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A Novel Method for Electricity Price Determination in Deregulated Markets

***Abstract**. The aim of the Dynamic Economic Emissions Dispatch (DEED) is to determine the optimal output of committed generating units whilst minimizing the units' fuel costs and emissions without violating practical power system operational constraints. In a deregulated market environment, the objective changes from solely minimizing fuel costs and emissions to include the maximization of the Independent System Operator's (ISO) profit. This formulation is known as the Profit Based Dynamic Economic Emissions Dispatch (PBDEED). In this paper, the PBDEED problem is investigated for the Nigerian electricity market which is a recently liberalised market. The model is solved in the Advanced Interactive Multidimensional Modelling System (AIMMS) environment using the price penalty factor approach and a comparison is made with the weighted sum approach. Obtained results indicate the suitability of our developed model.*

***Streszczenie.** Celem artykułu jest określenie optymalnego wyjścia powiązanych jednostek generatorów przy minimalizacji kosztów paliwa I emisji zanieczyszczeń bez wymuszania zmian w system,ie energetycznym. Przedstawiono problem PBDEED (Bazujący na zysku dynamiczny rozsył energii uwzględniający emisję zanieczyszczeń). (**Nowa metoda określania cen energii na zderegulowanym rynku**)*

Keywords: Dynamic Economic Emissions Dispatch, Profit Based Dynamic Economic Emissions Dispatch, Independent System Operator,
Słowa kluczowe: ekonomiczny rozsył energii, emisja zanieczyszczeń, ceny energii, zderegulowany rynek.

Introduction

The desire for reliable optimal operation of the power system at minimal cost has been the predominant preoccupation of power system operators and planners. The Dynamic Economic Emissions Dispatch (DEED) mathematical problem is one of such integral mathematical problem in power systems that seek to minimize the fuel costs of thermal generating units and minimize their emissions [1], [2]. This minimization of both fuel costs and emissions is done subject to the power balance constraint; which compels the sum of the total power generated by all generators to satisfy the total demand [3]. Other constraints often incorporated include ramp rate constraints [3], generator output limits [4], valve point effects [5], transmission line costs [6] etc. The decision variable is the output of the thermal generating units [7]. Topical research trends on DEED are concerned with applications in power systems including renewables [8], [9], [10], [11], [25] or the development of novel bio-inspired solution algorithms such as group based genetic algorithms [12], artificial bee colony algorithms [13], recurrent neural networks [14], Egyptian vulture algorithm [15] amongst others. Some research works combine two or more solution algorithms and deploy them to the DEED problem like [16] where the Lambda and Artificial Bee Colony Optimization are combined and [17] which combines the artificial bee colony algorithm and the golden section search method.

Recently power system operations in most nations have adopted deregulation [3]. Deregulation's effect varies depending on the state of the power system prior to deregulation. For some power systems, there is sufficient generating capacity and thus deregulation serves as a competition trigger which ideally leads to price reduction. For power systems with inadequate generating capacity [26], [28], [29] deregulation serves to spur investments which should lead to increased generating capacity and eventually to price reductions. In essence, the goal of deregulation is to improve system operations, increase efficiency and service delivery. In a deregulated clime, the DEED problem changes to Profit Based DEED [3] and is another topical area of research interest. The GENCO's foremost objective is to maximize profit and minimize emissions with the option of not supplying the total requested demand. Other practical operating constraints can be incorporated. Deregulation has been achieved in many nations of the world being first introduced in the United States in 1970's, Europe in the 1980's and most recently in Sub Saharan Africa. In 2005, Nigeria moved to deregulate her electric power industry which had been characterised by inefficiency and frequent service interruptions. This led to the introduction of the Electric Power Sector Reform (EPSR) Act 2005 as the first step towards Power sector reform [18]. Subsequent steps saw the unbundling of the Nigerian power utility into eighteen independent companies made up of 11 distribution companies, 6 generating companies and a single transmission company [18].

