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Highly Nonlinear Systems Estimation using Extended and Unscented Kalman Filters

***Abstract**. The main idea of this study is to evaluate the estimation performance of extended and unscented Kalman filters (EKF and UKF). So, these latter are introduced to estimate the dynamic states of a similar model operating with identical covariance matrices in the same situation. The mean square error (MSE) criterion is used to quantify the estimation error between the actual and the estimated values. The simulation results obtained with Matlab/ Simulink software confirm the superiority and efficiency of UKF over EKF, especially when the system is highly non-linear under process and measurement noises, such is the case of the inverted double pendulum mounted on a cart (DIPC).*

Streszczenie.**). Główną ideą tego badania jest ocena wydajności estymacji rozszerzonych filtrów Kalmana (EKF i UKF). Te ostatnie zostały wprowadzone w celu oszacowania stanów dynamicznych podobnego modelu działającego z identycznymi macierzami kowariancji. Kryterium błędu średniokwadratowego (MSE) służy do ilościowego określenia błędu oszacowania między wartościami rzeczywistymi i szacunkowymi. Wyniki symulacji uzyskane za pomocą oprogramowania Matlab i Simulink potwierdzają wyższość i wydajność UKF nad EKF, zwłaszcza gdy system jest wysoce nieliniowy. (**Nieliniowe oszacowanie systemów przy użyciu filtrów Kalmana)

Keywords: Nonlinear Systems, Double Inverted Pendulum, Extended and Unscented Kalman filters, State Estimation.
Słowa kluczowe: filtr Kalmana, system nieliniowy, dynamika.

Introduction

In many scientific and teaching fields, the automatic systems often use special case studies, which are representative of large classes of applications [1, 2]. Furthermore, with experience, the knowledge of these cases has been refined and they now provide an ideal basis for a valid comparison of different approaches. The inverted pendulum is one of the most important systems of control engineering and is the subject of much research [3, 4]; because such systems can be used to accurately describe many real-life problems, such as balancing artificial limbs, launching rockets, trajectory control, etc. In fact, a double inverted pendulum on a cart (DIPC) system is an extension of the only inverted pendulum mounted on a cart with a high level of nonlinearities, which is widely used for testing new control methods [5, 6].

The quality of a sensorless drive is mostly determined by the quality of its state observer. The Kalman filter based estimation technique was originally developed for linear systems in the presence of noise [7], but later the extended Kalman filter (EKF) which applies the Kalman filter to nonlinear system by linearizing all nonlinear models, has become a most widely used estimation algorithm for estimating the state variables of nonlinear systems [5, 6]. However, in practical applications, the EKF is difficult to implement, difficult to tune, and the reliability is limited [8-10] due to its many drawbacks such as the need for a Jacobi matrix computation which provides a high computational cost. Though, this can lead to significant errors for highly nonlinear systems due to the propagation of uncertainty in the nonlinear system. As the sampling time increases, the precision of the estimate decreases considerably [10, 11].

Whereas, the performance of the EKF is poor in some situations, its performance is acceptable if the system nonlinearity is not severe. Since a DPIC is a highly non-linear system [1, 3], so to overcome these limitations, the Unscented Kalman Filter (UKF) as a new variant of the Kalman filter was proposed by Julier and Uhlmann in 1996 [6]. Their idea is to generate several sampling points (Sigma points) around the current state estimate based on its covariance. Using an unscented transformation (UT), the UKF can estimate an accurate state values without the linearization process and it is not necessary to calculate Jacobian. In fact, the UKF algorithm has superior implementation properties to the EKE [11, 12].

The authors in reference [4] found the UKF easier to approximate the Gaussian distribution associated with each state vector variable, rather than approximated nonlinear function transformation. Use unscented Kalman filter (UKF) based estimation technique a set of points, called sigma points, are generated which track the true mean and covariance of the process [4, 13].

Indeed, the main of this study is to estimate all the state variables of the DPIC and to compare the dynamic state estimation in order to evaluate the performances of two different estimators, namely: the extended Kalman filter and the unscented Kalman filter, especially when they are particularly applied for a highly non-linear system with the presence of state and measurement noises. UKF has been shown to be able to predict the state of such system and is more accurately than EKF.

This paper is organized as follows. First, in section II, we provide the theoretical background of Extended and Unscented Kalman Filters algorithm. Next, the continuous and discrete DPIC models are presented in section III. Then, estimated results for different states using EKF and UKF are shown in section IV. Finally, in section V some conclusions to this paper are discussed.

