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# Iterative control of the static compensator

***Abstract**. The shunt static compensator with the iterative control is presented. The compensator generates reactive current computed in the real time with the use of the iterative secant method. Simulation results for the circuit composed of the three phase grid and DC/AC converter with PWM space vector control are presented. The control signal is acquired from the grid voltage. The RMS value of grid voltage is chosen as the compensator aim.*

***Streszczenie.** Opisano układ i sterowanie równoległego kompensatora statycznego. Kompensator generuje prąd bierny obliczany w czasie rzeczywistym z wykorzystaniem metody siecznych. Zaprezentowano wyniki symulacji obwodu złożonego z modelu sieci trójfazowej oraz przekształtnika DC/AC za sterowaniem PWM metodą wektora przestrzennego. Sygnały sterujące uzyskuje się z mierzonej wartości skutecznej napięcia sieci. Funkcją celu jest zadany poziom napięcia w węźle dołączenia kompensatora do sieci. (**Sterowanie kompensatora statycznego z użyciem algorytmu iteracyjnego**).*

**Keywords:** static compensation, iterative control, grid voltage stabilization.
**Słowa kluczowe:** kompensator statyczny, sterowanie iteracyjne, stabilizacja napięcia sieci energetycznej.

## Introduction

Power flow studies are important in designing and in determining the best operation of existing systems. Electric utility companies use very elaborate programs for power flow studies to obtain information concerning the system design and operation. Very advanced arrangements are installed in order to control power flow in a electric grid [1]. The possibility of generating or absorbing controllable reactive power with various power electronic switching converters has long been recognized [2]. These converters do not use capacitor or reactor banks to produce reactive power. They operate as alternating voltage or current sources.

The development of the power theory of electrical circuits and systems has more than century-long history with great variety of different attempts aimed at its formulation, with a lot of discussions and controversy [3]. In general, p-q theory can be used for both harmonic and reactive power compensation and it works effectively even under unbalanced conditions [4]. For balanced three-phase systems d-q transformation enables to transform the system to the stationary frame.

The local system weaknesses such as low voltages or over-voltages can be reduced by electronic power compensators. The STATCOM regulates voltage at its terminal by controlling the amount of reactive power injected into the power system. Such arrangements can be controlled by the local node voltage. The considered compensator is oriented only on the fundamental harmonic, higher harmonics are beyond the scope of this paper. Assuming that harmonic contents in the voltage is small the control can be based on the RMS value. The complex RMS values are denoted by capital letters *U* and *I* , vertical bars describe modules of these complex values $|U|$ and $|I|$. The control signal is achieved with the use of the iterative algorithm. Such approach is classified as the repetitive method.

## Thevenin grid representation

Let us consider the 2-port model of the single-phase grid as shown in Fig. 1.

Assume that considered nodes *ab* belong to the load type with no generator connected. The Thevenin impedance $R + jX$ and complex RMS voltage $E$ are unknown the system parameters. The compensator is represented by controlled current source $I$ depending on voltage $U$

(1) $$I = jbU$$

where $U$ is complex RMS voltage at the compensator nodes. Control parameter $b$ is real and it is computed using the iterative algorithm based on measurements of voltage $|U|$. The compensator with control parameter $b$ should maintain voltage $|U|$ on the given reference level $U_d$ . For circuit in Fig. 1, we obtain

(2) $$|U| = \frac{|E|}{\sqrt{(1+bX)^2 + (bR)^2}}$$

Voltage $|U|$ depends on control parameter *b* and system parameters $R$ and $X$. The compensator increases the grid voltage for $bX < 0$ and it decreases for $bX > 0$ .