Since the Nigerian power system was always experiencing frequent blackouts and service disruptions, deregulation was espoused as a sure means of attracting investment in the power sector [18]. Towards this end, the Multi Year Price Order (MYTO) was introduced [19]. MYTO is a price model that claims to combine the benefits of both price cap and incentive based prices. The overreaching goal is to set optimal prices that allows for proper expansion and funding of the power system. However, the adoption of MYTO and electricity deregulation in Nigeria has not yielded the anticipated benefits [20]. Market participants' claim that the price does not allow for profitable operations [20]. A key motive behind this research is to determine the optimal energy price under a deregulated environment that will allow for profitable operations. This paper therefore proposes a PBDEED model for the Nigerian electricity market. Price Penalty Factors (PPF) are used to solve the mathematical model. Although PPF's have been used in conventional economic dispatch formulations [14], [21], [22], their formulation for PBDEED is not in the literature. The optimal output of the generators is determined and furthermore the optimal energy price required for profit is also determined. The model is solved using AIMMS [23]. To the best of our knowledge, no research work has attempted to model the PBDEED problem for the Nigerian electricity market and determine the optimal energy price for GENCO's to make a profit. This research work therefore contributes to the literature in the following major ways:

(i) The formulation of the PBDEED for a practical deregulated electricity market including transmission line losses

(ii) Solution of the multi-objective PBDEED with various price penalty factors and comparison of the various price penalty factors with the weighted sum approach. The CONOPT solver on the AIMMS platform is used to solve the resulting mathematical model.

(iii) Investigating the effect of various weighting factors on key power system parameters.

(iv) Determination of the optimal energy price necessary for system participants to be profitable.

The remainder of this paper is organised thus: In the next section, the mathematical formulations for the PBDEED is detailed after which the solution methodology used for the solving the resulting mathematical model is given. Results and discussions are given next, after which the paper is concluded.

PBDEED Mathematical Formulation

The first objective function of the PBDEED is to maximize the GENCO's profit. This is defined as the difference between the revenue accrued from electricity sales and fuel costs of thermal generators. This is given in equation (1). The second objective function is to minimize harmful emissions of the thermal generators and is given by equation (2). The complete mathematical formulation is given below [3]:

(1) $$\max \sum_{t=1}^{T}\sum_{i=1}^{I} (\lambda \times P_{i,t}) - \sum_{t=1}^{T}\sum_{i=1}^{I} C_i(P_{i,t})$$

(2) $$\min \sum_{t=1}^{T}\sum_{i=1}^{I} E_i(P_{i,t})$$

With

(3) $$C_i(P_{i,t}) = a_i + b_i P_{i,t} + c_i P_{i,t}^2$$

(4) $$E_i(P_{i,t}) = e_i + f_i P_{i,t} + g_i P_{i,t}^2$$

subject to the following network constraints:

(5) $$\sum_{i=1}^{I} P_{i,t} \le D_t + loss_t$$

(6) $$P_{i,\min} \le P_{i,t} \le P_{i,\max}$$

(7) $$-DR_i \le P_{i,t+1} - P_{i,t} \le UR_i$$

where

(8) $$loss_t = \sum_{i=1}^{I}\sum_{k=1}^{K} P_{i,t} B_{i,k} P_{k,t}$$

$P_{i,t}$ is the power generated from generator *i* at time *t ;* C_i is the fuel cost of generator*i* ; E_i is the emissions for generator *i* ; D_t is the total system demand at time *t* ; $loss_t$ is the total system losses at time *t* ; λ is energy price/price; $P_{i,min}$ and $P_{i,max}$ are the minimum and maximum capacity of generator *i* respectively ; DR_i abd UR_i are the maximum ramp down and up rates of generator *i* respectively; a_i, b_i and c_i are the fuel cost coefficients of generator *i* respectively; e_i, f_i and g_i are the emission cost coefficients of generator *i* *r*espectively; $B_{i,k}$ is the i*k* th element of the loss coefficient square matrix of size *I*; I and T are the total number of thermal generators and the total generating scheduling horizon respectively .

Both the fuel costs and emissions function are represented by quadratic functions (equations (3) and (4)). Equations (5)-(7) are the constraints of the mathematical model. Equation (5) depicts the power balance constraint under PBDEED. The GENCO's have the option of supplying less than the required demand if this will lead to maximal profits. Equation (6) compels the output of the thermal generators to be within allowable practical limits whilst equation (7) ensures that the generator output within consecutive time intervals is within allowable limits. Equation (8) gives the total transmission line losses which are represented using the B loss coefficient method. The B loss coefficient method is the most common method for transmission line calculations in which the power network losses are a quadratic function of generator outputs [30]. This method is underpinned by assumptions that lead to a slight loss of accuracy, however the methods is still the most widely used method [30].

