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doi:10.15199/48.2020.09.05

Apparent Power Effectiveness for the Assessment of the Efficiency of the Cable Transmission Line in the Supply System with Sinusoidal Current

***Abstract**. Using the theory of electrical engineering for the load sinusoidal voltage and current, the power losses in the transmission line represented by an L-type equivalent circuit are determined. It is found out that for a cable transmission line the total power loss depends on both the real values of voltage and current and the difference between the initial phases of the latter. This results in a change in power losses in the line under conditions of constant real values of voltage and current – apparent power.*

***Streszczenie.** Określono analitycznie straty mocy w linii przesyłowej, reprezentowane przez równoważny obwód typu L. Ustalono, że dla linii transmisji kablowej całkowita strata mocy zależy zarówno od skutecznych wartości napięcia i prądu, jak i od różnicy między początkowymi fazami tej ostatniej. Prowadzi to straty mocy w warunkach stałych wartości prądu i napięcia. (**Ocena strat w linii przesyłowej kablowej w systemie zasilania prądem sinusoidalnym**).*

Keywords: instantaneous electrical power, energy process indicators, apparent power, line losses.
Słowa kluczowe: chwilowa moc elektryczna, wskaźniki procesu energetycznego, straty mocy.

Introduction

In electrical and electromechanical systems, instantaneous electrical power is a physical quantity that reflects the information about energy process. Scientists use instantaneous power, as a certain composition of currents and voltages, as instrument for identifying the parameters of electrical equivalent circuits [1] in electrical engineering, electrical power engineering and electromechanics objects. The relation of the circuit parameters to the components of power is determined based on the Kirchhoff's laws, using the harmonic analysis methods. This approach allows you to get the relation of instantaneous power of a circuit element and the mode parameters with instantaneous power at the controlled unit [2]. Methods for calculating instantaneous reactive, instantaneous active powers and power losses have been developed based on the principle of instantaneous power balance [3].

The analysis of the recent research

In electric power industry, when analysing the voltage levels in the units of energy-consuming systems, the relation of voltage with reactive power is used. Taking into account this connection, the problems of compensation for reactive power and voltage in the units of electric power system are solved. These tasks become relevant in the context of increasing use of alternative electric energy sources [4]. Consequently, the change of the power loss in the elements of electric energy transport and distribution complex in the conditions of incomplete compensation for reactive power is determined. This necessitates the additional control of reactive power in the controlled unit in order to stabilize the voltage level and reduce the losses in the transmission line. Therefore, automatic regulation systems of instantaneous power are developed [5].

Because of fundamental research [6], the known influence of reactive power on the losses of elements of power transmission system is specified. In particular, for determining the "real efficiency" of transmission line [6], the use of the root mean square (RMS) value of reactive power is substantiated.

Given that instantaneous power reflects information about the energy process, it is considered as a certain information signal [7−11]. According to some scientists [9−11], the RMS [10] or effective [9, 11] values of instantaneous power deserve attention as an integral characteristic.

Problem formulation

Apparent power is widely used in electricity, and in some cases, this parameter is included in the published data of electric machines, for example, transformers. Determination of energy indices on the apparent power basis, inherent in most of the existing methods, undergoes a thorough critique [10]. The authors of [10] note that, in the proposed method, the components of the instantaneous power are distinguished, the RMS values of which determine alternative indicators. A similar procedure is also used in [11]. The theoretical positions set forth in the above-mentioned papers are confirmed by experimental research, which question the adequacy of the energy process indicators used at present in power systems. On the other hand, the indicators proposed in papers [9−11] were not properly approbated, and are not popular in the field of electrical engineering, and the apparent power is the dominant generalized indicator in energy process.

Purpose

The assessment of the apparent power nature influence on the level of losses in the cable transmission line of power supply system under the conditions of sinusoidal voltage and current.

