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Analysis of interactions in the circuit of the power system with nonlinear load and LC passive filter

***Abstract**. The paper deals with an AC circuit containing the nonlinear load and a LC passive filter. Nonlinear load voltage at the power terminals proportional to the signum function of current is considered. The current-voltage characteristic of such load is unambiguous (without hysteresis). A quality analysis of the circuit voltages and currents was carried out. Distribution of active and reactive power for fundamental and higher harmonics in the circuit were also performed.*

***Streszczenie.** W pracy analizowany jest obwód prądu przemiennego z przykładowym obciążeniem nieliniowym i filtrem biernym LC. Przyjęto obciążenie nieliniowe, którego napięcie na zaciskach zasilania jest proporcjonalne do funkcji signum prądu. Charakterystyka napięciowo - prądowa obciążenia jest jednoznaczna (bez histerezy). Przeprowadzono analizę jakościową przebiegów napięć i prądów obwodu. Wykonano analizy rozkładu mocy czynnej i biernej dla harmonicznej podstawowej i wyższych harmonicznych w obwodzie. **Analiza oddziaływań w obwodzie systemu elektroenergetycznego z obciążeniem nieliniowym i filtrem biernym LC**.*

Keywords: nonlinear load, higher harmonics, reactive power compensation, interaction analysis.
Słowa kluczowe: obciążenie nieliniowe, wyższe harmoniczne, kompensacja mocy biernej, analiza oddziaływań.

Introduction

In order to improve energy efficiency and electricity savings, it is necessary to reduce the interaction between the power system and nonlinear receivers. For this purpose, LC passive filters are commonly used. These filters compensate the reactive power in the circuit and may reduce the flow of higher harmonics of current into the power system. Nonlinear loads, that most often disturb the quality of power supply voltage are arc furnaces and rectifiers [1]. In [2] it has been shown that nonlinear load with unambiguous current-voltage characteristics has a total reactive power equal to zero, and the reactive power of the first harmonic of this load is converted into the reactive power of the higher harmonics and fully transferred to the equivalent reactance of the supply circuit. This property is also characteristic for rectifiers. The phenomenon of power conversion in circuits with nonlinear loads and LC passive filters has not been analysed in the literature so far. Usually the nonlinear load is replaced by a simplified model of a current source. There is assumed that the nonlinear receiver is a generator of higher harmonics of current [3],[4],[5],[6]. In order to take into account the conversion phenomena, the AC circuit with LC passive filter and nonlinear load is considered. There was assumed that the voltage at terminals of nonlinear load is proportional to the current signum function. It is a model of the electric arc and a bridge rectifier.

Model of analyzed circuit

The analysed AC circuit is shown in Fig.1. The circuit contains bridge rectifier supplied by a sinusoidal voltage source with the amplitude E_s and the angular frequency ω. The inductance L_s and resistance R_s represent the impedance of the supply system. The LC passive filter is connected to the PCC point, and represented by: inductance L_f, capacity C_f and resistance R_f. The impedance of the load supply system is represented by the inductance L_1 and the resistance R_1. It is assumed that the inductance L_s is much smaller than the inductance L_1. Including a capacitor C_p makes it easier to solve the modelled circuit in Simulink. The algebraic loop problem occurs in the model if the capacitor C_p is not included. Applying a very small value of the capacitor resolves this problem. The value of capacitance C_p was assumed much smaller than capacitor C_f. For such relation, the impact of capacity C_p on circuit operation is insignificant. If the ripple output voltage U_c are very small, it may be assumed that the current-voltage characteristics $U_b(I_1)$ is unambiguous (without hysteresis). This characteristic may be described as the signum function of current I_1: $U_b(I_1)=(U_o+2U_d)\cdot sign(I_1)$, where: U_o - is the constant component in the output voltage of rectifier, U_d - is the diode voltage of bridge rectifier.