Solution Methodology

The resulting mathematical problem (PBDEED) is a multi-objective problem which can be transformed into a single objective function using either the price penalty factor technique or the weighted sum technique. In this work, the aim is to investigate various price penalty factors and determine their effect on the PBDEED problem. Results obtained from the price penalty factors are benchmarked against results obtained via the weighted sum approach method. Four price penalty factors are investigated in this work. They are:

- The Max-Max price penalty factor
- The Min-Max price penalty factor
- The Max-Min price penalty factor
- The Min-Min price penalty factor.

The four price penalty factors are represented by equations (9)-(12) respectively.

(9) $$W_i = \frac{(a_i + b_i P_{i,\max} + c_i P_{i,\max}^2) - (\lambda \times P_{i,\max})}{e_i + f_i P_{i,\max} + g_i P_{i,\max}^2} \quad (₦/lb)$$

(10) $$W_i = \frac{(a_i + b_i P_{i,\min} + c_i P_{i,\min}^2) - (\lambda \times P_{i,\min})}{e_i + f_i P_{i,\max} + g_i P_{i,\max}^2} \quad (₦/lb)$$

*11) $$W_i = \frac{(a_i + b_i P_{i,\max} + c_i P_{i,\max}^2) - (\lambda \times P_{i,\max})}{e_i + f_i P_{i,\min} + g_i P_{i,\min}^2} \quad (₦/lb)$$

(12) $$W_i = \frac{(a_i + b_i P_{i,\min} + c_i P_{i,\min}^2) - (\lambda \times P_{i,\min})}{e_i + f_i P_{i,\min} + g_i P_{i,\min}^2} \quad (₦/lb)$$

The objective function when using price penalty factor is:

(13) $$\min \left[\sum_{t=1}^{T}\sum_{i=1}^{I} [C_i(P_{i,t}) - (\lambda \times P_{i,t})] + \sum_{t=1}^{T}\sum_{i=1}^{I} W_i \times E_i(P_{i,t}) \right]$$

where W_i is the price penalty facto in *₦/lb,* whilst the objective function using the weighted sum approach is:

(14) $$\min \left[w \sum_{t=1}^{T}\sum_{i=1}^{I} [C_i(P_{i,t}) - (\lambda \times P_{i,t})] + (1-w) \sum_{t=1}^{T}\sum_{i=1}^{I} E_i(P_{i,t}) \right]$$

where *w* and (*1-w*) and are the weighting factors and the condition for both weighting factors is that:

(15) $$w + (1-w) = 1$$

Table 1 gives the data for thermal generators in Nigeria. The fuel cost coefficients, emission coefficients, and generator limits are actual data for the Nigerian system. The ramp rate (up and down), B loss coefficients and load profile are adapted from [3]. T=24 and I=4. The load profile is given in Fig.1 and B loss coefficient is given in eq. (16).

(16) $$B = 10^{-4} \times \begin{bmatrix} 0.420 & 0.051 & 0.045 & 0.057 \\ 0.051 & 0.180 & 0.039 & 0.048 \\ 0.045 & 0.039 & 0.195 & 0.051 \\ 0.057 & 0.048 & 0.051 & 0.213 \end{bmatrix} \quad per \quad MW$$

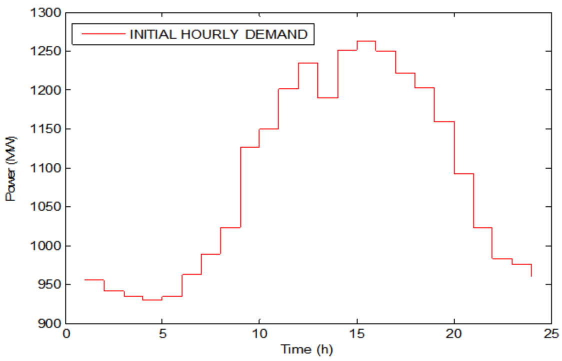

Fig. 1. Load Profile

The model is solved using AIMMS [23], [24], [27]. AIMMS is an Algebraic Modelling Language (AML) that allows for the solution of mathematical optimization problems. The problems can be represented in an algebraic format and solved using an assortment of various solvers. The CONOPT solver is the solver of choice for this application.

