{\max}$ = 0.01. The phase trajectory of the system after 9 s approached sufficiently close to the stabilized periodic orbit, which allowed the use of control procedure (Fig. 3b). In case of increasing the parameter d_{max} to the value 0.1, the control procedure could be started immediately (Fig. 3c – 3d). The obtained results clearly show the advantages of the developed multipoint algorithm. The high accuracy of the linear approximation (5) in the large environment of the fixed-point allows for a significant reduction of the control start time. In the absence of disturbances in the system, successful stabilization is possible even for a very small control signal. Modifying the control parameters at any time of sampling allows for the successful execution of the control process with a much larger allowable parameter change ($\mathbf{\Delta_p} > 0{,}1\mathbf{p}_0$). It is extremely important when controlling complex chaotic systems in the presence of strong disturbances and noise.

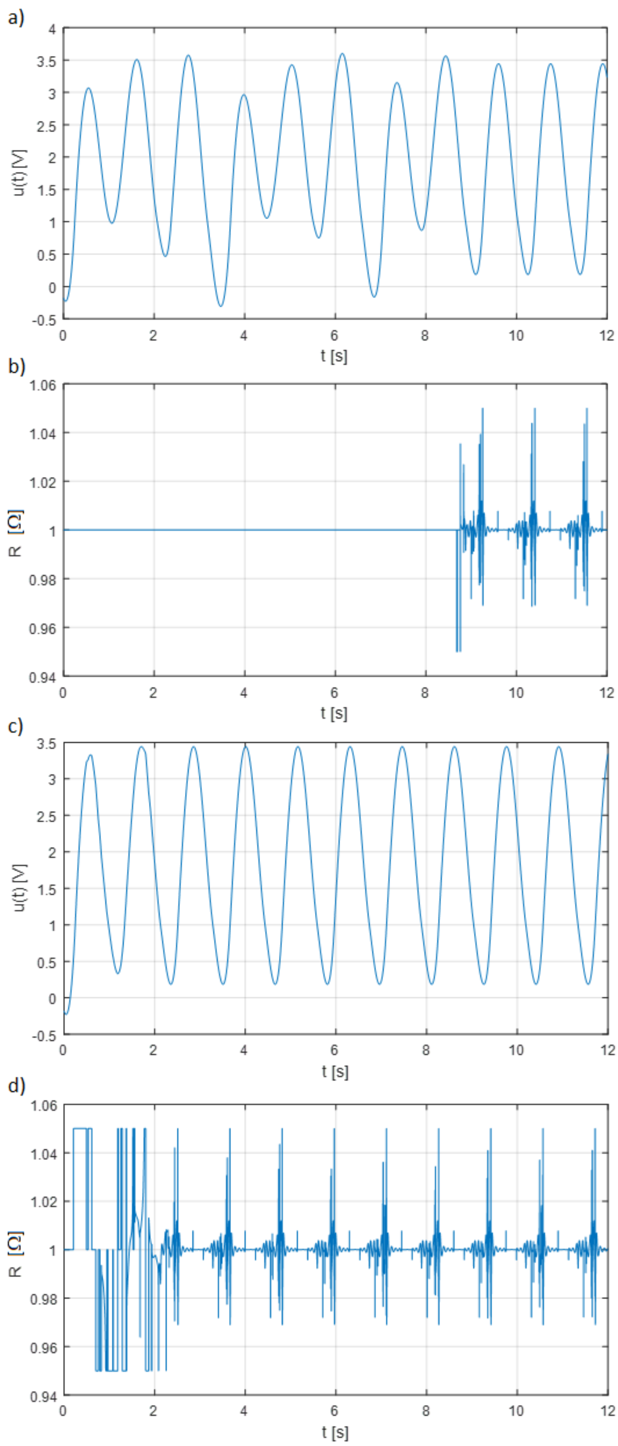

Fig.3. Stabilization of a short periodic orbit, immersed in a strange attractor: a), b) for d_{max} = 0.01; c), d) for d_{max} = 0.1