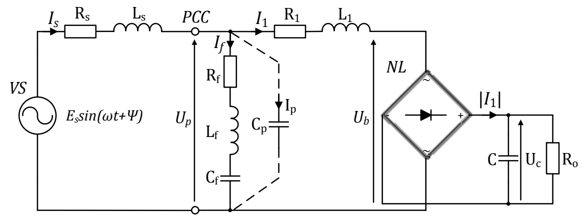

Fig.1. AC circuit model with nonlinear load and LC passive filter

To simplify and reduce number of parameters the analysis was carried out using dimensionless variables. For this purpose reference variables in the form of reactance ωL_1 and supply voltage amplitude E_s were used. Additional the time scaling $\tau=\omega t$ was introduced. Therefore, the circuit equations may be written following:

(1) $$\frac{di_s}{d\tau}=\frac{1}{x_s}\left(e_s\sin(\tau+\psi)-i_s r_s-u_p\right)$$

(2) $$\frac{di_1}{d\tau}=\frac{1}{x_1}\left(u_p-i_1 r_1-u_b(i_1)\right)$$

(3) $$\frac{di_f}{d\tau}=\frac{1}{x_f}\left(u_p-i_f r_f-\frac{1}{c_f}\int i_f d\tau\right)$$

(4) $$u_p=\frac{1}{c_p}\int i_p d\tau \quad where: i_p=i_s-i_f-i_1$$

where dimensionless variables are written:

(5) $$\tau=\omega t;\ X_1=\omega L_1;\ I_m=E_s/X_1;\ e_s=\frac{E_s}{E_s}=1;\ Y_k=\omega C_k;$$
$$i_k=I_k/I_m;\ u_k=\frac{U_k}{E_s};\ r_k=\frac{R_k}{X_1};\ x_k=\frac{\omega L_k}{X_1};\ c_k=X_1 Y_k;$$

where: k – denote circuit part and parameter index.

The MATLAB/Simulink system was used to analyse the circuit under consideration in Fig.1. An operational diagram of circuit was created in Simulink on the basis (1)-(4).

Analysis of interactions in circuit

In this section the power factor PF and total harmonic distortion THD of voltages and currents in circuit were analysed. These quantities are defined following [7]:

$$PF = \frac{P}{S};\ THD_U = \frac{\sqrt{\sum_{n=2}^{n_{\max}} U_n^2}}{U_1};\ THD_I = \frac{\sqrt{\sum_{n=2}^{n_{\max}} I_n^2}}{I_1} \qquad (6)$$

where: P,S – respectively active and apparent power; U_1, I_1 – rms value of fundamental component voltage and current; U_n, I_n - rms value of nth harmonic component voltage and current; n – harmonic order (n = 1,2,3,…,max).

The continuous operation mode of the rectifier was analyzed. Parameters of simulation were following: u_o = 0.5, r_s = r_1 = r_f = 0.01 and $x_f = 0$. The obtained results refers to case when the value of the variable x_f is equal to zero. It is common case occurring in the power system circuits with nonlinear loads and reactive power compensation systems [5]. For above assumptions the power factor PF of the sinusoidal voltage source (VS) as function x_s and c_f is shown in Fig.2. The maximum value of PF occurs for c_f equal to approx. 0.5, but only for small values x_s. An increase in the inductance of the power supply system may significantly reduce the power factor. The influence of the stiffness of supply network is particularly visible at x_s > 0.05 i.e. when the inductance of the power supply system L_s is greater than 5% of the inductance L_1.

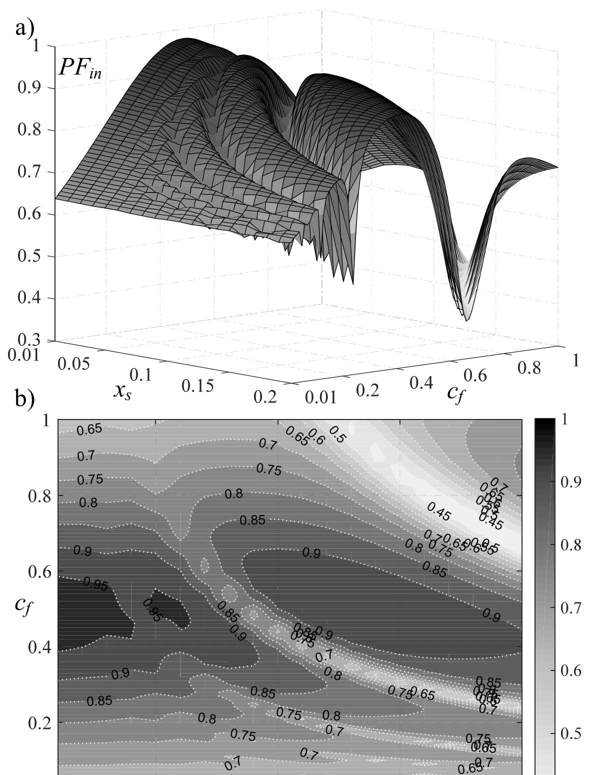

Fig.2. The power factor PF of supply voltage source in function x_s and c_f : a) 3D plot and b) contour plot