Results and Discussions

There are four PPF investigated. Table 2 shows the results of the PBDEED formulation for the Nigerian system with the average energy price/price of N 22.54863/kWh according to the MYTO [19]. Results shown are the total profits for the GENCO's, fuel cost, emissions, total power generated and total transmission line losses. From Table 2, it is obvious that there is no difference between results obtained by any of the PPF and the weighted sum approach. However, it is glaring that the price doesn't allow the GENCO's to be profitable as they are operating with a total monetary loss of N819177 (negative profit). This is therefore the reason why GENCO's complain that they do not make a profit and are therefore unable to invest in much needed power system infrastructure. It is therefore necessary to investigate at what price/energy price the GENCO's are able to make a profit. To determine this, a simple sensitivity analysis of the PBDEED mathematical model with respect to the electricity price is performed. The motive is to determine at what price the GENCO's flip from a loss to a profit. From Table 3 it is shown that at N 63 the GENCO's operate at a monetary loss however they make profit when the price is increased to N 64 (the exception is for the Min/Min price factor where the GENCO's flip from loss to profit at N 64 to N 65). Comparing all four PPF's it shows that the Min/Max price factor returns the least monetary loss and highest profit. However, at N 64/N 65 it will be impossible for GENCO's to fund expansion projects (the resulting profit is a pittance), therefore we investigate what happens when the energy price/price is increased to N 150. Again the Min/Max price factor returns the highest profit, lowest fuel cost and lowest fuel emissions. The energy generated and losses are equal amongst all four PPF's. For benchmark purposes, all four PPF's are compared with the weighted sum approach (last row of Table 2, Table 3 and Table 4). Furthermore, the effect of varying weighting factors on key power system parameters is investigated. The key power system parameters include the profit, fuel cost, emissions, total energy generated and total losses. This analysis is only done when the energy price/price is increased to N 150 and these results are shown in Figure 2, Figure 3 and Figure 4. From Figure 2 and Figure 3, it is shown that as the weighting factor for profit increases, the amount of emissions reduces. This is to be expected from the nature of equations (14) and (15) as one objective increases, the other objective reduces. Figure 4 shows that the energy generated follows the trend of emissions (as the emissions is essentially a function of generated power).

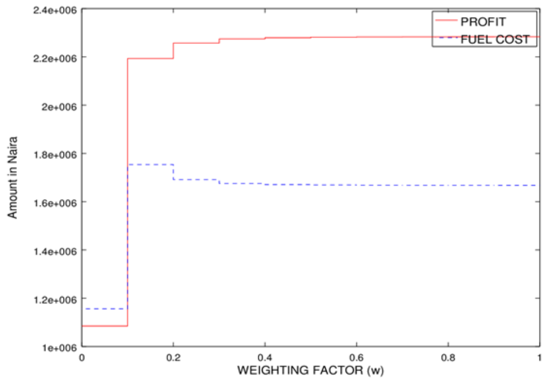

Fig. 2. Impact of Varying Weighting Factor on Profit and Fuel Cost

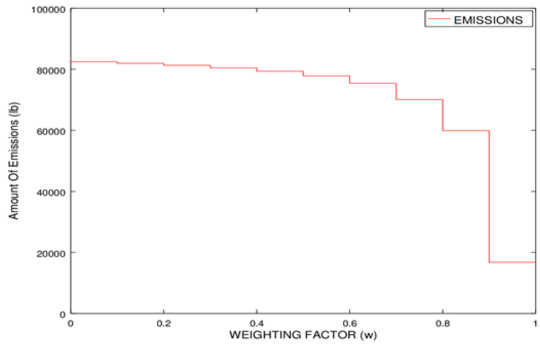

Fig. 3. Impact of Varying Weighting Factor on Emissions

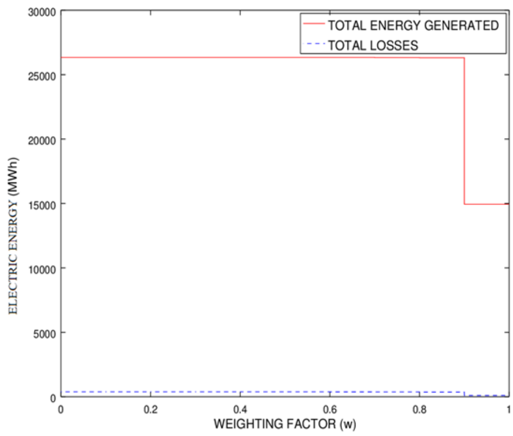

Fig. 4 Impact of Varying Weighting Factor on Total Energy Generated and Power Loss