The Kalman Filter (KF)

The Kalman filter introduced by Rudolf Emil Kalman in 1960 [5], is one of the most interesting mathematical developments in the theory of linear estimation. It is a state observer in a stochastic environment, when the variances of the noises are known; it is a linear estimator minimizing the variance of the estimation error. In this section we will focus on the two variants of the Kalman filter, namely the EKF and the UKF.

Extended Kalman Filter (EKF)

The extended Kalman filter is an optimal recursive estimation algorithm for nonlinear systems that are disturbed by random noise [13]. In this paper we look for finding the best linear estimate of the state vector x_k of the DPIC which evolves according to the following stochastic discrete-time nonlinear dynamic:

(1) $$\begin{cases} x_{k+1} = f(x_k, u_k, w_k) \\ y_k = h(x_k, v_k) \end{cases}$$

where *f(.)* is the state evolution function representing the machine dynamics, *h(.)* represents the relationship between the state vector and the observation y_k, u_k the machine input at time k and w_k and v_k are the process and measurement white Gaussian noise vectors with zero mean and with associated covariance matrices $Q = E\left[w_k w_k^T\right]$ and $R = E\left[v_k v_k^T\right]$, respectively.

To allow application of Kalman filter to the nonlinearity (1), this later must be linearized by using first order Taylor approximation near a desired reference point $(\hat{x}_k, \hat{w}_k = 0, \hat{v}_k = 0)$, which gives us the following approximated linear model [14]:

(2) $$\begin{cases} x_{k+1} \approx f(\hat{x}_k, u_k, 0) + F_k(x_k - \hat{x}_k) + W_k(w_k - 0) \\ y_k \approx h(\hat{x}_k, 0) + H_k(x_k - \hat{x}_k) + V_k(v_k - 0) \end{cases}$$

where F_k, W_k, H_k and V_k are the Jacobean matrices defined by :

(3) $$\begin{cases} F_k = \left.\dfrac{\partial f(x,0)}{\partial x}\right|_{x=\hat{x}_k}, \; W_k = \left.\dfrac{\partial f(\hat{x}_k, w)}{\partial w}\right|_{w=0}, \\ H_k = \left.\dfrac{\partial h(x,0)}{\partial x}\right|_{x=\hat{x}_k}, \; V_k = \left.\dfrac{\partial h(\hat{x}_k, v)}{\partial v}\right|_{v=0} \end{cases}$$

Thus, the EKF algorithm can be given by the following recursive equations:

(4) $$\begin{aligned} \hat{x}_{k+1/k} &= f(\hat{x}_{k/k}, u_k, 0) \\ P_{k+1/k} &= F_k P_{k/k} F_k^T + W_k Q W_k^T \\ K_k &= P_{k+1/k} H_k^T (H_k P_{k+1/k} H_k^T + V_k R V_k^T)^{-1} \\ \hat{x}_{k+1/k+1} &= \hat{x}_{k+1/k} + K_k(y_k - h(\hat{x}_{k+1/k}, 0)) \\ P_{k+1/k+1} &= P_{k+1/k} - K_k H_k P_{k+1/k} \end{aligned}$$

Where $\hat{x}_{k+1/k}$ is the a priori state prediction vector, $\hat{x}_{k+1/k+1}$ is the a posteriori state prediction vector, $P_{k+1/k}$ is the a priori prediction error covariance matrix, $P_{k+1/k+1}$ is the a posteriori prediction error covariance matrix and K_k is the Kalman gain.

Unscented Kalman filter (UKF)

UKF is an enhancement over the EKF algorithm. In reference [10] S. J. Julier and J. K. Uhlmann proposed completely new solution of estimation theory problem based on Unscented Transformation. These authors found that it is possible to simplify the UKF algorithm by eliminating the need for calculating Jacobians at each step and working point. This filter, like its classical form is based on two cycles performed procedures: prediction and correction. For the general formulation of the UKF, the n-dimensional state with mean and covariance are approximated by *(2n+1)* weighted sigma points. With respect to the nonlinear model describing by Eq. (1), the estimation of states using UKF algorithm [1-4, 15] can be done as follows:

Step 1: Initialization $\hat{x}_{0/0}$, $P_{0/0}$, R *and* Q

Step 2: Sigma points Calculation

Given the state vector and the covariance matrix estimates $\hat{x}_{k/k}$ *and* $P_{k/k}$ at time k respectively, we calculate a set of $2n+1$ sigma points x^i (with corresponding weights W^i) can be calculated as follows:

(5) $$\begin{cases} x^0_{k/k} = \hat{x}_{k/k} & \text{i=0} \\ W^0 = \lambda/(n+\lambda) \\ x^i_{k/k} = \hat{x}_{k/k} + (\sqrt{(n+\lambda)P_{k/k}})_i, & \text{i=1...2n} \\ W^i = \lambda/2(n+\lambda) \\ x^{i+n}_{k/k} = \hat{x}_{k/k} - (\sqrt{(n+\lambda)P_{k/k}})_{i-n}, & \text{i=n+1...2n} \\ W^{i+n} = 1/2(n+\lambda) \end{cases}$$

where n is the state vector dimension; λ is a positive scaling parameter.

Step 3: State Prediction (Time update)

Using the sigma points calculated in the first step, the state $\hat{x}_{k+1/k}$ and the covariance matrix $P_{k+1/k}$ are predicted as:

(6) $$\begin{aligned} x^i_{k+1/k} &= f(x^i_{k/k}, u_k) \\ \hat{x}_{k+1/k} &= \sum_{i=0}^{2n} W^i x^i_{k+1/k} \\ P_{k+1/k} &= \sum_{i=0}^{2n} W^i (x^i_{k+1/k} - \hat{x}_{k+1/k})(x^i_{k+1/k} - \hat{x}_{k+1/k})^T + Q \end{aligned}$$

These updated points are evaluated one by one using measurement function defined in Eq. (1) as

(7) $$\begin{aligned} y^i_{k+1/k} &= h(x^i_{k+1/k}, u_k) \\ \hat{y}_{k+1/k} &= \sum_{i=0}^{2n} W^i y^i_{k+1/k} \end{aligned}$$

Step 4: Correction (Measurement update equations)

Cross-covariance of the state and measurements and measurement covariance matrix can be obtained as:

(8) $$\begin{aligned} P^{xy}_{k+1/k} &= \sum_{i=0}^{2n} W^i \left[(x^i_{k+1/k} - \hat{x}_{k+1/k})(y^i_{k+1/k} - \hat{y}_{k+1/k})^T\right] + R \\ P^{yy}_{k+1/k} &= \sum_{i=0}^{2n} W^i \left[(y^i_{k+1/k} - \hat{y}_{k+1/k})(y^i_{k+1/k} - \hat{y}_{k+1/k})^T\right] \end{aligned}$$

After the Kalman gain K_{k+1} and the state $\hat{x}_{k+1/k+1}$, covariance matrix $P_{k+1/k+1}$, can be calculated as follows:

(9) $$\begin{cases} K_{k+1} = P^{xy}_{k+1/k}(P^{yy}_{k+1/k})^{-1} \\ \hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + K_{k+1}(y_{k+1} - \hat{y}_{k+1/k}) \\ P_{k+1/k+1} = P_{k+1/k} - K_{k+1} P^{yy}_{k+1/k} K^T_{k+1} \end{cases}$$

where y_{k+1} is the set of measurement at time t_k

In order to demonstrate the effectiveness of UKF algorithm compared to EKF algorithm, in the next section we tried to estimate all the state variable of a DIPC using these two Kalman filters variants.

Double inverted Pendulum mathematical model

Consider a double pendulum which is mounted to a cart, as shown in Fig. 1. The system consists to a cart which can move right or left on the rail freely, a lower pendulum hanged on the cart, and an upper pendulum, which is linked with the other end of the lower pendulum [15-17].

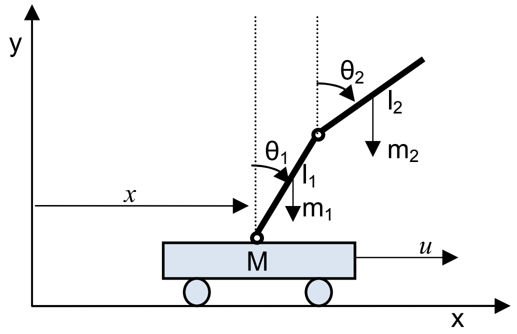

Fig.1. Schematic diagram of double inverted pendulum system.