Basic material and research results

Most calculations of the modes of electric power engineering are performed using the voltage and current values, considering them sinusoidal. Let us assume that the voltage and current in the electric circuit are:

$$\begin{cases} u = \sqrt{2}U\sin\left(\omega t + \psi_u\right); \\ i = \sqrt{2}I\sin\left(\omega t + \psi_i\right), \end{cases} \tag{1}$$

where U, I – voltage and current RMS values; ψ_u, ψ_i − voltage and current phase shift; ω − angular frequency. Based on instantaneous power, active P, reactive Q and apparent S power are introduced:

$$\begin{aligned} p = ui &= \sqrt{2}U\sin\left(\omega t + \psi_u\right)\sqrt{2}I\sin\left(\omega t + \psi_i\right) = \\ &= UI\left[\cos\left(\psi_u - \psi_i + 0\right)\right] - UI\left[\cos\left(\psi_u + \psi_i + 2\omega t\right)\right] = \\ &= P\cos\left(0\right) - Q\sin\left(0\right) - S\cos\left(2\omega t + \psi_u + \psi_i\right). \end{aligned}$$

Define the square norm of instantaneous power on the interval of repeatability as the RMS value as follows:

$$P_{RMS} = \|p\| = \sqrt{\frac{1}{T}\int_0^T p^2 dt} =$$
$$= UI\sqrt{\cos\left(\psi_u - \psi_i\right)^2 + \frac{\cos\left(\psi_u + \psi_i\right)^2}{2} + \frac{\sin\left(\psi_u + \psi_i\right)^2}{2}} =$$
$$= S\sqrt{\frac{1}{2} + \cos\left(\psi_u - \psi_i\right)^2}.$$

Consider the rationality of using apparent power from the standpoint of its impact on the level of losses in the power systems elements. To do this, we use an equivalent circuit of the cable transmission line as shown in Figure 1. Let us assume that the load current and voltage (1) are at the end of the line. We will suppose that the energy process measurement occurs at the end of the line. We represent the load current and voltage via the orthogonal components as follows:

$$\begin{cases} u_1 = u = U_{1a}\sin\left(\omega t\right) + U_{1b}\cos\left(\omega t\right); \\ i_1 = i = I_{1a}\sin\left(\omega t\right) + I_{1b}\cos\left(\omega t\right), \end{cases}$$

where $U_{1a} = \sqrt{2}U\cos\left(\psi_u\right)$, $U_{1b} = \sqrt{2}U\sin\left(\psi_u\right)$ – voltage orthogonal components; $I_{1a} = \sqrt{2}I\cos\left(\psi_i\right)$, $I_{1b} = \sqrt{2}I\sin\left(\psi_i\right)$ – current orthogonal components.

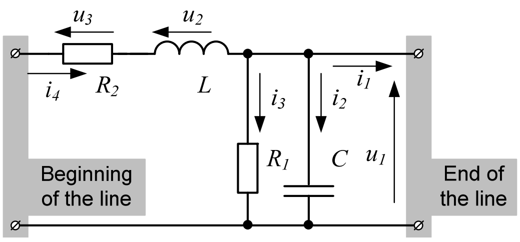

Fig.1. Transmission line equivalent circuit

Using the first and second Kirchhoff laws, we define the distribution of currents and voltages in the circuit elements. Capacitive current at the end of line

$$i_2 = C\frac{du_1}{dt} = CU_{1a}\omega\cos\left(\omega t\right) - CU_{1b}\omega\sin\left(\omega t\right) =$$
$$= I_{2a}\sin\left(\omega t\right) + I_{2b}\cos\left(\omega t\right),$$

resistance current at the end of line

$$i_3 = \frac{u_1}{R_1} = \frac{U_{1a}}{R_1}\sin\left(\omega t\right) + \frac{U_{1b}}{R_1}\cos\left(\omega t\right) =$$
$$= I_{3a}\sin\left(\omega t\right) + I_{3b}\cos\left(\omega t\right),$$

then line current

$$i_4 = i_1 + i_2 + i_3 = \left(I_{1a} + \frac{U_{1a}}{R_1} - CU_{1b}\omega\right)\sin\left(t\right) +$$
$$+\left(I_{1b} + \frac{U_{1b}}{R_1} + CU_{1a}\omega\right)\cos\left(\omega t\right) = I_{4a}\sin\left(\omega t\right) + I_{4b}\cos\left(\omega t\right).$$

