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Reliability Evaluation for Multi-State Repairable Systems with Hybridization the Markov Stochastic Process and the Universal Generating Function

***Abstract**. Multistate reliability models are generally complicated and the state space has a large number of states. Evaluating the performance distribution of complex series parallel to the repairable multi-state system with dependent linear components. They are much more complex and present major difficulties in defining the system and the performance of the MSS multistate reliability assessment system. A new approach is introduced to extend the classical theory of reliability based on the binary hypothesis to the repairable multi-state system (MSS). Generally, some of the stochastic processes of traditional methods did not provide an assessment of the reliability of the MSS system due to the enormous states of the system. This article is based on the hybridization of the Markov stochastic process and the universal generation function technology (UGF) which deals with the most sophisticated and realistic models ranging from perfect operation to complete failure in which components and systems can take many states. We consider the case where the performance and probability distributions of certain components depend on the linear of another component or group of components.*

***Streszczenie.** Zaprezentowano nowa metodę rozszerzająca klasyczną teorię niezawodności bazującą na hipotezie naprawialnych systemów wielostanowych MSS. Artykuł bazuje na hybrydyzacji stochastycznego procesu Markova i technologii uniwersalnej funkcji UGF. Rozważono przypadek kiedy gęstość prawdopodobieństwa pewnych składowych zależy liniowo od innych składowych. (**Określenie niezawodności w wielostanowych naprawialnym systemie z hybrydyzacją procesu Markova i użyciem funkcji UGF**)*

Keywords: Stochastic process, System repairable,Multi-state (MSS), Universal generating moment function (UMGF), linear dependence.
Słowa kluczowe:niezawodność, proces Markova, uniwersalna funkcja UGF.

Introduction

One of the most important problems during the design phase in many complex industrial applications system is the reliability assessment, reliability engineers are called upon to evaluate the reliability of the developing system.

Traditionally the classical reliability theory is based on the binary assumption that the system is either working perfectly or completely failed. However, in many real life situations we are actually able to distinguish among various levels of performance for power systems. Less effort has been devoted to develop methods for analyzing reliability or (availability) of multi-state systems. In this case, it is important to develop MSS reliability theory. Most of research works in MSS reliability analysis extend the binary case to the multi-state case. In order to assess a complex reparable MSS system reliability [1,2] a stochastic process and universal generating function methods are suggested. Generally, the methods of MSS reliability assessment are based on four different approaches: (i) The structure function approach,. (ii) The Monte-Carlo simulation technique (iii) the stochastic process (Markov) approach and (iv) The universal generating function (UGF) approach. According to the Markov method, the model construction of MSS state- space diagram with all transitions between states is very difficult to build. The main disadvantage to use this model in MSS Reliability evaluation is the dimension damnation of the state- space. The problems reencountered are: (i) identifying all states and transitions correctly is very difficult assignment, (ii) solving model with hundreds of states can be hard task and (iii) Applied only to relatively small MSS. There are a many real industrial applications in with a system should be considered to be a MSS as in manufacturing systems, telecommunications and power. For the MSS, for each component a several outage levels performances corresponding to the degradations. Therefore, the reliability analysis of MSS is much more complex than the case of binary state system.

The calculation of many MSS optimization problems where reliability measures have to be evaluated for a large number of states, the UGF technique is widely used. This technique reduces the space diagram and allows you to find the entire distribution of MSS performance of its components. The generator function, which is analogous to the Laplace transform, allows us, using a simple procedure, to obtain different probabilistic characteristics: moment of distribution, mathematical variance, etc. The UGF [2,3] is particularly effective in solving combinatorial problems. Allows us to write computational algorithms to solve combinatorial problems which include various compositions which cannot be reduced. The UGF allows us to build calculation algorithms for many reliability analysis problems.

In reality, the nature of the interactions between the components of the system and the physical nature of the components differ. So far, many studies have dealt only with the mutually statically independent between the components or the group of components. Some studies consider the opposite case.

This article suggests extending the combination of UGF to the case where the transition rate and the performance distribution of the surviving components are influenced by the failing components. Our suggested approach is valid in the case of a linear dependence between the components [4, 7]. In this work, a UGF technique extended to the case of dependent components will be described with an illustrative example

Markov model for multistate component

The methods of the stochastic process (Markov or Semi-Markov) are widely used for the reliability analysis of SMS and are more universal [8, 9]. In fact, these approaches have been used and successfully implemented for assessing the reliability of multi-state power systems and certain types of industrial application systems even before MSS is theoretically defined. Stochastic process methods can only be applied to a relatively small MSS due to exhaustive state systems (the number of states increases considerably with the increase in the number of system components) [10, 11].

