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The magnetic field calculation in electromechanical systems with saturated ferromagnetic structural elements

***Abstract**. A method for calculating the characteristics of the magnetic field in electromechanical systems with thin bridges based on the reduction of the initial-boundary value problem for Maxwell's equations to an equivalent system of integro-differential equations with its subsequent numerical solution is developed.*

***Streszczenie.** Opracowano metodę obliczania charakterystyk pola magnetycznego w układach elektromechanicznych z cienkimi mostkami ferromagnetycznymi, polegającą na redukcji problemu początkowo-brzegowego dla równań Maxwella do równoważnego układu równań całkowo-różniczkowych z jego późniejszym rozwiązaniem numerycznym (**Metoda obliczania charakterystyk pola magnetycznego w układach elektromechanicznych z cienkimi mostkami ferromagnetycznymi**).*

Keywords: brushless direct current motor, thin ferromagnetic bridges, magnetic permeability, nonlinearity, secondary sources method.
Słowa kluczowe: bezszczotkowy silnik prądu stałego, cienky mostek ferromagnetyczny, nieliniowość, metoda źródła wtórnego.

Introduction

Many electrical devices (electrical machines and devices, elements of automation and computer technology) contain ferromagnetic elements. Operation nominal modes of such devices are usually realized at magnetic induction values, which do not lead to significant saturation of soft magnetic materials. But sometimes, especially in low power high-speed electromechanical systems, the magnetic induction value can exceed the saturation limit. In this case, when calculating the magnetic field characteristics, it is necessary to take into account the nonlinear dependence $B(H)$. Subject to significant excess of magnetic induction in individual structural elements of the electric machine, their magnetic permeability may approach the magnetic constant μ_0. For further analysis, if we determine these areas and put the magnetic permeability for them equal to μ_0, it is possible to re-arrange the nonlinear problem of calculating the magnetic field characteristics in such devices to solve a linear problem. This approach is more relevant if it is necessary to perform multivariate calculations in search of optimal geometric, physical or mode parameters of electrical devices.

To calculate the magnetic field characteristics can be used one of the methods: finite difference method (FDM) [1], the finite element method (FEM) [2-7], the integral equations method [8-13], combined methods, etc.

The advantage of the method of integral equations in comparison with FDM and FEM is that the unknowns (secondary sources density) are distributed, in the general case, only in the volume and at the boundary of ferromagnetic bodies. Thus, the area of search for a solution is significantly smaller than in FDM and FEM, when it is necessary to search solutions in the whole, generally speaking, unlimited area [10 - 13].

At present, there are two main approaches to formulating a mathematical description of a magnetic system based on integral equations: the boundary integral equations method (or the secondary sources method) [8] and the spatial integral equations method [9].

The aim of the article is to develop a method for calculating the magnetic field characteristics in electromechanical systems with thin bridges based on the reduction of the initial-boundary problem for Maxwell's equations to an equivalent integro-differential equations system with its subsequent numerical solution.

As an example, consider an electric motor with an implicit pole stator without grooves and an open pole rotor with permanent magnets (Fig. 1). The design feature of the presented motor is the presence of a thin ferromagnetic bridge instead of a wedge, which forms the stator smooth surface. This design of the stator is due on the one hand to the possibility of forming stator windings with a denser winding of turns, on the other hand to simplify the technology of laying stator windings from the stator outside. This is achieved by forming a special shape of the stator plates in the form of "stars" and rings, followed formation the stator package by them and the windings.

In the nominal mode of the electric machine operation, the ferromagnetic material of the bridges can enter a state of magnetic saturation, which leads to a significant reduction in shunting the magnetic flux of stator currents and permanent magnets, because their magnetic permeability approaches the magnetic constant μ_0.