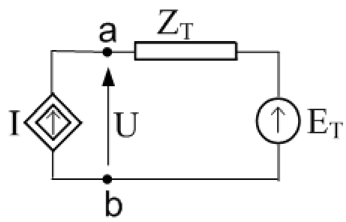


Fig.1. Thevenin representation of the 2-port

Let the dimensionless quantities be defined

(3) $$h = \frac{|U|}{|E|},\ r = \frac{R}{|X|},\ p = bX \text{ and } c = \frac{|X|}{|E|}|I|$$

Current *c* in (1) is expressed by the ratio of current $|I|$ to short-circuit current $\frac{|E|}{|X|}$. For circuit shown in Fig. 1, voltage gain *h* depends on control parameter *p* as follows

(4) $$h = \frac{1}{\sqrt{(1+p)^2 + (pr)^2}}$$

Current $c$ is expressed as

(5) $$c = \frac{|p|}{\sqrt{(1+p)^2 + (pr)^2}}$$

Relations given in (2) and (3) for chosen resistance value $r = 0.2$ are illustrated in Fig. 2.

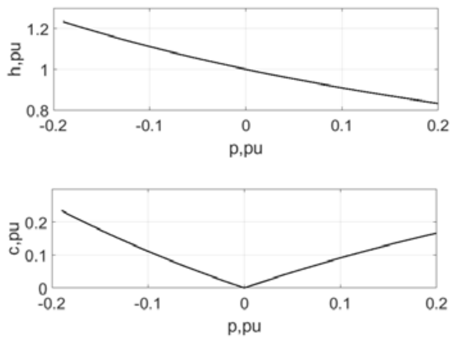

Fig.2. Voltage gain *h* and compensator current *c* as functions of control parameter *p*

The characteristics are obtained for Thevenin model of grid, with the assumption that the compensator current does not influent on parameters $R$, $X$ and $E$. The iterative algorithm presented in next sections is performed on the measurements of the actual grid voltage, this algorithm omits the model parameters $R, X, E$ [5].

### Iterative algorithm

Let desirable voltage shown Fig. 1 be

(6) $$|U| = U_d$$

The problem can be formulated as optimization problem. Find the control parameter *b* such that $(||U| - U_d|)$ is minimal, while only voltage $|U|$ is available. The problem can be solved by the use of the iterative algorithm similar to the secant method.

*Step 0.* Measure voltage $|U_0|$ for zero compensator current

(7) $$b_0 = 0$$

*Step 1.* Measure voltage $|U_1|$ for optional chosen nonzero compensator parameter $b_1$ such that $|I_1| = |b_1||U_0| < I_{\max}$.

*Step k.* Measure voltage $|U_k|$ and compute parameters $b_k$

(8) $$b_k = \frac{|U_0| - U_d}{|U_0| - |U_{k-1}|} b_{k-1}$$

for $k = 2, ..., K$, where final index *K* is such that $(||U_K| - U_d|) < U_{\min}$. The compensation criterion is fulfilled.

The algorithm given in the form of equations (7) and (8) for $k = 2$ is illustrated in the example presented below. Presented simulation is done with the use of PLECS program.

*Numerical example 1.*

The PLECS model of the compensator is shown in Fig. 3.