If all failures and repair times are distributed exponentially, then the stochastic process of distributed failure performance can be represented by the Markov model [12, 13]. We assume that the component has k different states as shown in Figure 1. For Markov process, each transition from the state *s* to any state m ($s,m=1,...,k$) has its own transition intensity designed as a_{sm} .

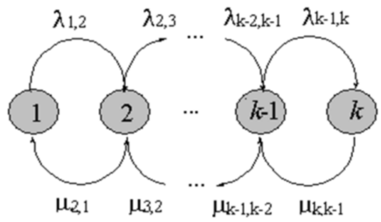

Fig.1. Markov Model Diagram of Repairable MSS Component

If $m \prec s$, then $a_{sm}=\lambda_{sm}$ define the transition failure rate for the failure that cause the component transition from the state s to state m with performance losses. If $m \succ s$, then $a_{sm}=\mu_{sm}$ define the corresponding transition repair rate from the state m to state *s* with best performance. For every component j $1\le j\le n$ we designate $p_{sm}(t)$ the probability that the Markov process which starts from the initial state k (k the best state) at instant $t=0$ will be at state m at instant t .The probabilities $p_{s,m}(t)$, $s,m=1,2,...,k$ can be found from the solution in the following homogenous Markov process as:

(1) $$\frac{\partial p_s(t)}{\partial t}=-p_s(t)\sum_{i=1 i\ne s}^{k}a_{si}+\sum_{i=1,i\ne s}^{k}p_i(t)a_{is}$$

(2) $$\sum_{i=1}^{n}p_i=1$$

In our case, all transitions are caused by failures and repairs of the component corresponding to the transition intensities a_{is} and are expressed by the component's failure and repair rates [14, 15,16].

Therefore, the corresponding system of differential equations (Kolmogorov's Equations) may be written as:

(3) $$\frac{\partial p_1(t)}{\partial t}=-\mu_{12}p_1(t)+\lambda_{21}p_2(t)$$

(4) $$\frac{\partial p_2(t)}{\partial t}=-\mu_{12}p_1(t)-(\lambda_{21}+\mu_{23})p_2(t)+\lambda_{32}p_3(t)$$

(5) $$\frac{\partial p_k(t)}{\partial t}=-\mu_{k-1,k}p_{k-1}(t)-\lambda_{k,k-1}p_k(t)$$

We assume the initial state is the best state k with a high performance. Therefore, solving the Kolmogorov equations systems equation (3) under the initial condition $p_k(0)=1$, $p_1(0)=p_1(0)=...=p_{k-1}(0)=0$, the states probabilities $p_s(t), s=1,...,k$ can be obtained.

Generic Model for components with dependency

The last few years have seen the appearance of a number of books presenting different methods of quantitative estimation reliability, where the components of production systems are always considered to be independent. Until now, little research supposes the dependence between the components or the group of components without supporting the distributions of rate of charges and transitions on the surviving components. In reality, a wide range of complex systems is characterized by different topologies, where different kinds of interactions exist between system components and different physical kinds of components. This is one of the main assumptions which is true in many technical systems [17-21].

We define the repartition function on surviving components by:*h(s)* ,*1<h(s)<s* where *s* is the number of surviving system component.

If $s=1$: *I* independency.

$s\ne 1$: *L* Linear Dependency.

Load repartition Transition rate repartition

Consider the general case where failures can lead to total failure or a reduction in component performance, in which case different levels of capacity degradation must be taken into account. We denote by *L (s, t)* the total instantaneous load of the homogeneous system and *l (s, t)* the repair of the charge of the individual components.

(6) $$l(s,t)=\frac{L(t)}{s}$$

s: Number of surviving component.

(7) $$l(n,0)=\frac{L(0)}{n}$$

n: Total number of system component.

Transition rate repartition

But before solving the system of equations must know how to calculate the failure rate during dependence must then use the dependence function *g (k).* We suppose the value of :

(8) $$\lambda^{+}=\frac{\lambda(t)}{g(k)}$$

Where *k*: is the number of surviving components.
Also for this failure rate is proportional to the component overload during failures.

If $k=1$: then it's independence.