The largest distortion of the voltages and currents in the circuit are particularly visible when the power system becomes less rigid. As a result of these interactions, the power factor of the circuit may be much lower than expected.

For non-rigid power supply system capacitor bank to reactive power compensation causes an increase of currents and voltages distortion in the circuit. These distortion may be much greater than ones before compensation. This is due to the resonances occurring in the circuit [5]. For example, total harmonic distortion THD of current i_s and voltage u_p are shown respectively in Fig.3 and Fig.4. The peaks are characteristic. For current i_s maximum value of THD may be greater than 200%. Whereas for c_f and x_s equal to zero, it is only 12%.

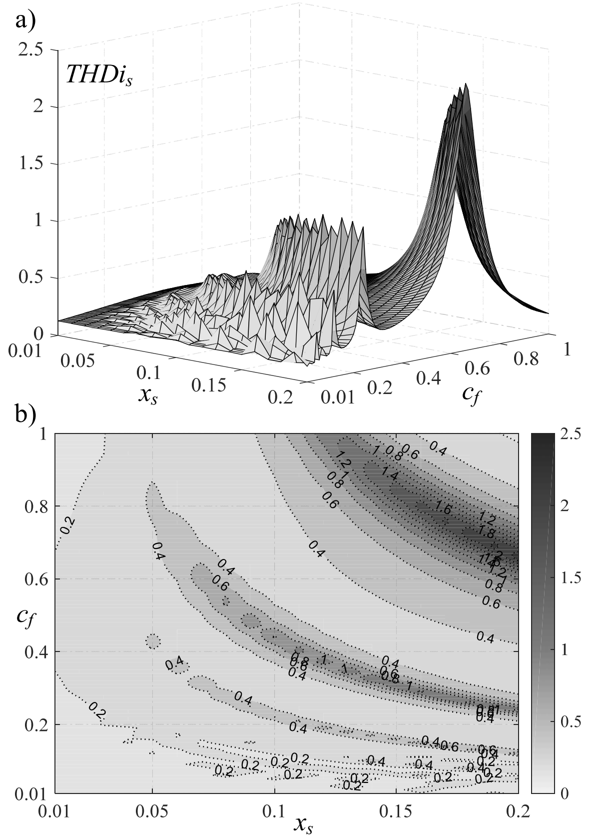

Fig.3. Total harmonic distortion of supply source current i_s in function x_s and c_f : a) 3D plot and b) contour plot

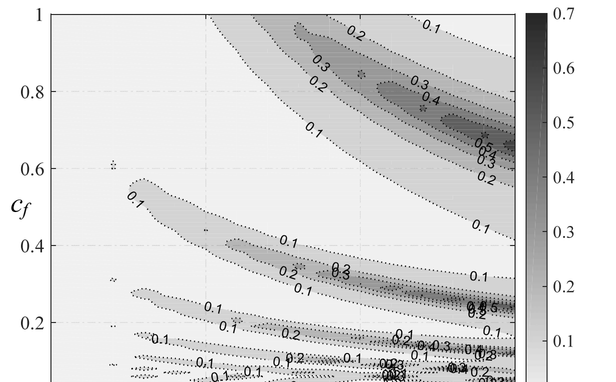

Fig.4. Total harmonic distortion of voltage u_p in function x_s and c_f

These distortion are observed also in current i_1. Total harmonic distortion THD of current i_1 is presented in Fig.5. The values of this coefficient are much smaller than for current i_s (Fig.3), and its value may only reach approx. 35%. The THD fluctuation for voltage u_b may be equal to approx. 20%, whereas without power compensation THD of voltage u_b is constant and equal to 47%.