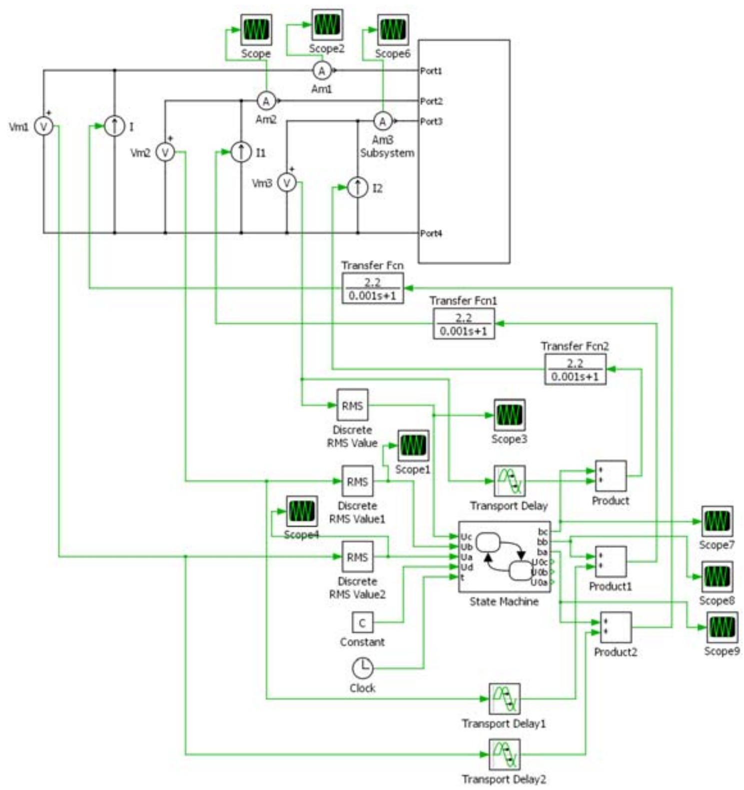

Fig.3. PLECS compensator model with current controlled sources

The Subsystem seen in the upper part of the figure contains the model of the three-phase grid as shown in Fig.4. AC voltage sources allow to obtain three sets of voltages: positive sequence, negative sequence and zero sequence. Voltmeters Vm1, Vm2 and Vm3 measure the voltage waveforms between three-phase lines and neutral line. These three voltages are the only signals available for the compensator control. Current controlled sources I, I1 and I2 represent the compensator. The time varying control signals for these sources are elaborated in the structure seen in the lower part of Fig. 3. The algorithm is performed in State Machine. This block is shown in Fig. 5. RMS voltage values $|U_a|, |U_b|, |U_c|$ obtained in the RMS blocks constitute three input signals for State Machine. RMS blocks work in the sliding mode. Parameters $b_a, b_b, b_c$ computed in State Machine constitute three time varying signals which are multiplied by grid voltage waveforms $u_a, u_b, u_c$ delayed by the quarter of period. Such delay realized in Transport Delay blocks causes that generated current is reactive. The resulting signals control current sources *I,I1,I2*.

State machine seen in Fig. 5 contains four states which are in duty in four time intervals. State and State1 realize *Step 0* and *Step 1* of the algorithm. State3 and State4 realize the next two algorithm steps.