$k\ne 1$: It's dependence.

We can write that:

(9) $$\lambda^{+}_{Ind}=\frac{\lambda(t)}{g(k)}$$

With *g(k)=1* And

(10) $$\lambda^{+}_{dep}=\frac{\lambda(t)}{g(k)}$$

With $g(k)\ne 1\ et\ 1\le g(k)\le k$

We can take $\frac{1}{g(k)}=\left(1+\frac{n-k}{k}\right)$

Dependency case depending on the rate of the overload on the components.

(11) $$\lambda^{+}_{dep}=\left(1+\frac{n-k}{k}\right)\lambda(t)=\frac{n}{k}\lambda(t)$$

A linear dependence

Combined UGF Technique to the case of components with linear dependency

In recent years, a number of books have appeared, presenting various methods for the quantitative estimation of systems made up of components with different operating levels (Reinschke and El-Neweihi). Generally reducible systems are envisaged.

The procedure used in this work is based on the universal z transform, which is a modern mathematical technique introduced in Ushakov [5, 6, 16,]. This method, which is practical for digital implementation, has proven to be very effective for combinatorial problems of large dimension. In the literature, the universal z transform is also called the u transform. The UGF of an independent discrete random variable is defined as a polynomial [22- 26]:

(12) $$u(z)=\sum_{j=1}^{k}P_j z^{g_j}$$

where the variable g has k possible values and P_j is the probability that g is equal to g_j. The probabilistic characteristics of the random variable g can be found using the function u(z). In particular, if the discrete random variable g is the MSS stationary output performance, the availability A is given by the probability $\mathrm{Proba}(g \geq W)$ which can be defined as follows [11, 18, 19, 20]:

(13) $$\mathrm{Proba}(\mathrm{g} \geq \mathrm{W}) = \Phi\left(u(z)z^{-\mathrm{W}}\right)$$

Where: Φ is a distributive operator :

(14) $$\Phi(Pz^{\sigma-\mathrm{W}}) = \begin{cases} P, \ if\ \sigma \geq \mathrm{W} \\ 0, \ if\ \sigma < \mathrm{W} \end{cases}$$

(15) $$\Phi\left(\sum_{j=1}^{k}P_j z^{g_j-\mathrm{W}}\right) = \sum_{j=1}^{k}\Phi\left(P_j z^{g_j-\mathrm{W}}\right)$$

It can be easily shown that equations (12)–(13) meet condition $\mathrm{Proba}(g \geq W) = \sum_{g_j \geq \mathrm{W}} P_j$. By using the operator Φ, the coefficients of polynomial $u(z)$ are summed for every term with $g_j \geq W$, and the probability that g is not less than some arbitrary value W is systematically obtained.Based on determined states probabilities for all components, the performance stochastic processes corresponding to the output performance for each system component j is $g_{j1}, g_{j2},\ldots,g_{jkj}$ correspond, $p_{j1}(t), p_{j2}(t),\ldots,p_{jkj}(t)$ The UGF for each individual component should be given as:

(16) $$u_j(t,z) = p_{j1}(t)z$$

(17) $$u_j(t,z) = p_{j1}(t)z^{g_{j1}} + p_{j2}(t)z^{g_{j2}} + .. + p_{jkj}(t)z^{g_{jkj}}$$

Reliability indices for entire mass

When the UGF of entire MSS has been obtained, the following reliability indices can evaluated easily.

The entire MSS availability $A(t)$ at instant $t > 0$ is evaluated as:

(18) $$A(t) = \delta\left(U(Z,t)\right) = \sum_{j=0}^{Msys} P_j^{sys}(t)\delta(g_j^{syt} > 0$$

For an arbitrary constant demand W, the MSS availability a $A(t,w)$ at instant $t > 0$ is:

(19) $$A(t,w) = \delta\left(U(Z,t),w\right) = \sum_{j=0}^{Msys} P_j^{sys}(t)\delta(g_j^{syt} \geq w$$

Illustrative example

The electrical flow is transmitted from point C to point E. The performance of the transformers is measured by (kV). Elements 1 and 2 are repairable and each has two possible states. A total failure state for both elements corresponds to an output performance of 0 kV and the operational state corresponds to a performance of 230 kV and 345 kV, respectively Fig 2, so that.

$$G_1 \in \{g_{11}, g_{12}\} = \{0,\ 230kV\} \quad G_2 \in \{g_{21}, g_{22}\} = \{0,\ 345kV\}$$

