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Quality estimation for models of a generalized Wiener process

***Abstract**. In the paper, we study quality of statistical simulation of a generalized wiener process. In order to construct a model, we use spectral representation of a generalized Wiener process in the form of stochastic integral. The model is constructed with a given accuracy and reliability in the space of integrable functions. For estimation of the model quality we use estimates of the Hurst parameter, as well as estimates of correlation function of the increments of realizations obtained.*

Streszczenie.** W pracy badamy jakość statystycznej symulacji uogólnionego procesu Wienera. Do budowy modelu wykorzystujemy widmową reprezentację uogólnionego procesu Wienera, w postaci całki stochastycznej. Model budowany jest z określoną dokładnością i niezawodnością. Do oceny jakości modelu stosujemy oszacowania parametru Hursta oraz funkcji korelacji uzyskanych realizacji.*(Szacowanie jakości dla modeli uogólnionego procesu Wienera***).

Keywords: generalized Wiener process, Hurst parameter, simulation of sub-Gaussian processes.
Słowa kluczowe: uogólniony proces Wienera, parametr Hursta, symulacja procesów sub-gaussowskich.

Introduction

In recent years, the methods for statistical simulation of stochastic processes and fields are widely used in many areas of natural and social sciences, such as physics, engineering, meteorology, biology, sociology, mathematics, in which they provide the basis for a thorough analysis and decision making. Among the classes of random processes a special place is occupied by Wiener and generalized Wiener processes. The list of branches, where the methods of statistical simulation of Wiener and generalized Wiener processes (fractional Brownian motion) are applied, is expanding every day. Thus, the models of Wiener and generalized Wiener random processes are used in financial mathematics [1,2], in solving problems of computational mathematics [3,4], in queuing systems [3], in problems of traffic modelling in the theory of telecommunication networks [5-8]. In many scientific papers, the problems of reliability and accuracy of simulation of these processes in different functional spaces are considered [9-13].

In studies of stochastic models, it is advisable to use statistical simulation methods to obtained realizations of random variables and processes [11,23]. With the use of modern computing and information technologies, it is possible to build computational experiments to investigate the behaviour of necessary processes, both individual realizations and in average. When we use realizations of random processes, the quality of these realizations plays a very important role [14-18].

The purpose of this paper is to investigate the quality of statistical simulation of generalized Wiener processes under analysis of models of processes and phenomena, whose behaviour can be described with the use of such processes. In order to estimate the quality of statistical simulation methods the Hurst index estimate is used [19,20,24].

In the paper, we construct a model of generalized Wiener process with parameter $\alpha\in(0,2)$, which approximates such process with given reliability $1-\varepsilon$, $0<\varepsilon<1$ and accuracy $\delta>0$ in the space $L_p([0,T])$. In order to construct this model, we use representation of a generalized Wiener process in the form of stochastic integral [11,14]. Estimates of the model accuracy and reliability in different functional spaces were studied in [21,22,25,26].

Simulation of a generalized Wiener process with given reliability and accuracy in the space $L_p([0,T])$

Let (Ω,B,P) be a standard probability space, T be some parametric set $(T=[0,T],\ T=[0,\infty])$.

Definition 1. A stochastic process $\{W_\alpha(t),\ t\in T\}$ is called generalized Wiener process with parameter $\alpha\in(0,2)$, if it is a Gaussian process with $W_\alpha(0)=0$, $EW_\alpha(t)=0$, and correlation function $R(t,s)=1/2(|t|^\alpha+|s|^\alpha-|t-s|^\alpha)$.

Generalized Wiener process with parameter $\alpha\in(0,2)$ can be represented in the form of stochastic integral [20]:

(1) $$W_\alpha(t)=\frac{A}{\sqrt{\pi}}\left(\int_0^\infty \frac{\cos(\lambda t)-1}{\lambda^{\frac{\alpha+1}{2}}}d\zeta(\lambda)-\int_0^\infty \frac{\sin(\lambda t)}{\lambda^{\frac{\alpha+1}{2}}}d\eta(\lambda)\right),\quad t\in[0,T],$$

where: $\zeta(\lambda)$, $\eta(\lambda)$ are independent real standard Wiener processes with $E\zeta(\lambda)=E\eta(\lambda)=0$, $E(d\zeta(\lambda))^2=E(d\eta(\lambda))^2=d\lambda$, and