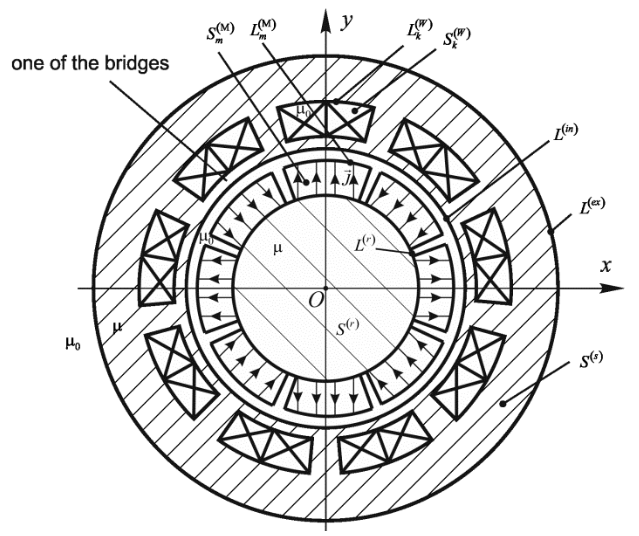

Fig. 1. The electric motor cross section with a smooth stator

Model for research

The electric motor stator $S^{(s)}$ and rotor $S^{(r)}$ are laminated and made of soft magnetic material (Fig. 1). The stator $S^{(s)}$ is a ferromagnetic tube with an external border $L^{(ex)}$ and $L^{(in)}$. Along the tube channels are made for the location of the winding $S_k^{(W)}$, k=1,2...,N_W, N_W is the number of coils in the winding; $L_k^{(W)}$ is the boundary between the k-th coil and the stator.

The cylindrical rotor is located in the middle of the stator. Permanent magnets $S_m^{(M)}$, $m=1,2,...,N_M$, (N_M is the number of permanent magnets) are glued to the rotor boundary $L^{(r)}$, that have been magnetized homogeneously to the magnetization $\vec{J}$ in the *Oxy* plane as shown in Fig.1.

When problem is set, we neglect the hysteresis and anisotropy of ferromagnetic materials and assume that the magnetic induction depends on the magnetic field strength as $B=B(H)$, where B, H is magnetic induction, magnetic field strength. If we enter the magnetic permeability of the material μ, the specified dependence can be written as $\vec{B}=\mu(H)\vec{H}$.

Taking into account the fact that the stator windings are supplied with currents of a given shape with low frequency, we can neglect the bias currents.

The problem is considered in a plane-parallel approximation. A mathematical model describing electromagnetic processes in an electric motor is written for unknown vector magnetic potential having only a z-component $\vec{A}=(0,0,A_z)$.

Earlier in [15], a three-dimensional boundary value problem was formulated to calculate the characteristics of the magnetic field in BLCM taking into account the magnetic properties of the medium. In [13], based on the secondary sources method, the boundary value problem of calculating the plane-parallel magnetic field characteristics in BLCM taking into account the nonlinearity of the steel magnetic characteristic is reduced to a system of integral equations for fictitious magnetic charges. In the kernels of integral equations there is a function $\nabla_Q\mu(Q)$, the calculation of which complicates the numerical solution of these equations.

Therefore, in [14], using the Green's identity and the properties of the magnetic charges simple layer potential, the kernels of the integral equations change in the direction of decreasing the components containing the function $\nabla_Q\mu(Q)$:

(1) $$\sigma(Q)-\frac{1}{\pi}\oint_L \sigma(M)K_1(M,Q)dL_M = =\frac{1}{\pi}\int_S \rho(M)K_2(M,Q)dS_M + F^{\sigma}(Q),$$

(2) $$\rho(Q)+\frac{1}{2\pi}\int_S \rho(M)K_3(M,Q)dS_M = =-\frac{1}{2\pi}\oint_L \sigma(M)K_4(M,Q)dL_M - F^{\rho}(Q),$$

where:

$$K_1(M,Q)=\lambda(Q)\frac{\vec{r}_{MQ}\vec{n}_Q}{r_{MQ}^2}-\frac{1}{L}\int_L \lambda(P)\frac{\vec{r}_{MP}\vec{n}_P}{r_{MP}^2}dL_P,$$

$$K_2(M,Q)=\lambda(Q)\frac{\vec{r}_{MQ}\vec{n}_Q}{r_{MQ}^2}-\frac{1}{L}\int_L \lambda(P)\frac{\vec{r}_{MP}\vec{n}_P}{r_{MP}^2}dL_P-\frac{\pi}{L},$$

$$F^{\sigma}(Q)=2\mu_0\left[\lambda(Q)\vec{H}^{(B)}(Q)\vec{n}_Q-\frac{1}{L}\int_L \lambda(P)\vec{H}^{(B)}(P)\vec{n}_P dL_P\right]$$