It is given that no friction exists in the pendulum system then the dynamic equations of the double inverted pendulum system are obtained from Lagrange's equation of motion as:

$$(10)\quad \begin{cases} (M+m_1+m_2)\ddot{x}-(m_1+2m_2)l_1\ddot{\theta}_1 \cos\theta_1 - m_2 l_2 \ddot{\theta}_2 \cos\theta_2 = \\ u+(m_1+2m_2)l_1\dot{\theta}_1^2 \sin\theta_1 + m_2 l_2 \dot{\theta}_2^2 \sin\theta_2 \\ -(m_1+2m_2)l_1\ddot{x}\cos\theta_1 + 4(\frac{m_1}{3}+m_2)l_1^2\ddot{\theta}_1 + 2m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_2-\theta_1) = \\ (m_1+2m_2)g\,l_1 \sin\theta_1 + 2m_2 l_1 l_2 \dot{\theta}_2^2 \sin(\theta_2-\theta_1) \\ -m_2\ddot{x}l_2\cos\theta_2 + 2m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_2-\theta_1) + \frac{4}{3}m_2 l_2^2 \ddot{\theta}_2 = \\ m_2 g l_2 \sin\theta_2 - 2m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_2-\theta_1) \end{cases}$$

These dynamic equations (10) can be put in the following compact form:

$$(11)\quad \begin{cases} a_{11}\ddot{x}+a_{12}\ddot{\theta}_1+a_{13}\ddot{\theta}_2=b_1 \\ a_{21}\ddot{x}+a_{22}\ddot{\theta}_1+a_{23}\ddot{\theta}_2=b_2 \\ a_{31}\ddot{x}+a_{32}\ddot{\theta}_1+a_{33}\ddot{\theta}_2=b_3 \end{cases}$$

$$(12)\quad \begin{cases} a_{11}=M+m_1+m_2 \\ a_{12}=a_{21}=(m_1+2m_2)l_1\cos\theta_1 \\ a_{13}=a_{31}=m_2 l_2 \cos\theta_2 \\ a_{22}=4(m_1/3+m_2)l_1^2 \\ a_{23}=a_{32}=2m_2 l_1 l_2 \cos(\theta_2-\theta_1) \\ a_{33}=4m_2 l_2^2/3 \end{cases}$$

and

$$(13)\quad \begin{cases} b_1=u+(m_1 l_1+2m_2 l_1)\dot{\theta}_1^2\sin\theta_1+m_2 l_2\dot{\theta}_2^2\sin\theta_2 \\ b_2=(m_1+2m_2)\,g\,l_1\sin\theta_1-2m_2 l_1 l_2\dot{\theta}_2^2\sin(\theta_2-\theta_1) \\ b_3=m_2 l_2 g\sin\theta_2+2m_2 l_1 l_2\dot{\theta}_1^2\sin(\theta_2-\theta_1) \end{cases}$$

The position x and velocity $\dot{x}$ of the cart, the angle θ_1 and angular velocity $\dot{\theta}_1$ of the lower pendulum and the angle θ_2 and angular velocity $\dot{\theta}_2$ of the upper pendulum are the state vector, then we can write:

$$(14)\quad \begin{aligned} X &= \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}^T \\ &= \begin{bmatrix} x & \dot{x} & \theta_1 & \dot{\theta}_1 & \theta_2 & \dot{\theta}_2 \end{bmatrix}^T \end{aligned}$$

Then the nonlinear dynamic model of the DIPC can be put in the continuous general nonlinear representation as follows:

$$(15)\quad f(X(t),u(t)) = \begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_5(t) \\ \dot{x}_6(t) \end{pmatrix} = \begin{pmatrix} x_2(t) \\ \ddot{x}(t) = -\frac{a_{12}}{a_{11}}\ddot{\theta}_1(t) - \frac{a_{13}}{a_{11}}\ddot{\theta}_2(t) + \frac{b_1}{a_{11}} \\ x_4(t) \\ \ddot{\theta}_1(t) = \frac{a_{13}a_{21}-a_{11}a_{23}}{a_{11}a_{22}-a_{12}a_{21}}\ddot{\theta}_2(t) - \frac{a_{21}}{a_{11}a_{22}-a_{12}a_{21}}b_1 + \frac{a_{11}}{a_{11}a_{22}-a_{12}a_{21}}b_2 \\ x_6(t) \\ \ddot{\theta}_2(t) = \frac{\alpha}{\gamma}b_1 - \frac{\beta}{\gamma}b_2 + \frac{a_{11}(a_{11}a_{22}-a_{12}a_{21})}{\gamma}b_3 \end{pmatrix}$$