(2) $$A^2=\left\{\frac{2}{\pi}\int_0^\infty \frac{\cos(\lambda t)-1}{\lambda^{\alpha+1}}d\lambda\right\}^{-1}=\left\{-\frac{2}{\pi}\Gamma(-\alpha)\cos\left(\frac{\alpha\pi}{2}\right)\right\}^{-1}.$$

Let us consider the interval $[0,\Lambda]$, $\Lambda>0$ and represent the process $W_\alpha=\{W_\alpha(t),\ t\in[0,T]\}$ in the following form:

(3) $$W_\alpha(t)=W_\alpha(t,[0,\varepsilon])+W_\alpha(t,[\varepsilon,\Lambda])+W_\alpha(t,[\Lambda,\infty]),$$

where $0<\varepsilon<\Lambda$ and

(4) $$W_\alpha(t,[a,b])=\frac{A}{\sqrt{\pi}}\left(\int_a^b \frac{\cos(\lambda t)-1}{\lambda^{\frac{\alpha+1}{2}}}d\zeta(\lambda)-\int_a^b \frac{\sin(\lambda t)}{\lambda^{\frac{\alpha+1}{2}}}d\eta(\lambda)\right).$$

Let $0=\lambda_0<\lambda_1<...<\lambda_M=\Lambda$ be such a partition of the interval $[0,\Lambda]$, that $\lambda_1=\varepsilon$. We shall construct a model of the process W_α in the form of the following series:

(5) $$S_M(t,\Lambda)=\frac{A}{\sqrt{\pi}}\left(\sum_{i=1}^{M-1}\frac{\cos(\lambda_i t)-1}{\lambda^{\frac{\alpha+1}{2}}}\left(\zeta(\lambda_{i+1})-\zeta(\lambda_i)\right)-\sum_{i=1}^{M-1}\frac{\sin(\lambda_i t)}{\lambda^{\frac{\alpha+1}{2}}}\left(\eta(\lambda_{i+1})-\eta(\lambda_i)\right)\right)=\frac{A}{\sqrt{\pi}}\left(\sum_{i=1}^{M-1}\frac{\cos(\lambda_i t)-1}{\lambda^{\frac{\alpha+1}{2}}}X_i-\sum_{i=1}^{M-1}\frac{\sin(\lambda_i t)}{\lambda^{\frac{\alpha+1}{2}}}Y_i\right),$$

where: $\{X_i,Y_i\}$, $i=1,2,...,M-1$, are independent Gaussian random variables and $EX_i=EY_i=0$, $EX_i^2=EY_i^2=\lambda_{i+1}-\lambda_i$, $t\in[0,T]$, $M\in N$.

Definition 2. The model $S_M=\{S_M(t,\Lambda),\ t\in[0,T]\}$ approximates the stochastic process $W_\alpha=\{W_\alpha(t),\ t\in[0,T]\}$ with given reliability $1-\varepsilon$, $0<\varepsilon<1$, and accuracy $\delta>0$ in the space $L_p([0,T])$, $p\geq 1$, if

(6) $$P\left\{\left(\int_0^T\left|W_\alpha(t)-S_M(t,\Lambda)\right|^p dt\right)^{\frac{1}{p}}>\delta\right\}\leq\varepsilon .$$

Since the deviation process $X_M(t,\Lambda)=W_\alpha(t)-S_M(t,\Lambda)$, $t\in[0,T]$, is a centred Gaussian random process, then it is a strictly sub-Gaussian and Theorem 1, given below, can be applied to this process. As a result, we shall obtain conditions, under which the model $S_M=\{S_M(t,\Lambda),\ t\in[0,T]\}$ approximates the process $W_\alpha=\{W_\alpha(t),\ t\in[0,T]\}$ with given reliability $1-\varepsilon$, $0<\varepsilon<1$ and accuracy $\delta>0$ in the space $L_p([0,T])$, $p\geq 1$.

Theorem model $S_M=\{S_M(t,\Lambda),\ t\in[0,T]\}$ approximates the process $W_\alpha=\{W_\alpha(t),\ t\in[0,T]\}$ with given reliability $1-\varepsilon$, $0<\varepsilon<1$ and accuracy $\delta>0$ in the space $L_p([0,T])$, $p\geq 1$, if

(7) $$\int_0^T\left(\frac{A^2}{\pi}\left\{\frac{t^2\lambda_1^{2-\alpha}}{2-\alpha}+\frac{2}{\alpha\Lambda^\alpha}+\frac{2t^2}{3}\sum_{i=1}^{M-1}\frac{(\lambda_{i+1}-\lambda_i)^3}{\lambda_i^{\alpha+1}}\right\}\right)^{\frac{p}{2}}dt<\delta^p\min\left\{\frac{1}{p^{\frac{p}{2}}},\frac{1}{\left(-2\ln\frac{\varepsilon}{2}\right)^{\frac{p}{2}}}\right\},$$

where: $M\in N$ and $0=\lambda_0<\lambda_1<...<\lambda_M=\Lambda$ is a partition of the interval $[0,\Lambda]$.