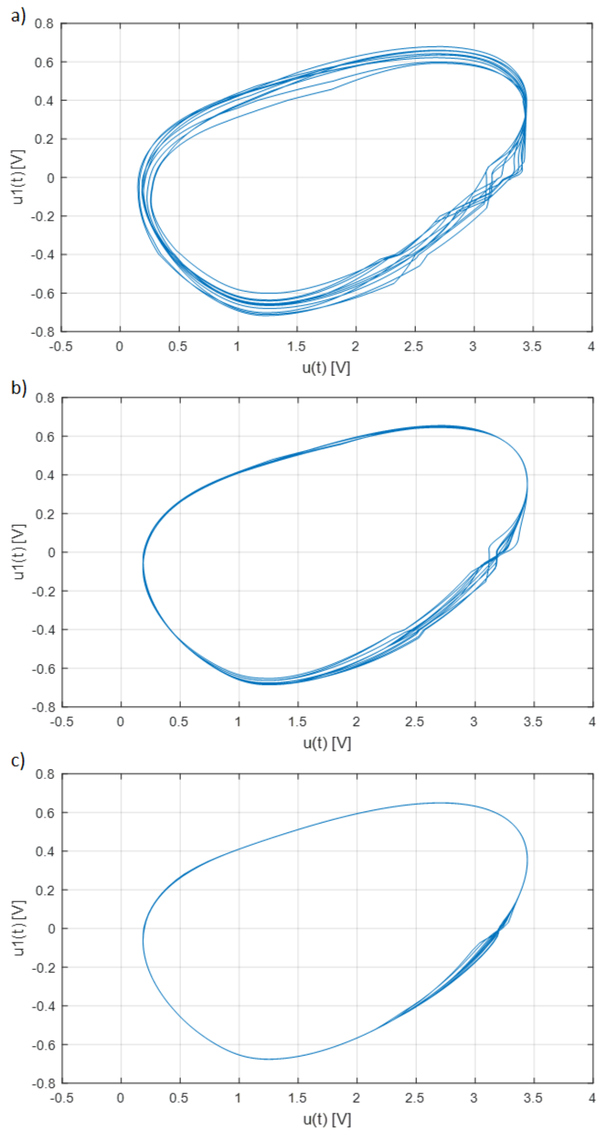

Fig.4. Stabilization of vibrations in the Chua's system in the presence of strong disturbances: a) $\mathbf{p} = R$; b) $\mathbf{p} = [R\ C_1]^T$; c) $\mathbf{p} = [R\ C_1\ L]^T$

In order to investigate the effect of the strong disturbances on the quality of the stabilization, a quasi-periodic voltage trajectory were connected in series with the coil [4]. The maximum permissible modification of control parameters was $\mathbf{\Delta}_{\mathbf{p}_{max}} = 0.1\mathbf{p}_0$. Figure 4 shows a phase portrait in coordinates $(u(t), u_1(t))$. In the case of modification of R parameter (Fig. 4a), stabilization was successful, however much better results were received when two control parameters $\mathbf{p} = [R\ C_1]^T$ were changed (Fig. 4b). We managed to obtain the best results when changing three parameters
$\mathbf{p} = [R\ C_1\ L]^T$ (Fig. 4c). It should be noted that the trajectory would perfectly match the stabilized periodic orbit in the absence of any disturbances in the considered system. The advantage of the developed multipoint method is a possibility to change several control parameters, which allows for the effective elimination of chaotic oscillations in complex non-linear systems of higher orders.

In the case of controlling real systems, a very important aspect is the possibility of effective stabilization of the phase trajectory in the presence of noise. Fig. 5a - 5b shows the results of the elimination of chaotic vibration in the Chua's system in the presence of noise at the level of 5%.