This continuous non-linear model of DIPC (Eq. 15) must be discredited using Euler forward method, which can be written in non-linear discrete stochastic form as follows:

$$(16)\quad f(X_k,u_k,w_k) = \begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \\ x_5(k+1) \\ x_6(k+1) \end{pmatrix} = \begin{pmatrix} x_2(k) \\ -\frac{a_{12}(k)}{a_{11}}\ddot{\theta}_1(k) - \frac{a_{13}(k)}{a_{11}}\ddot{\theta}_2(k) + \frac{b_1(k)}{a_{11}} \\ x_4(k) \\ \frac{a_{13}(k)a_{21}(k)-a_{11}a_{23}(k)}{a_{11}a_{22}-a_{12}(k)a_{21}(k)}\ddot{\theta}_2(k) - \frac{a_{21}(k)b_1(k)}{a_{11}a_{22}-a_{12}(k)a_{21}(k)} + \frac{a_{11}b_2(k)}{a_{11}a_{22}-a_{12}(k)a_{21}(k)} \\ x_6(k) \\ \frac{\alpha(k)}{\gamma(k)}b_1(k) - \frac{\beta(k)}{\gamma(k)}b_2(k) + \frac{a_{11}(a_{11}a_{22}-a_{12}(k)a_{21}(k))}{\gamma(k)}b_3(k) \end{pmatrix} T_s + \begin{pmatrix} x_1(k)+w_1 \\ x_2(k)+w_2 \\ x_3(k)+w_3 \\ x_4(k)+w_4 \\ x_5(k)+w_5 \\ x_6(k)+w_6 \end{pmatrix}$$

and

$$(17)\quad h(X_k,v_k) = \begin{bmatrix} x_1(k)+v_1 & x_3(\mathrm{k})+v_2 & x_5(k)+v_3 \end{bmatrix}^T$$

with

$$(18)\quad \begin{cases} b_1(k)=u(k)+(m_1 l_1+2m_2 l_1)x_4^2(k)\sin x_3(k)+ \\ m_2 l_2 x_6^2(k)\sin x_5(k) \\ b_2(k)=(m_1+2m_2)\,g\,l_1\sin x_3(k)- \\ 2m_2 l_1 l_2 x_6^2(k)\sin(x_5(k)-x_3(k)) \\ b_3(k)=m_2 l_2 g\sin x_5(k)+ \\ 2m_2 l_1 l_2 x_4^2(k)\sin(x_5(k)-x_3(k)) \end{cases}$$

$$(19)\quad \begin{cases} a_{12}(k) = a_{21}(k) = (m_1 + 2m_2) l_1 \cos x_3(k) \\ a_{13}(k) = a_{31}(k) = m_2 l_2 \cos x_5(k) \\ a_{23}(k) = a_{32}(k) = 2 m_2 l_1 l_2 \cos(x_5(k) - x_3(k)) \end{cases}$$

Where the explicit expressions of $\alpha(k)$, $\beta(k)$ and $\gamma(k)$, are given as follows:

$$(20)\quad \begin{cases} \alpha(k) = (a_{31}(k) a_{12}(k) a_{21}(k) - a_{31}(k) a_{11} a_{22} - \\ a_{12} a_{21} + a_{11} a_{23}(k) a_{21}(k)) b_1(k) \\ \beta(k) = (a_{11} a_{31}(k) a_{12}(k) - a_{32}(k)(a_{11})^2) b_2(k) \\ \gamma(k) = (a_{11} a_{31}(k) - a_{31}(k) a_{13}(k))(a_{11} a_{22} - a_{12} a_{21}) + \\ (a_{21}(k) a_{13}(k) - a_{11} a_{23}(k))(a_{32}(k) a_{11} - a_{12}(k) a_{31}(k)) \end{cases}$$

where T_s is the sampling time.

Simulation and results

This section shows the results of estimated states using EKF and UKF as discussed in section II. The parameters of the double inverted pendulum on cart used in the simulation are given by: $M=1.5Kg$, $m_1=0.5Kg$, $m_2=0.75Kg$, $L_1=0.5m$, $L_2=0.75m$ and $g=9.81m/s^2$. In addition, the simulation study was tested under Matlab/Simulink software environment with sampling time $T_s = 0.001$s.