Corollary 1. The model $S_M=\{S_M(t,\Lambda),\ t\in[0,T]\}$ approximates the process $W_\alpha=\{W_\alpha(t),\ t\in[0,T]\}$ with given reliability $1-\varepsilon$, $0<\varepsilon<1$ and accuracy $\delta>0$ in the space $L_2([0,T])$ if

(8) $$\frac{A^2}{\pi}\left\{\frac{T^3\lambda_1^{2-\alpha}}{3(2-\alpha)}+\frac{2T}{\alpha\Lambda^\alpha}+\frac{2T^3}{9}\sum_{i=1}^{M-1}\frac{(\lambda_{i+1}-\lambda_i)^3}{\lambda_i^{\alpha+1}}\right\}<\frac{\delta^p}{2}\min\left\{1,\left(-\ln\frac{\varepsilon}{2}\right)^{-1}\right\},$$

where: $M\in N$ and $0=\lambda_0<\lambda_1<...<\lambda_M=\Lambda$ is a partition of the interval $[0,\Lambda]$.

If we take uniform partition of the interval $[0,\Lambda]$:

(9) $$\Delta\lambda=\lambda_{i+1}-\lambda_i=\frac{\Lambda}{M},\quad \lambda_1=\frac{\Lambda}{M},\quad \lambda_i=\frac{i\Lambda}{M},$$

then in the case of $L_2([0,1])$ the following conditions on model parameters Λ and M follow from Corollary 1:

(10) $$\frac{A^2}{\pi}\left\{\frac{\Lambda^{2-\alpha}}{3(2-\alpha)M^{2-\alpha}}+\frac{2}{\alpha\Lambda^\alpha}+\frac{2}{9}\sum_{i=1}^{M-1}\frac{\Lambda^{2-\alpha}}{i^{1+\alpha}M^{2-\alpha}}\right\}<\frac{\delta^2}{2}\min\left\{1,\left(-\ln\frac{\varepsilon}{2}\right)^{-1}\right\}.$$

Baxter estimation of Hurst parameter of a generalized Wiener process

Let $W_\alpha(t)$ be a generalized Wiener process with Hurst parameter $\alpha\in(0,2)$. Suppose that the Hurst parameter α is unknown, but such that $\alpha\in(0,\alpha^*]$, where $\alpha^*\in(0,2)$ is fixed [14]. Also suppose that the stochastic process $W_\alpha(t)$ is observed at the points $\{k/a_n,\ 0\leq k\leq a_n\}$ where: $a_n\in N$, $n\geq 1$; $a_n\to\infty$, $n\to\infty$.

Let $\Delta W_{k,n}^{(p)}$ be the p-th order increment of the fractional Brownian motion $W_\alpha(t)$, where p is a natural number, $k=0,...,a_n-1$. Thus, in particular:

(11) $$\Delta W_{k,n}^{(1)}=W_{\frac{k+1}{a_n}}-W_{\frac{k}{a_n}},$$

(12) $$\Delta W_{k,n}^{(2)}=W_{\frac{k+2}{a_n}}-2W_{\frac{k+1}{a_n}}+W_{\frac{k}{a_n}},$$

(13) $$\Delta W_{k,n}^{(3)}=W_{\frac{k+3}{a_n}}-3W_{\frac{k+2}{a_n}}+3W_{\frac{k+1}{a_n}}-W_{\frac{k}{a_n}},$$

.................................,

(14) $$\Delta W_{k,n}^{(p)}=\sum_{i=0}^{p}(-1)^i C_p^i W_{\frac{k}{n}+\frac{p-i}{n}}.$$

Let us consider sequences of Baxter sums for the generalized Wiener process $W_\alpha(t)$:

(15) $$\widehat{S}_n^{(p)}=a_n^{\alpha-1}\sum_{k=0}^{a_n-1}\left(\Delta W_{k,n}^{(p)}\right)^2,\ S_n^{(p)}=a_n^{\alpha-1}\widehat{S}_n^{(p)},\ n\geq 1.$$

Put

(16) $$V_p(k,\alpha)=\frac{1}{2}\sum_{i,j=0}^{p}(-1)^{i+j+1}C_p^iC_p^j\left|k+(i-j)\right|^\alpha,\quad k\geq 0,\ p\geq 1$$

In particular: $V_1(0,\alpha)=1$; $V_2(0,\alpha)=4-2^\alpha$; $V_3(0,\alpha)=15+3^\alpha-6\cdot 2^\alpha$.