Table 1. Modified Nigeria System Data

I	a_i (\$/h)	b_i (\$/MWh)	c_i (\$/MW^{2}h)	e_i (lb/h)	f_i (lb/MWh)	g_i (lb/MW^{2}h)	$P_{i,\min}$ (MW)	$P_{i,\max}$ (MW)	DR_i (MW/h)	UR_i (MW/h)
1	6929	7.84	0.13	13.8593	0.32767	0.00419	137.5	550	120	80
2	525.74	6.13	1.2	13.8593	0.32767	0.00419	75	300	90	50
3	1998	56	0.092	40.2669	-0.54551	0.00683	135	540	100	65
4	12787	13.1	0.031	40.2669	-0.54551	0.00683	275	1100	90	50

Table 2. Optimal Results with Price of ₦ 22.54863/kWh

	PROFIT (PBDEED) (₦)	COST (PBDEED) (₦)	EMISSIONS (PBDEED) (lb)	POWER GENERATED (PBDEED) (MW)	LOSS (PBDEED) (MW)
Max/Max	-819177	1156053	16752	14940	101
Min/Max	-819177	1156053	16752	14940	101
Max/Min	-819177	1156053	16752	14940	101
Min/Min	-819177	1156053	16752	14940	101
w **=0.5**	-819177	1156053	16752	14940	101

Table 3. Optimal Results with Price of N63/kWh and N64/kWh

	PROFIT (PBDEED) (₦)	COST (PBDEED) (₦)	EMISSIONS (PBDEED) (lb)	POWER GENERATED (PBDEED) (MW)	LOSS (PBDEED) (MW)
Max/Max (₦63/₦64)	-18165 8171	1670931 1672859	91079 91323	26234 26266	395 396
Min/Max (₦63/₦64)	-8311 18015	1636524 1649457	78911 80784	25845 26054	368 378
Max/Min (₦63/₦64)	-19728 6470	1670155 1670155	91388 91388	26197 26197	394 394
Min/Min (₦64/₦65)	-40489 9095	1389757 1466065	46331 56716	21082 22695	228 271
w **=0.5** (₦63/₦64)	-14997 11300	1573114 1584561	66858 68322	24732 24935	327 333

Table 4. Optimal Results with Price of ₦150/kWh

	PROFIT (PBDEED) (₦)	COST (PBDEED) (₦)	EMISSIONS (PBDEED) (lb)	POWER GENERATED (PBDEED) (MW)	LOSS (PBDEED) (MW)
Max/Max	2275160	1677673	91642	26352	398
Min/Max	**2275780**	**1676977**	**91363**	**26352**	**398**
Max/Min	2275160	1677673	91642	26352	398
Min/Min	2275160	1677673	91642	26352	398
w **=0.5**	2281269	1669058	79361	26336	382

Conclusion

The PBDEED mathematical formulation is applied to the deregulated Nigerian electricity market. Four PPF's were formulated and tested on practical data with the weighted factor approach as a benchmark. Results indicate that the Min/Max PF returns the best performance as it had the highest profit, lowest fuel cost and lowest emissions. Using the developed formulation, obtained results further indicate that with the current energy price, the GENCO's are not making a profit and therefore will be unable to break even and fund expansion projects. Thus, deregulation will not have the needed effects. Experimental results indicate that only at N64/N65 will the GENCO's be profitable. This represents an energy increase of at least 188.89% which will still not be able to fund expansion projects for the GENCO's. The impact of varying the weighting factors on the profit, fuel cost, emissions, energy generated and losses were conducted and results show that an increase in GENCO's profit leads to a corresponding reduction in emissions and vice versa. Future work will include the investigation of Bid Based DEED incorporating transmission line costs for the Nigerian power system.

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