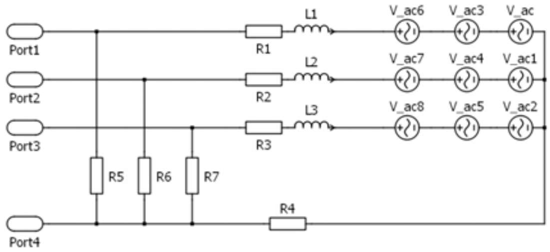


Fig. 4. Grid model

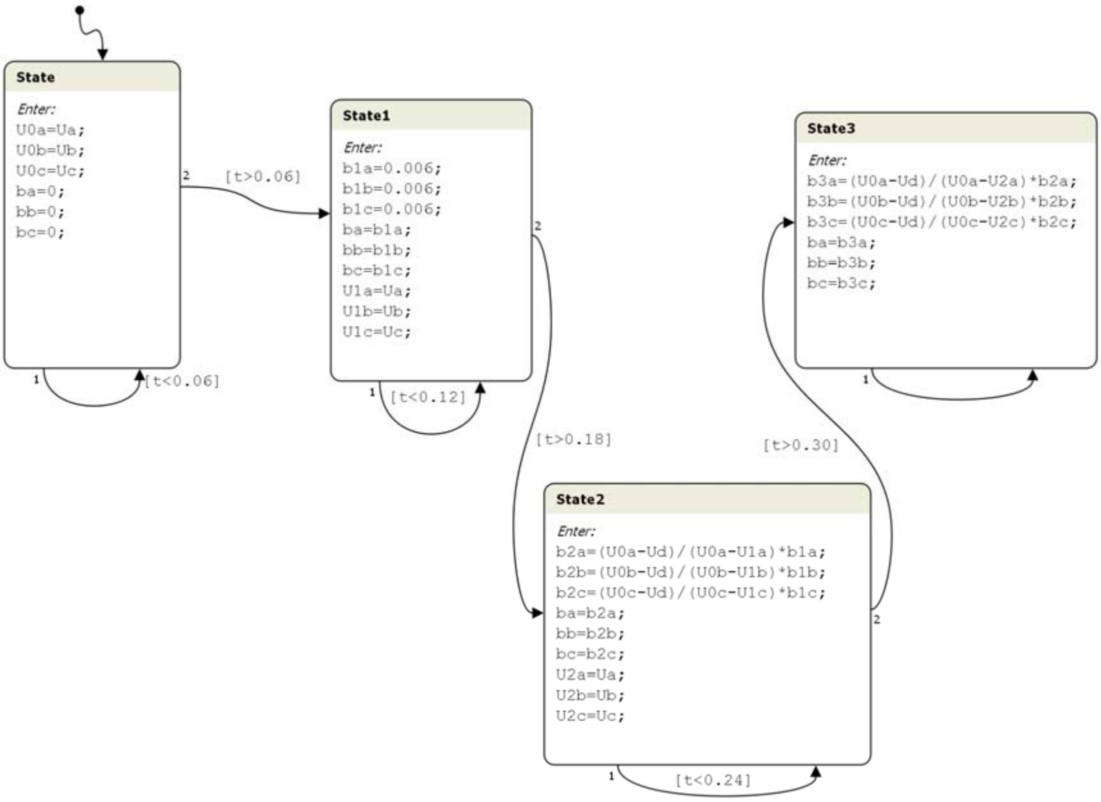


Fig. 5. State machine

Simulation results observed within time interval (0,0.4s) in the Scopes 9, 2 and 4 seen in Fig. 3 are presented in Figs. 6, 7, and 8, respectively. These results have been obtained for slightly unbalanced grid sources (Fig. 4). The amplitudes of voltage sources for positive, negative and zero symmetrical components were chosen as follows $|U_m^+| = 305V$, $|U_m^-| = 10V$, $|U_m^0| = 5V$. Demanded RMS grid voltage is chosen as $U_d = 230V$.

Simulation results are presented in Figs. 6,7,8. Control parameter $b_a$ is computed by state machine. This signal is multiplied by grid voltage waveform.

The obtained signal controls the current sources seen in Fig. 3. The resulting compensator current is shown in Fig. 7. As the final result the RMS grid voltage can observed in Fig. 8.

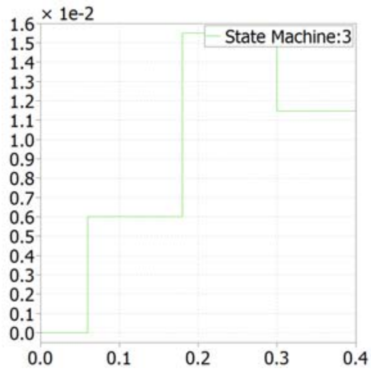


Fig. 6. Control parameter $b_a$, $1/\Omega$

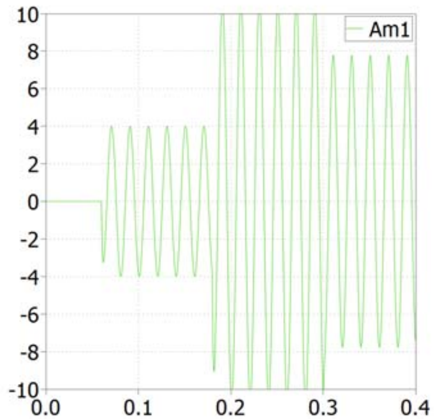

Fig. 7. Compensator current $i_a$, amper

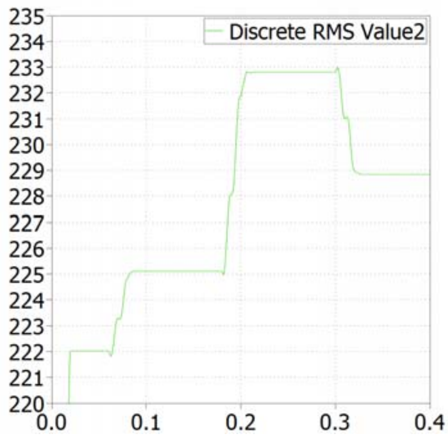

Fig. 8. RMS grid voltage $|U_a|$, volt

Similar three waveforms can be observed for the remaining two phases. Grid RMS voltages for $a,b,c$ phases before and after compensation are shown in Table 1.