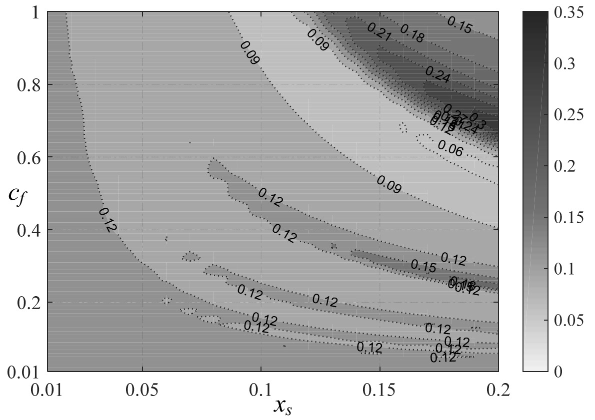

Fig.5. Total harmonic distortion of current i_1 in function x_s and c_f

Analysis of example currents and voltages waveforms in circuit

The total harmonic distortion THD of voltages and currents waveforms may be reduced if inductance L_f is connected in series with a capacitor C_f. Depending on the resonant frequency of such LC circuit higher harmonics are reduced [5].

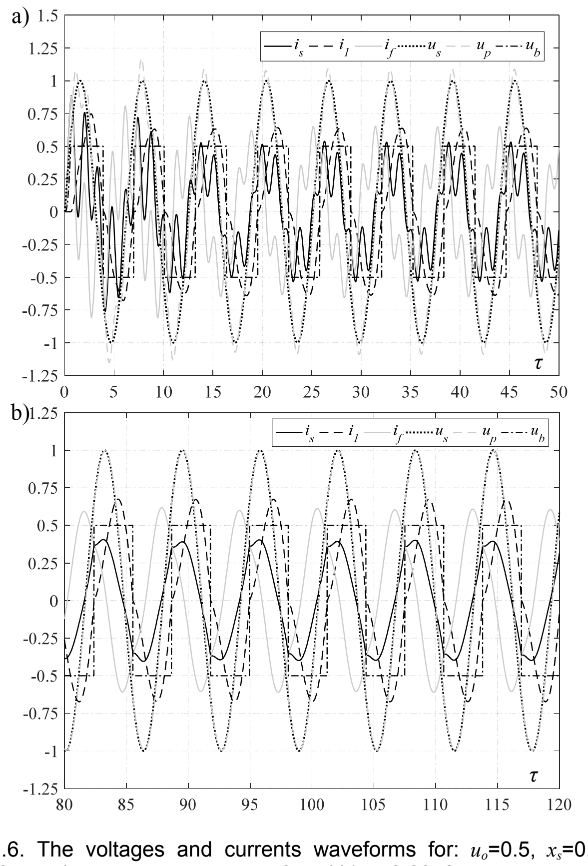

Fig.6. The voltages and currents waveforms for: u_o=0.5, x_s=0.1, c_f=0.5 and different value x_f : a) x_f=0 and b) x_f=0.2378

The example waveforms obtained for parameters: c_f=0.5, x_s=0.1, r_s=r_1=r_f=0.01 and u_o=0.5 are shown in Fig.6a and Fig.6b, respectively for x_f=0 (i.e. without inductance L_f) and x_f=0.2378 (with inductance L_f). Parameter x_f was calculated for resonant frequency order n_r equal to 2.9. Significantly smaller distortions for waveforms in Fig.6b are observed. Whereas the transients after switching on the supply voltage become longer than for x_f=0. Therefore, obtained waveforms are shown only in steady state, achieved after approx. 13 cycles. The period for the adopted time scale τ is equal to 2π.

The values of THD for analysed waveforms are presented in Table 1. For x_f=0.2378 the THD of current i_s decreased about ten times compared to x_f=0, whereas for current i_f approximately four times. The distortion of the voltage u_p is also much smaller than for x_f=0. The THD of current i_1 and voltage u_b are practically unchanged.