The direct calculation makes it possible to obtain the following formulas for mean and variance of the random variable $\widehat{S}_n^p$, $p\geq 1$:

(17) $$E\widehat{S}_n^{(p)}=V_p(0,\alpha);$$

(18) $$\operatorname{Var}\widehat{S}_n^{(p)}=\frac{1}{n}\left(2V_p^2(0,\alpha)+4\sum_{k=1}^{n-1}\left(1-\frac{k}{n}\right)V_p^2(k,\alpha)\right).$$

Theorem 2. The statistics

(19) $$\widehat{\alpha}_n^{(1)}=\left(1-\frac{\ln S_n^{(1)}}{\ln a_n}\right),\ n\geq 1$$

is strongly consistent estimate of the Hurst parameter α.

In order to find the variance $\mathrm{D}\,\widehat{S}_n^{(1)}$, we shall use the following lemma.

Lemma 1. [15] Let $W_\alpha(t)$ be a fractional Brownian motion with the Hurst parameter $\alpha\in(0,\alpha^*]$. Then for $\alpha^*\in(0,2)$ the following inequality holds:

(20) $$\sup_{\alpha\in(0,\alpha^*]}\mathrm{D}\,\widehat{S}_n^{(1)}\leq\frac{D_1}{a_n},$$

where:

$$D_1 = \begin{cases} 2\left(3+2\zeta\left(4-2\alpha^*\right)\right),\ \alpha^* \in \left(0,\ \frac{3}{2}\right), \\ 2\left(3+2\left(1+\ln a_n\right)\right),\ \alpha^* = \frac{3}{2}, \\ 2\left(3+2\frac{a_n^{2\alpha^*-3}}{2\alpha^*-3}\right),\ \alpha^* \in \left(\frac{3}{2},\ 2\right), \end{cases} \tag{21}$$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s},\ s>1.$$

Similarly, we can show that statistics $\hat{\alpha}_n^{(p)} = \left(1-\ln S_n^{(p)} / \ln a_n\right)$, $p\geq 2$, are strongly consistent estimates of the Hurst parameter α.

Lemma 2. Let $W_\alpha(t)$ be a generalized Wiener process with the Hurst parameter α. Then the following inequality holds:

$$\sup_{\alpha\in(0,2)} D\hat{S}_n^{(p)} \leq \frac{D_p}{a_n}, \tag{22}$$

where:

$$D_p = 2\left(K_p + \frac{1}{2}L_p + \frac{1}{2}M_p^2\zeta(4p-4)\right), \tag{23}$$

$$V_p(m,\alpha) = \frac{1}{2}\sum_{i,j=0}^{p}(-1)^{i+j+1}C_p^i C_p^j \left|m+(i-j)\right|^\alpha, \quad k\geq 0 \tag{24}$$

$$K_p = \sup_{\alpha\in(0,1)} V_p^2(0,\alpha),\ L_p = \sup_{\alpha\in(0,2)} V_p^2(1,\alpha), \tag{25}$$

$$M_p = \sup_{\alpha\in(0,2)} \left|\alpha(\alpha-1)(\alpha-2)\cdot\ldots\cdot(\alpha-(2p-1))\right|, \quad p\geq 2 \tag{26}$$

$\zeta(\cdot)$ is a Riemann zeta function.

From the representation:

$$V_p(k-l,\alpha) = \frac{1}{2}\sum_{i,j=0}^{p}(-1)^{i+j+1}C_p^i C_p^j \left|k-l+(i-j)\right|^\alpha,\ k,l\geq 0 \tag{27}$$

and from Lemma 2 for $p=2$ and $p=3$ it follows that $V_2(0,\alpha)=4-2^\alpha$; $V_3(0,\alpha)=15+3^\alpha-6\cdot 2^\alpha$ and $V_2(1,\alpha)=7-4\cdot 2^\alpha+3^\alpha$; $V_3(1,\alpha)=26-16\cdot 2^\alpha+6\cdot 3^\alpha-4^\alpha$.