Voltage drop under this current is:

$$u_2 = L\frac{di_4}{dt} = -L\omega\left(I_{1b} + \frac{U_{1b}}{R_1} + CU_{1a}\omega\right)\sin\left(\omega t\right) +$$
$$+L\omega\left(I_{1a} + \frac{U_{1a}}{R_1} - CU_{1b}\omega\right)\cos\left(\omega t\right) = U_{2a}\sin\left(\omega t\right) + U_{2b}\cos\left(\omega t\right);$$

$$u_3 = i_4 R_2 = R_2\left(I_{1a} + \frac{U_{1a}}{R_1} - CU_{1b}\omega\right)\sin\left(\omega t\right) +$$
$$+R_2\left(I_{1b} + \frac{U_{1b}}{R_1} + CU_{1a}\omega\right)\cos\left(\omega t\right) = U_{3a}\sin\left(\omega t\right) + U_{3b}\cos\left(\omega t\right).$$

To determine the losses in line, consider the instantaneous power of the circuit resistive elements. Resistor power R_2 :

$$p_{R2} = u_3 i_4 = P_{R2.a1-1}\cos\left(0\right) + P_{R2.a1+1}\cos\left(2\omega t\right) +$$
$$+P_{R2.b1-1}\sin\left(0\right) + P_{R2.b1+1}\sin\left(2\omega t\right),$$

where

$$P_{R2.a1-1} = \left(U_{3a}I_{4a} + U_{3b}I_{4b}\right) = R_2\left(I_{4a}^2 + I_{4b}^2\right) =$$
$$= R_2\left(I_{1a} + \frac{U_{1a}}{R_1} - CU_{1b}\omega\right)^2 + R_2\left(I_{1b} + \frac{U_{1b}}{R_1} + CU_{1a}\omega\right)^2;$$
$$P_{R2.b1-1} = \left(U_{3a}I_{4b} - U_{3b}I_{4a}\right) = R_2\left(I_{4a}I_{4b} - I_{4b}I_{4a}\right) = 0;$$
$$P_{R2.a1+1} = \left(U_{3b}I_{4b} - U_{3a}I_{4a}\right) = R_2\left(I_{4b}^2 - I_{4a}^2\right) =$$
$$= R_2\left(I_{1a} + \frac{U_{1a}}{R_1} - CU_{1b}\omega\right)^2 - R_2\left(I_{1b} + \frac{U_{1b}}{R_1} + CU_{1a}\omega\right)^2;$$
$$P_{R2.b1+1} = \left(U_{3b}I_{4a} + U_{3a}I_{4b}\right) = R_2\left(2I_{4b}I_{4a}\right) =$$
$$= 2R_2\left(I_{1b} + \frac{U_{1b}}{R_1} + CU_{1a}\omega\right)\left(I_{1a} + \frac{U_{1a}}{R_1} - CU_{1b}\omega\right).$$

Resistor power R_1 :

$$p_{R1} = u_1 i_2 = P_{R1.a1-1}\cos\left(0\right) + P_{R1.a1+1}\cos\left(2\omega t\right) +$$
$$+P_{R1.b1-1}\sin\left(0\right) + P_{R1.b1+1}\sin\left(2\omega t\right),$$

where

$$P_{R1.a1-1} = \left(U_{1a}I_{2a} + U_{1b}I_{2b}\right) = \frac{1}{R_1}\left(U_{1a}^2 + U_{1b}^2\right);$$
$$P_{R1.b1-1} = \left(U_{1a}I_{2b} - U_{1b}I_{2a}\right) = \frac{1}{R_1}\left(I_{2a}I_{2b} - I_{2b}I_{2a}\right) = 0;$$
$$P_{R1.a1+1} = \left(U_{1b}I_{2b} - U_{1a}I_{2a}\right) = \frac{1}{R_1}\left(U_{1b}^2 - U_{1a}^2\right);$$
$$P_{R1.b1+1} = \left(U_{1b}I_{2a} + U_{1a}I_{2b}\right) = \frac{2}{R_1}\left(U_{1b}U_{1a}\right).$$