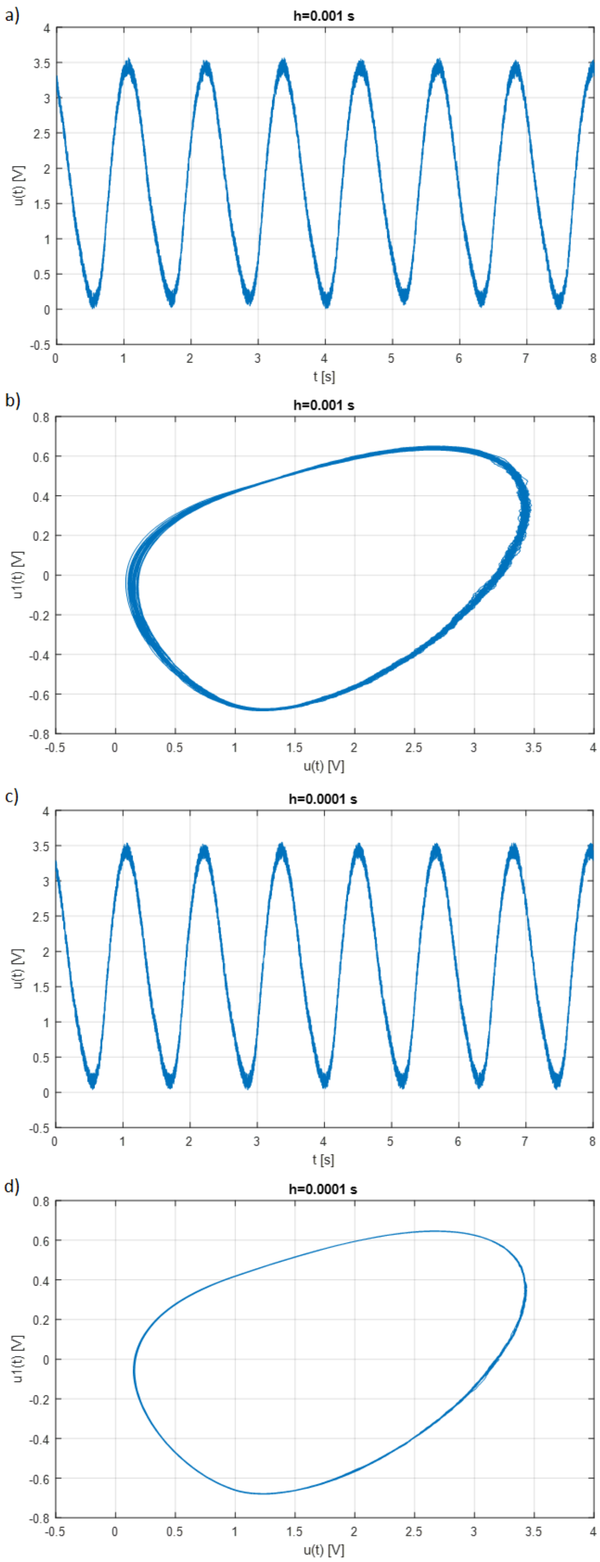

Fig.5. Stabilization of a short periodic orbit in the presence of 5% noise in the considered system: a), b) for h = 0.001s; c), d) for h = 0.0001s