Estimated and actual states (cart position, upper and lower pendulum angle and angular velocity) of DIPC are shown in Figures 2 to 7 which verify that estimated states are almost identical to the actual values and also show the similar behaviour.

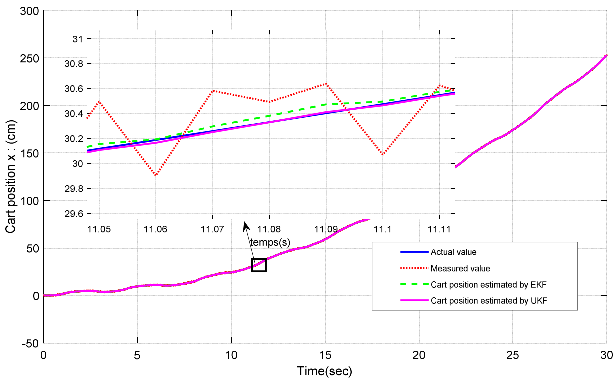

Fig. 2. Actual, measured and estimated values of cart position x

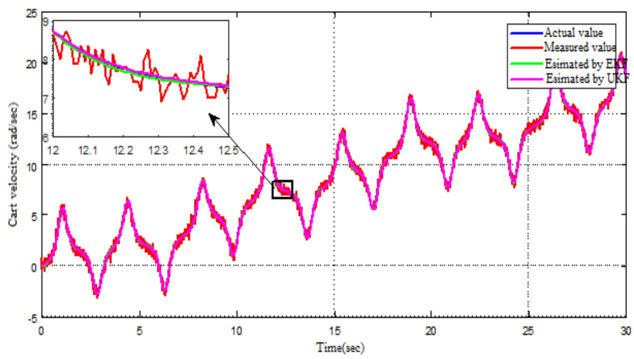

Fig. 3. Actual, measured and estimated values of cart velocity

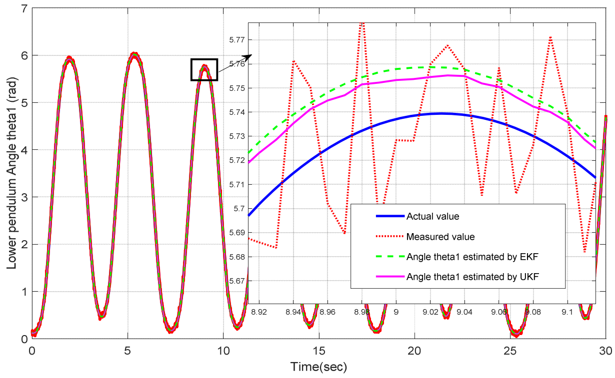

Fig. 4. Actual, measured and estimated values of lower pendulum angle θ_1

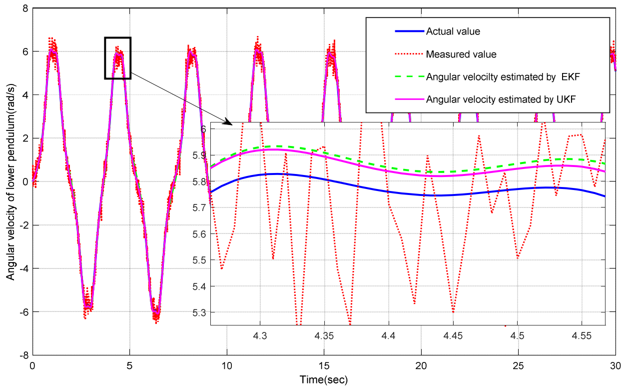

Fig. 5. Actual, measured and estimated angular velocity of lower pendulum

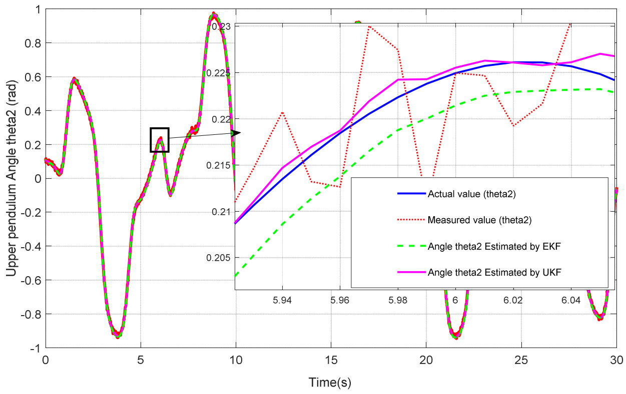

Fig. 6. Actual, measured and estimated upper pendulum angle θ_2

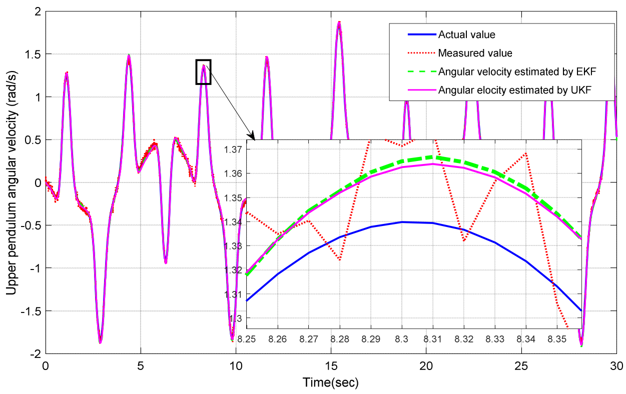

Fig. 7. Actual, measured and estimated values of upper pendulum angular velocity

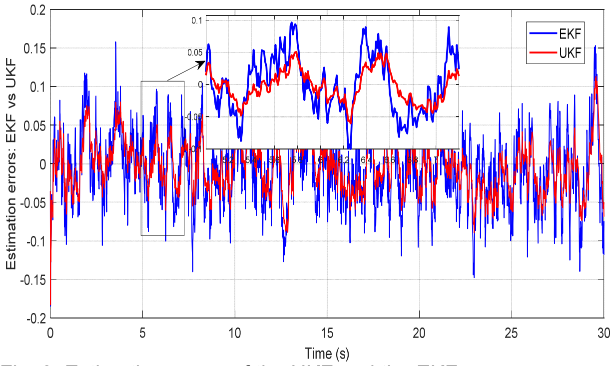

Fig. 8. Estimation errors of the UKF and the EKF

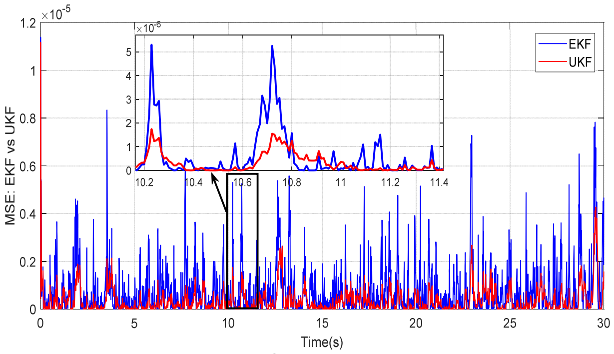

Fig. 9. Mean-square errors of the UKF and the EKF

The mean square error (MSE) criterion is used to quantify the estimation error. The simulation results shown in Fig.2, Fig.4 and Fig.6 demonstrate that the UKF outperforms the EKF when estimating the lower angle, the upper angle and the cart position. On the other hand, these two filters have almost the same performance when estimating their respective time derivatives as shown in Fig.3, Fig.5 and Fig.7.

The results in Fig. 8 and Fig. 9 above show that the unscented Kalman filter is more accurate than the extended Kalman filter. The accuracy is evaluated by taking the MSE between the estimated state value and the actual value of each variable state in presence of system process and measurement noises.

Conclusion

In this paper, the dynamic equations of the double inverted pendulum system are obtained from Lagrange's equation of motion. Using the developed discrete mathematical model of DIPC, all their dynamic states are estimated by two varieties of Kalman filters, extended and unscented Kalman filters. The UKF consistently achieves a better level of accuracy than the EKF at a comparable level of complexity. As illustrated in previous studies, the EKF and UKF are both sufficient tools for non linear systems. However, as verified by the simulation results, the UKF algorithm is very promising method compared to EKF. The estimation error and the MSE values illustrate that UKF has slightly better results than the EKF. The results obtained by simulation shows how UKF can contribute to improving the performance of the state estimation better than the EKF.

State estimation with EKF and UKF methods can be used for monitoring and controlling the dynamic state variables of various strongly nonlinear systems.

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