$$K_3(M,Q)=\frac{\vec{r}_{MQ}\nabla_Q\mu(Q)}{\mu(Q)r_{MQ}^2}-\frac{1}{S}\int_L \ln\frac{\mu(P)}{\mu_0}\frac{\vec{r}_{MP}\vec{n}_P}{r_{MP}^2}dL_P + +\frac{2\pi}{S}\ln\frac{\mu(M)}{\mu_0},$$

$$K_4(M,Q)=\frac{\vec{r}_{MQ}\nabla_Q\mu(Q)}{\mu(Q)r_{MQ}^2}-\frac{1}{S}\int_L \ln\frac{\mu(P)}{\mu_0}\frac{\vec{r}_{MP}\vec{n}_P}{r_{MP}^2}dL_P + +\frac{\pi}{S}\ln\frac{\mu(M)}{\mu_0}+\frac{2\pi}{S},$$

$$F^{\rho}(Q)=\mu_0\left[\frac{\vec{H}^{(Vort)}(Q)\nabla_Q\mu(Q)}{\mu(Q)}- -\frac{1}{S}\int_S \frac{\vec{H}^{(Vort)}(P)\nabla_P\mu(P)}{\mu(P)}dS_P\right],$$

$\sigma_M(Q)$ is the density of a simple layer of magnetic charges at the point Q of the ferromagnetic bodies boundary *L*;

$$L = L^{(r)} \cup L^{(in)} \cup L^{(ex)} \cup \bigcup_{k=1}^{N_W} L_k^{(W)};$$

$\sigma_M(M)$ is similarly at the point *M*; $\lambda(Q)=[\mu(Q)-\mu_0]/[\mu(Q)+\mu_0]$, $\mu(Q)$ is the magnetic permeability at the point Q of the ferromagnetic medium, which is depended of the intensity of the magnetic field; μ_0 is magnetic permeability of the external medium to the ferromagnetic bodies, $\mu_0=4\pi\cdot10^{-7}$ H/m; $\vec{r}_{QM}$ is the position vector, which is directed from the integration point M to the observation point Q; $\vec{n}_Q$ is normal to the boundary L, which is directed from the ferromagnetic bodies to the outside; $\rho_M(Q)$ is the volume density of magnetic charges at the point Q of cross-section S of ferromagnetic bodies *S*;

$$S = S^{(r)} \cup S^{(s)};$$

$\vec{H}^{(Vort)}(Q)$ is the magnetic field intensity created as permanent magnets, and the currents in the windings.

The kernel of integral equation (2) includes a function$\nabla_Q\mu(Q)$. The advantage of the secondary sources method is that this function can be explicitly expressed through the density of magnetic field sources [15], which significantly simplifies the formation of the system of equations (1), (2) for further numerical solution.

To calculate the the magnetic field characteristics, we organize an iterative process of finding the magnetic permeability μ in a ferromagnetic medium, taking into account the nonlinear characteristic of the dependence $\mu(H)$. To do this, we use a system of integral equations (1), (2) in the approximation of an inhomogeneous medium. Let be known $\mu^{(n)}(Q)$ on the *n*-th iterative approximation step, where $Q\in S$, then

$$\lambda^{(n)}(Q)=\left(\mu^{(n)}(Q)-\mu_0\right)\Big/\left(\mu^{(n)}(Q)+\mu_0\right).$$

We determine the kernels and right parts of integral equations (1), (2) at the *n*-th iterative step of the magnetic permeability distribution search process. For this purpose in them instead of $\lambda(Q)$ we substitute $\lambda^{(n)}(Q)$ and instead of $\mu(Q)$ - $\mu^{(n)}(Q)$.