The losses in the circuit (Fig. 1) are determined by integrating the instantaneous power on the period of recurrence *T*. Because of integrating the power loss will be equal to the component of the instantaneous power of zero frequency [12]:

$$\Delta P_{R1} = P_{R1.a1-1} = \frac{1}{R_1}\left(U_{1a}^2 + U_{1b}^2\right);$$
$$\Delta P_{R2} = P_{R2.a1-1} = R_2\left(I_{1a}^2 + I_{1b}^2\right) + R_2C^2\omega^2\left(U_{1a}^2 + U_{1b}^2\right) +$$
$$+\frac{R_2}{R_1^2}\left(U_{1a}^2 + U_{1b}^2\right) + R_2C\omega\left(U_{1a}I_{1b} - U_{1b}I_{1a}\right) + 2\frac{R_2}{R_1}\left(U_{1a}I_{1a} + U_{1b}I_{1b}\right).$$

Thus, the total power losses in the circuit (Fig. 1) are presented as the sum of six components:

$$\Delta P = \Delta P_{R1} + \Delta P_{R2} =$$
$$= \Delta P_{R1}^{I} + \Delta P_{R2}^{II} + \Delta P_{R2}^{III} + \Delta P_{R2}^{IV} + \Delta P_{R2}^{V} + \Delta P_{R2}^{VI}. \quad (2)$$

The relations of the power loss components with the mode parameters are as follows:

$$\Delta P_{R1}^{1} = \frac{1}{R_1}\left(U_{1a}^2 + U_{1b}^2\right) = \frac{U^2}{R_1};$$

$$\Delta P_{R2}^{II} = R_2\left(I_{1a}^2 + I_{1b}^2\right) = R_2 I^2 ;$$

$$\Delta P_{R2}^{III} = R_2 C^2 \omega^2 \left(U_{1a}^2 + U_{1b}^2\right) = R_2 C^2 \omega^2 U^2 ;$$

$$\Delta P_{R2}^{IV} = \frac{R_2}{R_1^2}\left(U_{1a}^2 + U_{1b}^2\right) = \frac{R_2}{R_1^2} U^2 ;$$

$$\Delta P_{R2}^{V} = R_2 C\omega\left(U_{1a} I_{1b} - U_{1b} I_{1a}\right) = R_2 C\omega \sin\left(\psi_u - \psi_i\right) UI ;$$

$$\Delta P_{R2}^{VI} = 2\frac{R_2}{R_1}\left(U_{1a} I_{1a} + U_{1b} I_{1b}\right) = 2\frac{R_2}{R_1}\cos\left(\psi_u - \psi_i\right) UI .$$

We characterize the power loss components from the position of their dependence on the mode parameters. The power component ΔP_{R1}^{I}, ΔP_{R2}^{III}, ΔP_{R2}^{IV} depend on the load voltage RMS value. The power component ΔP_{R2}^{II} depends on the RMS value of the load current. The components of the power ΔP_{R2}^{V} and ΔP_{R2}^{VI} depend on the product of current and voltage RMS values $UI = S$, while the specified components also depend on the difference between the phase shift $\left(\psi_u - \psi_i\right) = \varphi$. Thus, the apparent power in the analysed case is not a sufficient indicator that characterizes the load and its effect on the line losses.