The failure rates and the repair rates for these two elements are:

$$\lambda_{2,1}^{(1)} = 7\ year^{-1}, \quad \mu_{1,2}^{(1)} = 100\ year^{-1} \quad for\ élément\ 1$$

$$\lambda_{2,1}^{(2)} = 10\ year^{-1}, \quad \mu_{1,2}^{(2)} = 80\ year^{-1} \quad for\ élément\ 2$$

Element 3 is a multi-state element with only minor failures and minor repairs. It can be in one of three states: a total failure state corresponding to a performance of 0, a partial failure state corresponding to a performance of 115 kV, and a fully operational state corresponding to a performance of 765 kV. So, $G_3(t) \in \{g_{31}, g_{32}, g_{33}\} = \{0, 115kv, 765kV\}$.The failure rates and repair rates for Element 3 are:

$$\lambda_{3,2}^{(3)} = 10\ year^{-1}, \quad \lambda_{2,1}^{(3)} = 7\ year^{-1},$$

$$\mu_{1,2}^{(3)} = 120\ year^{-1}, \quad \mu_{2,3}^{(3)} = 110\ year^{-1}$$

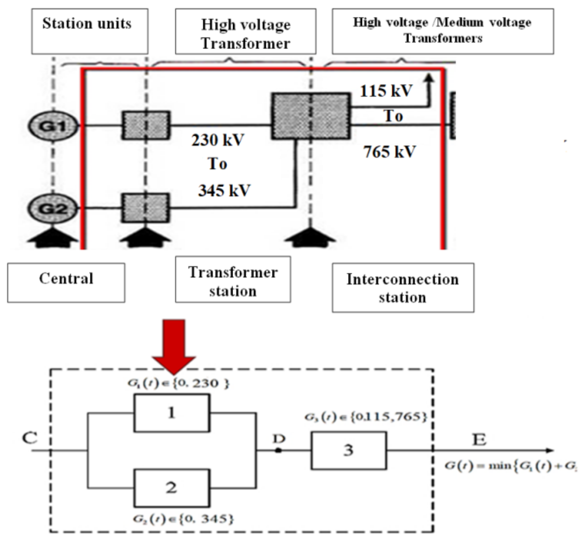

Fig.2. A multi-state system with 3 elements (Transformers).

The total flux between points C and D passing through the parallel transformers 1 and 2 is equal to the sum of the fluxes of each of these transformers. The flow from point D to E is limited by the output performance of element 3. However, this flow cannot be greater than the flow between points C and D. Therefore, the flow between points C and E (the performance system) is:

(20) $$G(t) = f\left(G_1(t), G_2(t), G_3(t)\right) = min\left\{G_1(t) + G_2(t), G_3(t)\right\}$$

According to the Markov method, we have separately constructed the following Kolmogorov equations for each subsystem: The subsystem consists of two parallel components.

(21) $$UGF1 = G_1(t)$$

(22) $$G_1(t) = p_{11}Z^{g11} + p_{12}Z^{g12} = p_{11}Z^{0} + p_{11}Z^{230}$$

(23) $$UGF2 = G_2(t)$$

(24) $$G_2(t) = p_{21}Z^{g21} + p_{22}Z^{g22} = p_{21}Z^{0} + p_{22}Z^{345}$$

(25) $$UGF3 = G_3(t)$$

(26) $$G_3(t) = p_{31}Z^{g31} + p_{32}Z^{g32} + p_{33}Z^{g33} = p_{11}Z^{0} + p_{11}Z^{150} + p_{32}Z^{765}$$

According to the configuration of components, the UGF of subsystem UGF12 consisted of Comp1 and Comp2 connected in parallel can be obtained using eq. (17).

(27) $$UGF12 = UGF1 \oplus UGF2 = G_{12}(t)$$

(28) $$G_{12}(t) = p_{11}Z^{0} + p_{11}Z^{230} \oplus p_{21}Z^{0} + p_{22}Z^{345}$$

Then, *UGFs* of the whole system is:

(29) $$UGFs = G_s(t) = UGF12 \oplus UGF3 = \sum_{1}^{5} Pi(t)Z^{gi}$$

Taking into account that the sum of probabilities of its states is equal to 1.