Table 1. Voltages before and after compensation

| | $\|U_a\|$ | $\|U_b\|$ | $\|U_c\|$ |
|---|---|---|---|
| Before comp. | 222.0 | 206.2 | 206.2 |
| After comp. | 228.8 | 230.6 | 232.8 |

Results presented in Table 1 are obtained after 3 iterative steps.

**Signals for PWM control of the DC/AC inverter**

A shunt compensator operates as a current source controlled by the grid voltage waveform. This current is obtained from DC/AC voltage inverter with the additional series inductor as shown in Fig. 9. The DC side of the inverter is connected to capacitor *C* with voltage $U_{dc}$.

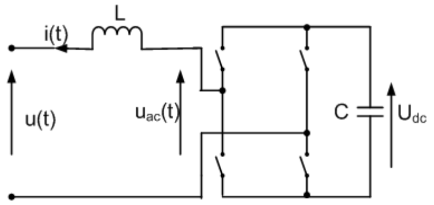

Fig. 9. General structure of the inverter

We assume that capacitor is charged and its voltage is maintained on a proper level higher than the peak value of the voltage $u_{ac}(t)$ at the AC inverter side. It is seen in Fig. 9, that voltage $u_{ac}(t)$ is equal to the sum of the grid voltage $u(t)$ and the voltage drop on the inductance *L*. This voltage drop follows from the compensator current obtained in the iterative process. As the result the complex value of the voltage fundamental harmonic at the AC converter side can be obtained as

(9) $$U_{ac} = U + Z_L I$$

where $Z_L = R_L + jX_L$, $X_L = \omega L$ and $\omega$ fundamental frequency. As $X_L \gg R_L$, we take $R_L = 0$ and obtain

(10) $$U_{ack} = U_{k-1}(1 - \omega L b_k)$$

From complex amplitude $U_{ack}$ the time function signal for DC/AC inverter control within *k*-th operation step can be obtained. The formula (10) contains compensator series inductance $L$. The modified algorithm presented in (11)-(18) enables one to omit inductance $L$.

Equation (10) defining time varying signal $u_{ac}(t)$ for PWM inverter control contains parameter $b_k$ computed in (7),(8). We change this parameter for new real parameter defined as

(11) $$w = 1 - bX$$

It was written below equation (2), that for $bX < 0$ the grid voltage is increased. It means that for new parameter *w* it can be stated that the compensator increases the grid voltage for $w > 1$ and deceases for $w < 1$.

Using (11), equation (10) is substituted for the following relation

(12) $$U_{ac} = wU$$

New algorithm can be formulated as follows.

*Step 0.* $w_0 = 1$.

*Step 1.* Optionally chosen $w_1 \neq 1$, such that

(13) $$|I_1| = \frac{w_1 - 1}{X_L}|U_1| \le I_{max}$$

*Step 2.*

(14) $$w_2 = 1 - \frac{|U_0| - U_d}{|U_0| - |U_1|}(1 - w_1)$$

*Step k.*

(15) $$w_k = 1 - \frac{|U_0| - U_d}{|U_0| - |U_{k-1}|}(1 - w_{k-1})$$

The set $w_k$ for $k = 0,1,2,...,K$ determines the set of voltage phasors

(16) $$U_{ack} = w_k U_{k-1}$$

Obtained parameters $w_k$ are real, so voltages $U_{ac}$ and $U$ are in phase. Power angle $\delta$ should be introduced in order to maintain the average capacitor voltage. Power angle $\delta$ can be substituted for the properly chosen of the control signal time delay.