Let us apply the voltage and current RMS value at end of the line unchanged. We represent the loss as the two components sum, the first of which depends on the voltage and current RMS value $\Delta P_{=}$ and in this case is not variable, and the second depends on the difference a current and voltage phases shift $\Delta P(\varphi)$:

$$\Delta P = \Delta P_{=} + \Delta P(\varphi).$$

In order to characterize the power and efficiency of the energy process in [9] it is proposed to use a generalized symmetry coefficient. For the case under consideration, the generalized symmetry coefficient at end of the line will look as follows:

$$K_c = \frac{P}{S} = \cos\left(\psi_u - \psi_i\right) = \cos(\varphi). \tag{3}$$

Several indicators are proposed in [11] to characterize the power and efficiency of the energy process. For the case considered, we use the closest in character to the previous indicator − active power degree:

$$q_P = \frac{P}{P_{RMS}} = \frac{S\cos(\psi_u - \psi_i)}{S\sqrt{\frac{1}{2} + \cos(\psi_u - \psi_i)^2}} = \sqrt{1 - \frac{1}{2\cos(\varphi)^2 + 1}}. \tag{4}$$

Consider a numerical example for which we calculate the power loss in the line (2) and the energy process (3, 4). We perform the loss analysis for the cable transmission line SBG 16.5 km long, with a voltage of 10 kV. Line parameters: $R_1 = 100\ kOhm$; $R_2 = 5.75\ Ohm$; $L = 1.798\ mH$; $C = 1.149\ \mu F$; $U = 10\ kV$; $I = 65\ A$.

Figure 2 shows the power loss ΔP curves in the line from the phase shift of current (ψ_i) for several phase shift of voltage (ψ_u) at constant apparent power. Changes in power losses (Fig. 2 a) are 21 % relative to the average value of 25.4 kW. At the same time, the unchanged value of the apparent power of the load is accompanied by a change in the RMS norm of the load power shown in Figure 2b. Using the value of apparent power (Fig. 2 b, line 6), the deviation of the RMS norm from this value, depending on the phase shift, is 22.5 % and 29.3 %.

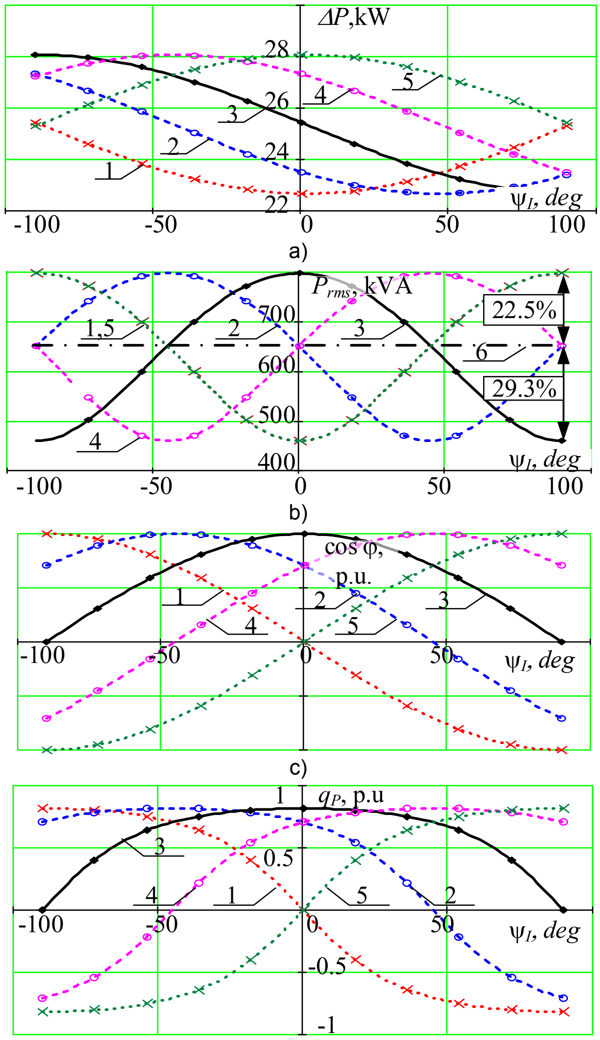

Fig. 2. Change of the indicators depending on the change of the current initial phase: a) line power loss; b) the load power RMS norm is ratio to apparent power; c) the generalized coefficient of power symmetry; d) active power degree. (1 – voltage phase shift -90°, 2 – voltage phase shift -45°, 3 – voltage phase shift 0°, 4 – voltage phase shift 45°, 5 – voltage phase shift 90°).