(30) $$\begin{cases} P_{11}1(t) + P_{12}(t) = 1 \\ P_{21}(t) + P_{22}(t) = 1 \\ P_{31}(t) + P_{32}(t) + P_{33}(t) = 1 \end{cases}$$

(31) $$G_s(t) = f\left(G_1(t), G_2(t), G_3(t)\right)$$

(32) $$G_s(t) = min\left\{G_1(t) + G_2(t), G_3(t)\right\}$$

We can find 2 * 2 * 3 = 12 differential equations if the simple stochastic Markov method has been realized. As a result, computing efforts will be consumed in number. However, the combined approach presented only needs to solve three differential equations of component:

two of order two and one of order three.

The further derivation of the state probabilities and the reliability indices of the *MSS* is based on the technique of the *UGF* universal generator function that can be implemented by simple mathematical calculation. Table 1 shows the possible states of the system

Table 1. The possible states of the system

Ns	G1 (0, 230)	G2 (0, 345)	G3 (0, 115, 765)	G_s
1	230	345	765	575
2	0	345	765	345
3	230	0	765	230
4	230	345	115	115
5	0	0	765	0
6	0	345	115	115
7	230	0	765	230
8	230	345	0	0
9	0	0	115	0
10	0	345	0	0
11	230	0	0	0
12	0	0	0	0

(33) $$\begin{cases} Pr\{G = 575\} = P_1(t) \\ Pr\{G = 345\} = P_2(t) \\ \Pr\{G = 230\} = P_3(t) + P_7(t) \\ Pr\{G = 115\} = P_4(t) + P_6(t) \\ \Pr\{G = 0\} = P_5(t) + P_8(t) + P_9(t) + P_{10}(t) + \\ P_{11}(t) + P_{12}(t) \end{cases}$$

The instantaneous MSS availability *A(t)* at *t>0* system mean performance can be calculated respectively by using eq. (15), (16), (17) and (18) based on the UGF of the entire MSS.The instantaneous MSS availability $A(t)\ at\ t > 0$

(34) $$A(t,w) = \Omega\left(G(Z,t)\right) = \sum_{i=1}^{5} P_i(t)\Omega(g_i > 0)$$

(35) $$A(t,w) = \sum_{i=1}^{5} P_i(t) = 1 - P_0$$

For constant demand $W = 575\,kV$, the *MSS* availability $A(t)\ at\ t > 0$.

(36) $$A(t,w) = \sum_{i=1}^{5} P_i(t)\Omega(g_i \geq 575) = P_1(t)$$

$W = 345\,kV$, the *MSS* availability $A(t)\ at\ t > 0$

(37) $$A(t,w) = \sum_{i=1}^{5} P_i(t)\Omega(g_i \geq 345) = P_2(t)$$

$W = 230\,kV$, the *MSS* availability $A(t)\ at\ t > 0$

(38) $$A(t,w) = \sum_{i=1}^{5} P_i(t)\Omega(g_i \geq 230) = P_3(t) + P_7(t)$$

$W = 115\,kV$, the *MSS* availability $A(t)\ at\ t > 0$

(39) $$A(t,w) = \sum_{i=1}^{5} P_i(t)\Omega(g_i \geq 115)$$

(40) $$A(t,w) = P_4(t) + P_6(t)$$

Results and discussion

These probabilities (performance distribution of the entire system)and availability of the system are presented in Figures 4(a) - 4(c) for three threshold demand (W) values of 575 kV, 345 kV , 230 kV and 115 kV respectively. We can notice that after a certain time, the system becomes stable (monotonic probabilities of the states) and the availability becomes asymptotically constant.

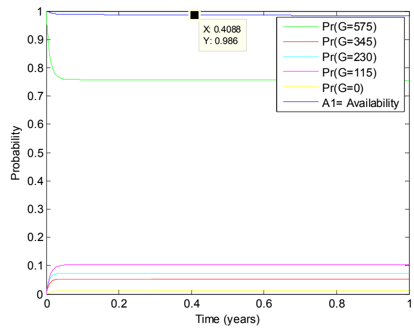

Fig.3. Availability and probabilities of different levels of performance for w=115 kV

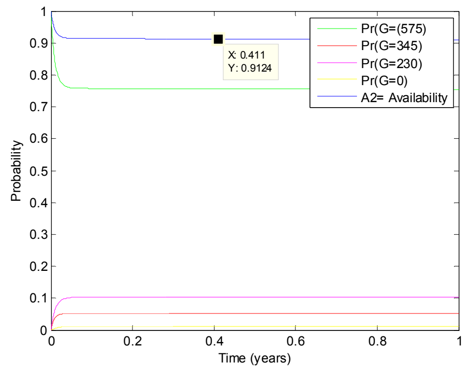

Fig.4. Availability and probabilities of different levels of performance for w=230 kV