*Numerical example 2.*

The PLECS block model of the inverter is shown in Fig. 10. The Sub1 seen in the upper part of Fig.10 contains the model of the three-phase three-wire grid shown in Fig.12.

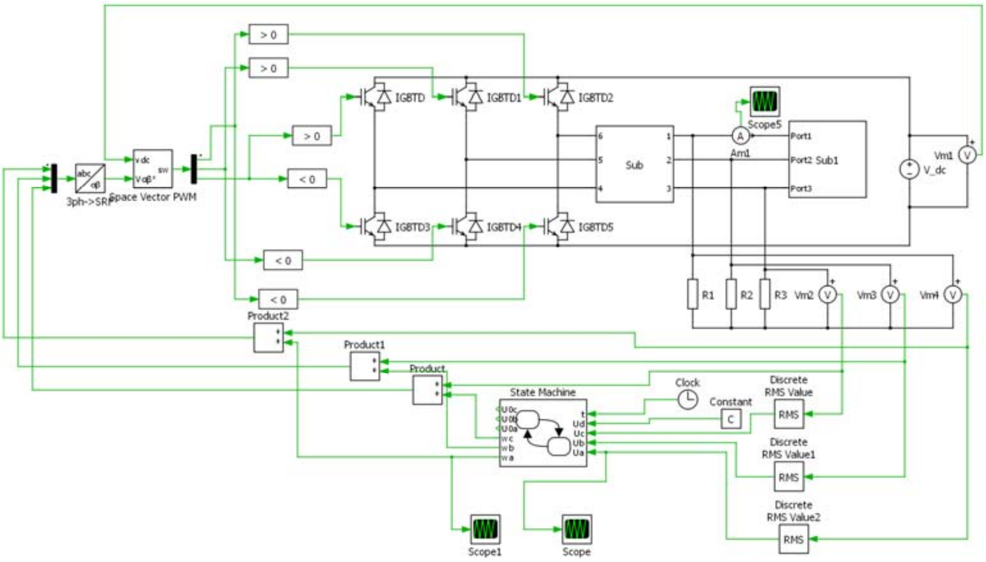


Fig.10. PLECS model

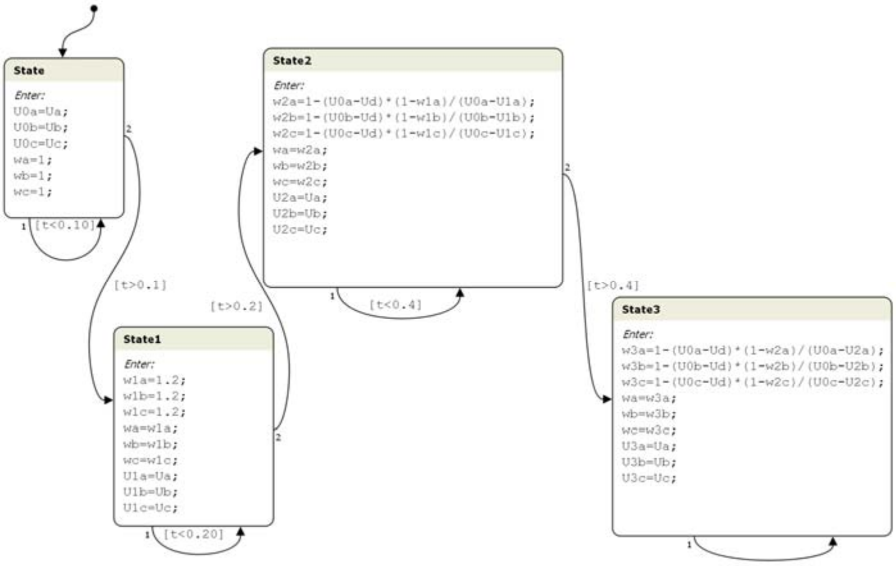


Fig.11. State machine

The amplitudes of voltage sources for positive, and negative symmetrical components were chosen as follows $\left|U_m^+\right| = 305V$ , $\left|U_m^-\right| = 5V$ . Demanded RMS grid voltage is chosen as $U_d = 230V$ . Subsystem Sub contains LCL filter connecting the inverter with the grid reduces current ripples.