Next, we find the density of the simple layer magnetic charges $\sigma^{(n)}(Q)$ and the density of the bulk magnetic charge $\rho^{(n)}(Q)$ in the *n*-th step, solving a system of linear algebraic equations (either by iterative method or by the direct method):

$$\sigma^{(n)}(Q)-\frac{1}{\pi}\oint_L \sigma^{(n)}(M)K_1^{(n)}(M,Q)dL_M =$$

$$=-\frac{1}{\pi}\int_S \rho^{(n)}(M)K_2^{(n)}(M,Q)dS_M + F^{\sigma(n)}(Q)\ ;$$

$$\rho^{(n)}(Q)+\frac{1}{2\pi}\int_S \rho^{(n)}(M)K_3^{(n)}(M,Q)dS_M =$$

$$-\frac{1}{2\pi}\oint_L \sigma^{(n)}(M)K_4^{(n)}(M,Q)dL_M - F^{\rho(n)}(Q)\ .$$

Having determined the densities of magnetic charges $\sigma^{(n)}(Q)$ and $\rho^{(n)}(Q)$, we find the magnetic field strength created by this system of charges

$$\vec{H}^{(Vort\ free)(n)}(M)=\frac{1}{2\pi\mu_0}\int_L \frac{\sigma^{(n)}(Q)\vec{r}_{MQ}}{r_{MQ}^2}dL_Q +$$

$$+\frac{1}{2\pi\mu_0}\int_S \frac{\rho^{(n)}(Q)\vec{r}_{MQ}}{r_{MQ}^2}dS_Q\ .$$

Then the resulting magnetic field strength:

$$\vec{H}^{(n)}(M)=\vec{H}^{(Vort\ free)(n)}(M)+\vec{H}^{(Vort)}(M)\ .$$

We find a new magnetic permeability distribution in a ferromagnetic medium:

$$\mu^{(n+1)}(Q)=\mu\left(H^{(n)}(Q)\right).$$

Continuing the described algorithm, we arrive at the desired magnetic permeability distribution taking into account the nonlinearity of the ferromagnetic material characteristics $\mu(H)$ and find the magnetic field characteristics.

The iterative process ends when the condition is reached

$$\varepsilon=\max_{\forall Q}\left|1-\mu^{(n-1)}(Q)\big/\mu^{(n)}(Q)\right|\le\varepsilon_0\ ,$$

where: ε_0 is the given error in determining the magnetic permeability in a ferromagnetic medium.

After analyzing the magnetic permeability distribution in ferromagnetic media, we separate areas with high magnetic permeability and with a magnetic permeability close to μ_0. We form new boundaries between them and consider the magnetic permeability of the regions they limit independed of the external magnetic field, i.e. it is constant in this regions. In this case, the problem of calculating the magnetic field characteristics is reduced to the following integro-differential equation, which is easy to obtain from (1), (2). In these equations it should be assumed that $\nabla_Q\mu(Q)=0$ and the bulk magnetic charges density ρ is zero:

(3) $$\sigma(Q)-\frac{1}{\pi}\oint_L \sigma(M)K_1(M,Q)dL_M = F^{\sigma}(Q)\ ,$$

where:

$$K_1(M,Q)=\lambda(Q)\frac{\vec{r}_{MQ}\vec{n}_Q}{r_{MQ}^2}-\frac{1}{L}\int_L \lambda(P)\frac{\vec{r}_{MP}\vec{n}_P}{r_{MP}^2}dL_P\ ,$$

$$F^{\sigma}(Q)=2\mu_0\left[\lambda(Q)\vec{H}^{(B)}(Q)\vec{n}_Q-\frac{1}{L}\int_L \lambda(P)\vec{H}^{(B)}(P)\vec{n}_P dL_P\right],$$

The solution of equation (3) is much simpler, and has a smaller dimension compared to the system of equations (1), (2), because the density of a magnetic charges simple layer is distributed only at the boundary of ferromagnetic bodies with different magnetic permeability.

The calculations results and their analysis

Fig. 2 shows the geometric parameters of the electric motor: 1 — stator winding; 2 — stator; 3 — homogeneously magnetized permanent magnets; 4 — rotor shaft. The material from which the rotor shaft and the ferromagnetic part of the stator are made is electrical steel, the magnetic properties of which were determined by the dependence $B(H)$. The permanent magnets are made of *NdFeB*, the coercive force of the magnet is H_c=10^3 kA / m, or B_r = 1.32 T (the direction of magnetization of the permanent magnets is shown in Fig.1).