Table 1. Total harmonic distortion for currents and voltages waveforms in circuit

x_f	THDi_s	THDi_1	THDi_f	THDu_s	THDu_p	THDu_b
0	0.485	0.121	0.402	0.000	0.084	0.473
0.2378	0.049	0.120	0.110	0.000	0.013	0.473

After taking into account the parameter x_f, the power factor PF is also improved at specific points of the analyzed circuit. The values of power factor PF and power factor of fundamental harmonics PF_1 are shown in Table 2. These were measured at the voltage source terminals (PF_{in}, PF_{1in}), the PCC point (PF_{PCC}, PF_{1PCC}) and the input terminals of nonlinear load (PF_{load}, PF_{1load}). The power factor PF significant increased for x_f=0.2378 in voltage source VS and PCC point. The power factor of nonlinear load PF_{load} increases slightly. After taking into account parameter x_f the power factor of the fundamental components don't change significantly.

Table 2. The power factor PF and power factor for fundamental components PF_1 in different part of circuit

x_f	PF_{in}	PF_{PCC}	PF_{load}	PF_{1in}	PF_{1PCC}	PF_{1load}
0	0.9000	0.8944	0.8630	1.0000	0.9991	0.9687
0.2378	0.9893	0.9827	0.8707	0.9906	0.9840	0.9739

The obtained power factor results are close to unity for the voltage source VS and PCC point. The value of this coefficient for nonlinear load remains practically constant, both when inductance L_f in a circuit occurs or not.

Power distribution in circuit

The distribution of active and reactive power in analysed circuit was carried out in MATLAB/Simulink system. The total active and reactive power were calculated following:

$$P = \frac{1}{T}\int_0^T U_{(t)} \cdot I_{(t)} dt\,; \quad Q = \frac{1}{\omega T}\int_0^T U_{(t)} \cdot \left(\frac{dI_{(t)}}{dt}\right) dt \tag{7}$$

The reactive power was defined as the product of voltage and current time derivative dI/dt and averaged over the period T [2]. The powers (7) may be written as sum of the power of first harmonic component and power of higher harmonics components:

$$P = P_{h1} + P_{hh}; \quad Q = Q_{h1} + Q_{hh} \tag{8}$$

where: P_{h1}, Q_{h1} – respectively the active and reactive power of the fundamental component; P_{hh}, Q_{hh} - respectively the active and reactive power of the sum of higher harmonics.

The power distribution in circuit was analysed for total power, first harmonic power and higher harmonics power. Calculating the total powers (P,Q) and the powers of the first harmonics (P_{h1},Q_{h1}), the powers of the higher harmonics (P_{hh},Q_{hh}) may be determined from (8). The power components were referenced to $E^2_s/\omega L_1$ and analysed using dimensionless variables. The analyse was carried out for the same parameters as previous section.

Figure 1a shows the distribution of reactive power in circuit for x_f=0. The total reactive power and the power of first harmonic of the voltage source VS are close to zero. This is due to the reactive power compensation in circuit. The total reactive power of nonlinear load NL is also close to zero, whereas the reactive power of the first harmonic and the reactive power of the higher harmonics of this load have similar values, but opposite signs. The reactive power conversion of first harmonic into the reactive power of higher harmonics is observed. Next, the reactive power of higher harmonics of nonlinear load NL is fully transferred to the equivalent reactance of the supply circuit.