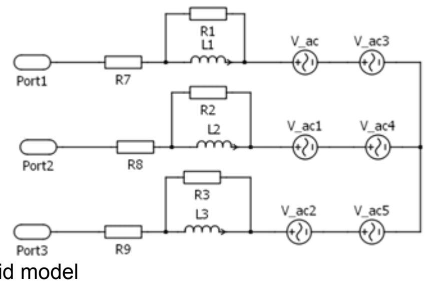


Fig. 12. Grid model

Fig.13 shows the sequence of states for the algorithm described in equations (14),(15) . Simulation results are presented in Figs. 13,14,15. Control parameter $w_a$ shown in Fig. 13 is computed by state machine. This signal is multiplied by the grid voltage waveform. The obtained signal is further proceeded in block Space Vector PWM seen in Fig. 10. The resulting compensator current is shown in Fig. 14. As the final result the RMS grid voltage can observed in Fig. 15.

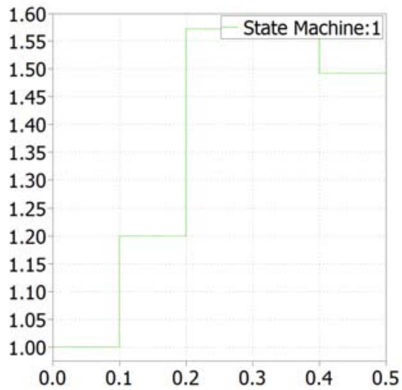


Fig. 13. Control parameter $w_a$

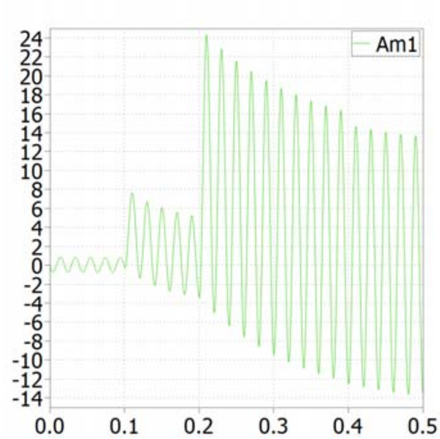


Fig.14. Compensator current $i_a$ , amper

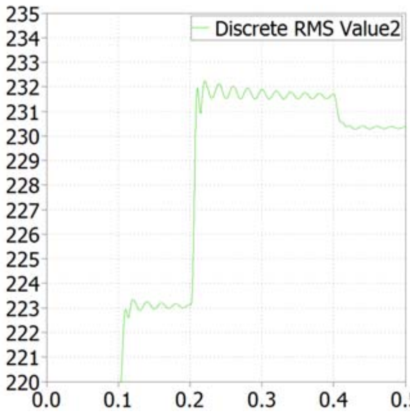


Fig. 15. RMS grid voltage $|U_a|$ , volt

Similar three waveforms can be observed for the remaining two phases. Grid RMS voltages for $a,b,c$ phases before and after compensation are shown in Table 2.

Table 1. Voltages before and after compensation

| | $\|U_a\|$ | $\|U_b\|$ | $\|U_c\|$ |
|---|---|---|---|
| Before comp. | 219.4 | 214.2 | 214.2 |
| After comp. | 230.4 | 230.5 | 228.5 |

**Conclusions**

The numerical results presented in the paper were obtained with assumption that grid AC voltage sources seen in Figs.4 and 12 are stiff. It means that these sources manage to deliver the demanded active power. The demanded active power increases while the grid voltage is increased due to the compensator action. The shunt compensator operates as a current source controlled by a grid voltage waveform. This current is obtained from DC/AC voltage inverter with the additional series inductor as shown in Fig. 9. The DC side of the inverter is connected to capacitor *C* with voltage $U_{dc}$ . We assume that capacitor is charged and its voltage is maintained on a proper level, higher than the peak value of the voltage $u_{ac}(t)$ at the AC inverter side .

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