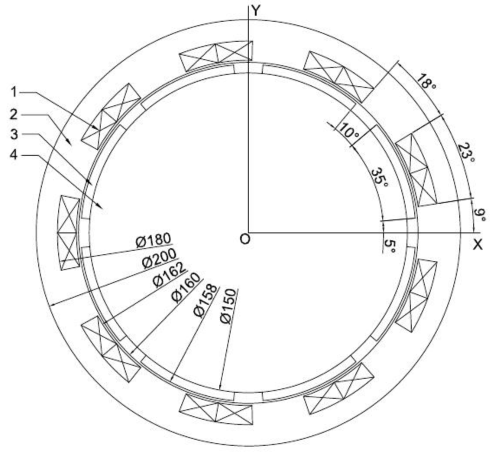

Fig. 2. The geometric parameters of the electric motor

The magnetic field calculation of the motor was performed in idle mode (the magnetic field of the permanent magnets). Fig. 3 shows the result of calculating the r-component of the magnetic induction in the gap between the permanent magnets and the inner part of the stator: a) a nonlinear problem for the magnetic system of an electric motor with bridges; b) a nonlinear problem for the magnetic system of an electric motor without bridges (their magnetic permeability was set equal to μ_0); c) a linear problem for the magnetic system of an electric motor with bridges (magnetic permeability of all ferromagnetic materials is equal to $\mu=10^3\mu_0$); d) a linear problem for the magnetic system of an electric motor without bridges (magnetic permeability of all ferromagnetic materials is equal to $\mu=10^3\mu_0$, for bridges is equal to μ_0). As the analysis of the obtained data shows, the mean square error of the dependences, which is presented in Fig. 3, b, d, compared to dependence shown in Fig. 3, a, is 14.5%, 14.7%, which is sufficient for the technical decision. The data shown in Fig. 3, c have a standard deviation of 30.8% from the data shown in Fig. 3, a. That is, in the case of solving a linear problem for the magnetic system of an electric motor with bridges, the calculation gives a very large error. At other rotor positions a calculations result have similar character.

Thus, the assumption of possible saturation of thin ferromagnetic bridges was justified. Further analysis of the electric motor operation allows setting the magnetic permeability of the bridges regions equal to μ_0, and for the rest of the stator and rotor ferromagnetic regions equal to the constant magnetic permeability μ.

This leads to the solution of the integral equation (3) for the surface density of a magnetic charges simple layer, which allows to reduce the search area of unknowns exclusively to the boundaries of ferromagnetic bodies.

A comparative analysis of calculations carried out with the same statement of the problem using the software product COMSOL Multiphysics [17], the standard deviation of the magnetic field induction in the working gap of the electric motor compared to the integral equations method was <3.5%.

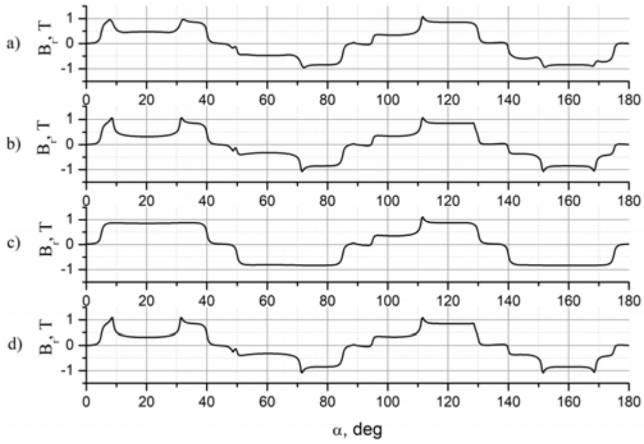

Fig. 3. Graph of magnetic induction r-components distribution in the working gap between the rotor and the stator

Conclusion

A method for calculating the magnetic field characteristics in electrical devices with thin bridges or magnetically saturated regions is proposed, which allows to identify areas with pronounced saturation of ferromagnetic elements, set their magnetic permeability equal to the magnetic permeability of air, and set its constant for ferromagnetic elements. This allows the complex problem for calculating the magnetic field characteristics in an electrical device with ferromagnetic elements with nonlinear magnetic characteristics reduce to the problem of calculating the magnetic field in a piecewise homogeneous medium with constant magnetic permeability. The solution of the last problem is reduced to the solution of the integral equation for the density of a magnetic charges simple layer distributed only along the boundary of ferromagnetic bodies with different magnetic permeability. This significantly reduces the search area for the solution in contrast to the finite difference method or the finite element method, where the solution is tryed, generally speaking, in all unlimited space.

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