In the case under consideration, with the increase of the phase shift ψ_i, the loss of power in the line decreases. It should be noted that the observed trend is due to the ratio of losses ΔP_{R2}^{V} and ΔP_{R2}^{VI}.

The amount of active power (Fig. 2d) has the maximum value $q_P = \sqrt{2/3} = 0.817$. Moreover, its sign depends on the direction of transmission of active power (consumption or generation). The zero value of the indicator is achieved in the absence of active power.

Determine the energy process effect on the line using the efficiency in the following form

$$\eta = \frac{P_{end.}}{P_{beg.}} = \frac{UI\cos(\psi_u - \psi_i)}{UI\cos(\psi_u - \psi_i) + \Delta P}. \tag{5}$$

The graphic dependence of the generalized coefficient of symmetry, active power quantity and the efficiency of the line at the voltage phase shift $\psi_u = 0^\circ$ is shown in Figure 3.

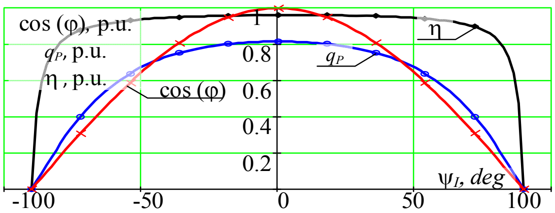

Fig. 3 The dependence of the power factor, active power amount and the efficiency of the line on the current phase shift

To determine the degree of relation curves shown in Figure 3 we use the methods of statistical analysis. For example, the Pearson correlation coefficient, commonly used to describe the correlation between variables *x* and *y* [13], is defined as follows

(6) $$r_{x,y} = \frac{\operatorname{cov}(x,y)}{\sigma_x \sigma_y},$$

where $\operatorname{cov}(x,y)$ – the covariance of variables *x* and *y*; σ_x, σ_y – standard deviation of the variables *x* and *y*. We calculate the Pearson correlation coefficient for two pairs of variables, $\eta,\ q_P$ and $\eta,\ \cos(\varphi)$; the results are shown Table 1.

Table 1. Calculation results of Pearson correlation coefficient

x, y	σ_η	σ_{q_P}	$\sigma_{\cos(\varphi)}$	$\operatorname{cov}(x,y)$	$r_{x,y}$
$\eta,\ q_P$	0.159	0.232	-	0.029	0.773
$\eta,\ \cos(\varphi)$	0.159	-	0.313	0.031	0.67

Thus, the correlation coefficient for both cases is in the range (0.5 – 0.8) corresponding to the moderate correlation [13], but $r_{\eta,\ q_P} > r_{\eta,\ \cos(\varphi)}$ in this way the active power amount most reflects the energy process from the point of view of its effect on line loss.

Conclusions and direction of further research

1. Apparent power under constant sinusoidal current and voltage RMS values is unchanged and does not depend on phase shift difference between current and voltage. Unlike the apparent power, the RMS value of instantaneous power depends on the difference between the current and the voltage phase shift, even if their RMS values are unchanged.

2. It has been determined that for the cable transmission line represented by the L-type equivalent circuit the total power loss depends on the voltage and current RMS values, and on the difference between the phases shift of latter's.

3. As a result of numerical calculation of the mode of line, made of cable SBG16 in the length of 5 km, it has been determined that at constant RMS values of load voltage, current and change of phase shift between them from -90° to +90° the power loss in the line changes by 21 %. In this case, the apparent load power remains unchanged.

4. The relation of the alternative indicators of energy power process, used in the experiment, with the power factor and apparent power has been determined.

5. As a result of the determination of the Pearson correlation coefficient for the efficiency of the line and the degree of the load active power, a moderate level of correlation of the indicator pairs has been determined for the efficiency and the power factor. The efficiency of the line and the degree of load active power have a higher level of correlation.

6. Further research may be performed in the direction of the substantiation of additional indicators of energy process distortion, taking into account the influence on the elements of electrical power engineering, electrical engineering and electromechanics.

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