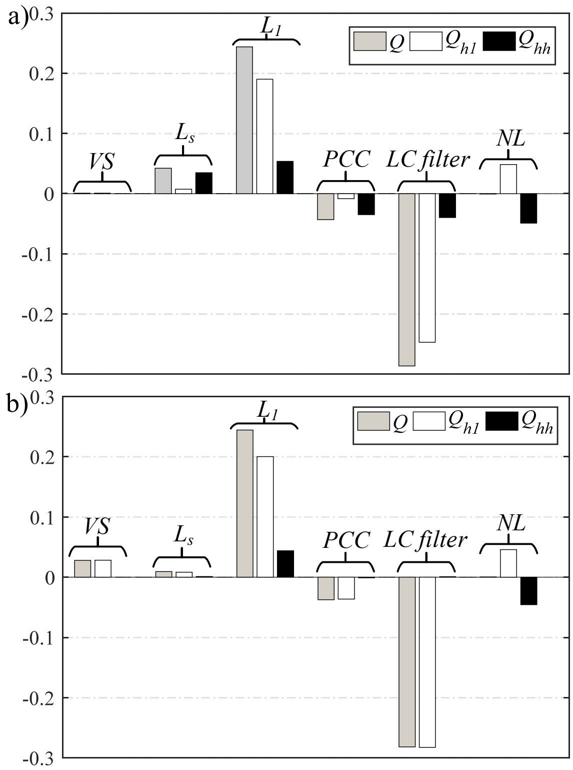

Fig.7. Distribution of reactive power in circuit for: a) x_f=0 and b) x_f=0.2378

For x_f=0.238 (Fig.7b) the reactive power of higher harmonics at the PCC point, inductance L_s and LC filter decreased. The reactive power of higher harmonics in nonlinear load does not change significantly in compare to x_f=0. Its value is comparable to the reactive power of first harmonic of load and reactive power of higher harmonics of the inductance L_1.

The parameter x_f has not a significant influence on active power in circuit. For x_f=0.238 very small changes of active power are observed in compare to x_f=0. Therefore, the distribution of active power shown in Fig.8 concerns only to the case if x_f=0.238. The active power of the higher harmonics on all elements is close to zero. In effect the total active power and the active power of the fundamental harmonic are comparable.

When a capacitor to reactive power compensation is used, the reactive power of higher harmonics increases in the circuit. The reactive power of higher harmonics is dissipated in all parts of circuit, excluding the voltage source, that is sinusoidal. After taking into account inductance L_f, this power is reduced to zero in selected elements and points of circuit. The power of higher harmonics of nonlinear load and inductance L_1 remained practically unchanged, even if inductance L_f is used.

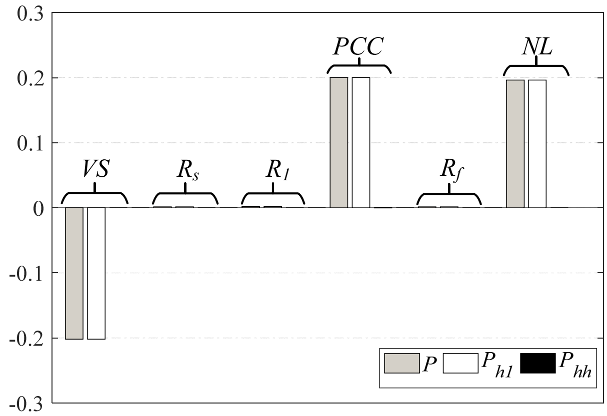

Fig.8. Distribution of active power in circuit for x_f = 0.2378

Conclusion

The model of circuit with nonlinear load and LC passive filter enabled quantitative analysis of power conversion phenomena and harmonics propagation. The analyses confirm influence of the mains inductance on increase of currents and voltages distortion in circuit.

The analyses indicate need to take into account the additional series inductance for capacitor banks in analysis of power factor and total harmonic distortion in circuit. The inductance of LC passive filter selected for 2.9th harmonic frequency order (in close to 3rd harmonic) allowed to significantly reduce power conversion phenomena occurring in the circuit. This inductance should be also taken into account in the analysis of other higher harmonics.

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REFERENCES

[1] Singh B., Chandra A.: Power Quality – Problems and Mitigations Techniques, John Wiley & Sons Ltd, 2015
[2] M. Wciślik: Powers Balances in AC Electric Circuit with Nonlinear Load, IEEE 2010
[3] R. Klempka: Designing a group of single-branch filters taking into account their mutual influence, Archives of electrical engineering, 2014, s. 81 – 91
[4] M. Włas: Engineering design of passive filter structures, Zeszyty Naukowe Wydziału Elektrotechniki i Automatyki Politechniki Gdańskiej Nr 28, s. 143-148, 2010
[5] A. Lange and M. Pasko: Selected methods of improving electrical energy quality with LC systems, Gliwice: Wydawnictwo Politechniki Śląskiej, 2015
[6] C. S. Mboving, Z. Hanzelka and R. Klempka: Different approaches for designing the passive power filters, Przegląd Elektrotechniczny, 11 2015, s. 102-108
[7] IEEE Recommended Practices and Requirements for Harmonic Control in Electrical Power Systems, IEEE Std 519-1992, 15/2004