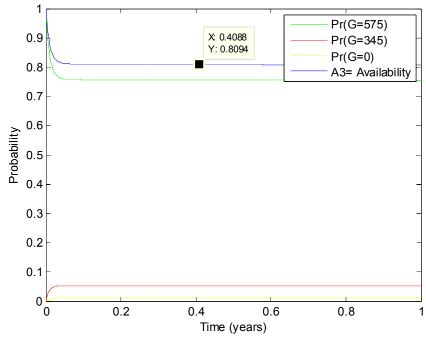

Fig.5. Availability and probabilities of different levels of performance for w=345 kV

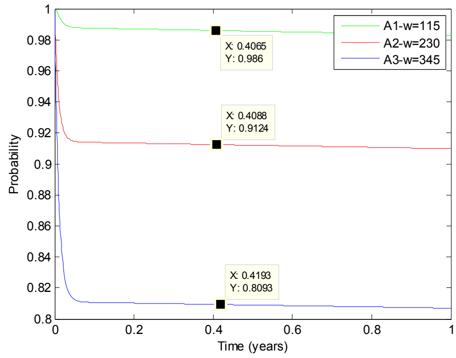

Fig.6. Probability of different performance

The system availability at different time stages within [0.1 : 0.4] years interval is shown in Figure 7.

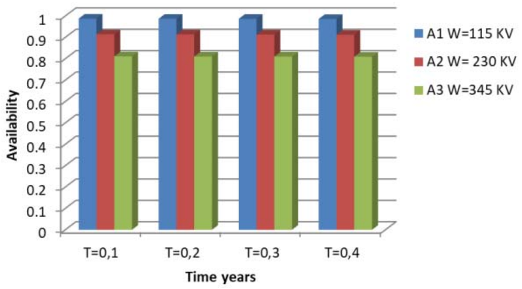

Fig.7. The system availability at different time stages

The system reliability at different time stages within [0.05-0.4]years interval is shown in table 2.

Table 2. The system reliability at different time stages

T (years)	0.05	0.1	0.15	0.2	0.3	0.4
R(t)	0.921	0.8399	0.7728	0.7076	0.603	0.491

The reliability curve up to 5 years for optimal design policy is shown in Figure 8 and 9.

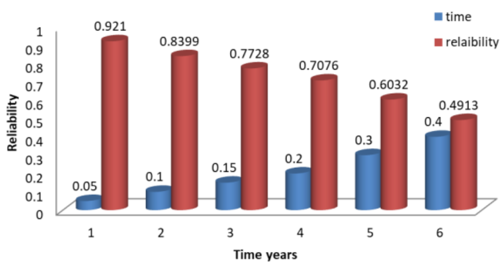

Fig.8. The reliability curve up to 5 years for optimal design policy

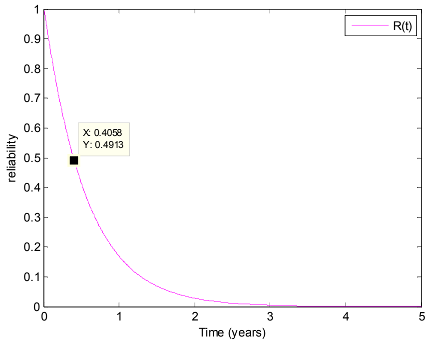

Fig.9. The reliability curve up to 5 yeas

Conclusion

The previous studies in series-parallel systems have not been considered the systems with repairable components. The considered system in this study has components with linear failure and repair rate, in this paper the system Availability and reliability evolution is studied for a fixed value of demand in function of time. The method based on the hybridization of the Markov stochastic process and the universal generating function UGF technology. Analysis and the results show the decreasing evolution of the system reliability in function of an increasing demand.

The analysis of the causes of failure and the construction of the degradation mode of each component of an electrical system gives us an overview of its total reliability. By using the universal generator function, the number of states of the multistate system is substantially deduced. As a result, models are more easily constructed to calculate the reliability of the system with greater accuracy.

The results of the study show us the causes of failure and the modes Degradation of electrical components and electrical system on which one can more accurately predict the system states or the lifetime of the system.

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