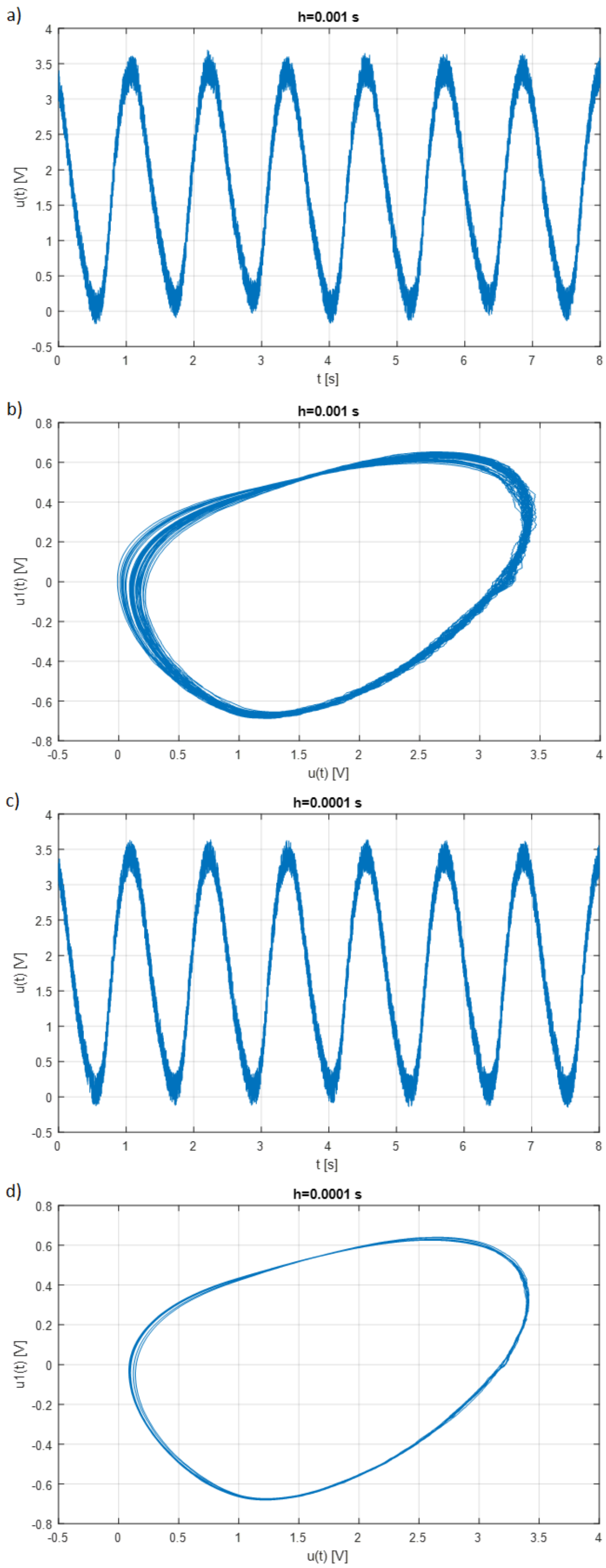

Fig.6. Stabilization of a short periodic orbit in the presence of 10% noise in the Chua's circuit: a), b) for h = 0.001s; c), d) for h = 0.0001s

In the case under consideration, the modification of the control parameters $\mathbf{p} = [R\ C_1]^T$ has been performed with the sampling step h = 0.001s. We have obtained much better results for h = 0.0001s (Fig. 5c – 5d). The high efficiency of the developed algorithms for the stabilization of periodic orbits is confirmed by the results obtained for the elimination of chaotic oscillations in the presence of 10% noise (Fig. 6).

The presented multipoint method is effective even in the case of controlling the system in the presence of noise at such a high level (Fig. 6a - 6b). The reduction of the sampling step leads to a significant improvement in the quality of stabilization, which is confirmed by the phase portrait shown in Fig. 6d. The obtained results confirm the advantages of the multipoint algorithm developed by the author, in which we can modify several parameters at any time of sampling. Frequent changes of control parameters are of great importance in the case of stabilization of a selected unstable periodic orbit in the presence of strong noise in a given system.

Summary

The advantage of the presented algorithm for stabilizing unstable periodic orbits is high efficiency and the ability to significantly increase the $d_{\max}$ parameter, which affects the moment of starting the control process. This procedure considerably reduces the transient process when the system is unstabilized and is of great importance when stabilizing the selected orbit in the presence of disturbances and noise. It is also very important that the period when the trajectory is uncontrolled is reduced to a minimum, because the change of parameters takes place at any time of sampling. Due to the possibility of frequent modification of the control signal, the linear approximation of the generalized Poincaré map is very accurate in the large environment of the periodic orbit, which allows the stabilization of selected unstable periodic orbits in the presence of very strong noise. In addition, the ability to change several control parameters allows the use of such control for any complex chaotic systems.

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