The estimates of coefficients in Lemma 2 for $p=2$ and $p=3$ are the following:

$$K_2 = \sup_{\alpha\in(0,2)} V_2^2(0,\alpha) = \sup_{\alpha\in(0,2)} \left(4-2^\alpha\right)^2 = 9, \tag{28}$$

$$L_2 = \sup_{\alpha\in(0,2)} V_2^2(1,\alpha) = \sup_{\alpha\in(0,2)} \left(7-4\cdot 2^\alpha+3^\alpha\right)^2 = 16, \tag{29}$$

$$M_2 = \sup_{\alpha\in(0,2)} \left|\alpha(\alpha-1)(\alpha-2)(\alpha-3)\right| = 0{,}563 \tag{30}$$

and

$$K_3 = \sup_{\alpha\in(0,2)} V_3^2(0,\alpha) = \sup_{\alpha\in(0,2)} \left(15+3^\alpha-6\cdot 2^\alpha\right)^2 = 100, \tag{31}$$

$$L_3 = \sup_{\alpha\in(0,2)} V_3^2(1,\alpha) = \sup_{\alpha\in(0,2)} \left(26-16\cdot 2^\alpha+6\cdot 3^\alpha-4^\alpha\right)^2 = 15^2, \tag{32}$$

$$M_3 = \sup_{\alpha\in(0,2)} \left|\alpha(\alpha-1)(\alpha-2)\cdot\ldots\cdot(\alpha-7)\right| = 129.738. \tag{33}$$

Taking into account that $\zeta(8-2\alpha)\leq\zeta(4)=\pi^4/90$ for $\alpha\in(0,2)$, we get the following inequality for $p=2$:

$$\sup_{\alpha\in(0,2)} D\hat{S}_n^{(2)} \leq \frac{2}{a_n}\left(9+\frac{16}{2}+\frac{1}{2}\cdot(0.563)^2\cdot\frac{\pi^4}{90}\right) \approx \frac{34.344}{a_n} \tag{34}$$

and noticing that $\zeta(12-2\alpha)\leq\zeta(8)=\pi^8/9450$ for $\alpha\in(0,2)$, we obtain the estimate in the case of $p=3$:

$$\sup_{\alpha\in(0,2)} D\hat{S}_n^{(2)} \leq \frac{2}{a_n}\left(100+\frac{225}{2}+\frac{1}{2}\cdot(129.738)^2\cdot\frac{\pi^8}{9450}\right) \approx \frac{1.733\cdot 10^4}{a_n}. \tag{35}$$

Theorem 3. Let $W_\alpha(t)$ be a fractional Brownian motion with the Hurst parameter α. Then $\hat{S}_n^{(p)} \to V_p(0,\alpha), p\geq 1$ with probability one as $n\to\infty$.

Fig. 1 shows the trajectories of the model for three values of the parameter α: the red trajectory $S_1(t)$ corresponds to the value $\alpha=0.6$, the blue one $S_2(t)$ corresponds to $\alpha=0.8$, and the green trajectory $S_3(t)$ corresponds to $\alpha=1.2$.

As it was expected, the greater α corresponds to the smoother trajectory of the process.

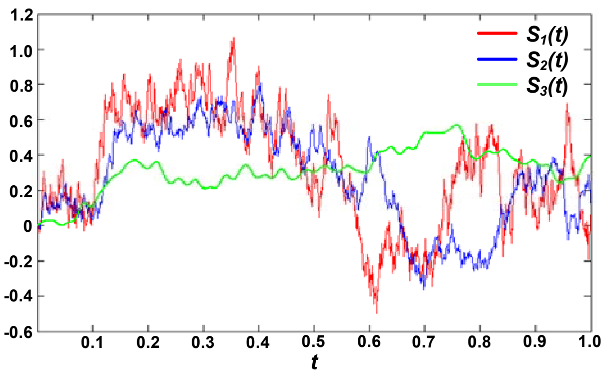

Fig. 1. Trajectories of the fractional Brownian motion

For trajectories, presented on Fig. 1, the estimates of the Hurst parameter α are found: for $S_1(t)$ the estimate is $\alpha=0.6$, for $S_2(t)$ the estimate is $\alpha=0.8$, and for $S_3(t)$ the estimate is $\alpha=1.2$.

Conclusions

In the paper, we suggest to use estimation of the Hurst parameter for studying the quality of statistical models. The model of generalized Wiener process is constructed with given reliability and accuracy in the space $L_p([0,T]),\quad p\geq 1$.

The results obtained in the paper are illustrated by example of simulation of generalized Wiener process in $L_2([0,1])$. The obtained estimates of the Hurst parameter confirmed the quality of the received realizations.

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