Tomasz RYMARCZYK[1,2], Katarzyna SZULC[3]

Research & Development Centre Netrix S.A.(1), University of Economics and Innovation (2)
Systems Research Institute, Polish Academy of Sciences (3)

doi:10.15199/48.2019.06.25

Minimization of Objective Function in Electrical Impedance Tomography by Topological Derivative

***Abstract**. The article presents the reconstruction of 2D objects studied using the topological derivative and level set function in electrical impedance tomography, which is a non-invasive imaging method in which an unknown physical object is examined using measurements on its edge. The internal distribution of conductivity is obtained on the basis of the measurements. The solution to the optimization problem is obtained by combining finite element methods and topological algorithms. The presented solution can be effectively used in applications based on electrical tomography.*

***Streszczenie.** W artykule przedstawiono rekonstrukcję badanych obiektów 2D z wykorzystaniem pochodnej topologicznej i funkcji zbiorów poziomicowych w elektrycznej tomografii impedancyjnej, która jest nieinwazyjną metodą obrazowania, w której nieznany obiekt fizyczny jest badany za pomocą pomiarów na jego krawędzi. Wewnętrzny rozkład konduktywności jest otrzymywany na podstawie pomiarów. Rozwiązanie problemu optymalizacji uzyskuje się przez połączenie metody elementów skończonych i algorytmów topologicznych. Prezentowane rozwiązanie może być skutecznie wykorzystywane w aplikacjach opartych na tomografii elektrycznej (**Minimalizacja funkcji celu w elektrycznej tomografii impedancyjnej za pomocą pochodnej topologicznej**).*

Keywords: Electrical Impedance Tomography, Inverse Problem, Topological Derivative
Słowa kluczowe: elektryczna tomografia impedancyjna; problem odwrotny, pochodna topologiczna

Introduction

Electrical impedance tomography (EIT) is a non-invasive problem of image reconstruction in which the distribution of conductivity in an object domain can be reconstructed using external voltage measurements. EIT is an imaging technique with existing and potential uses in engineering and medical problems. In this method, electric currents are injected into a conductive object through electrodes placed on its surface, and the resulting electrical potential on the electrodes is determined. In the opposite case, the objective function is minimized in an iterative procedure, using the measured and modeled data, the internal conductivity profile is calculated. Electric tomography can be used in industrial, medical and geophysical applications. Recently, there has been a dramatic increase in the field of chest imaging, with an emphasis on lung ventilation and heart rhythm monitoring. The finite element method was applied to domain modeling, and topological algorithms were used to solve the inverse problem [21]. The problem of topology optimization applied in the EIT is to find the conductivity distribution of the object, which minimizes the difference between electrical potentials obtained from measurements of electrodes at the boundary of the object and numerically simulated electrical potentials [3,4,8,14-16].

Topological Derivative

The optimization problem is usually defined as the minimization of a given activity [2,6,9-11,13,22]. Shape derivatives and topological derivatives have been included in the level determination methods to investigate problems related to shape optimization [17, 18]. The basic method of shape optimization is a topological derivative evaluated for a given functional shape defined in the geometric field and dependent on the classical solution of the elliptical problem of the limit of value. The topological derivative is defined as the first term of asymptotic expansion of a given functional shape with respect to a small parameter that measures the magnitude of single domain perturbation. Represents the variation of the functional shape when the domain is disturbed by openings, inclusions, defects or cracks. Topological methods are based on the differentiation of variational inequalities in relation to the operator's differential operator coefficients. This is sufficient to obtain a directional differentiation of variation inequalities in relation to limit variations in relation to changes in topology. When applying the inverse problems, partial differential equations are solved, which model the probing fields and which depend in one way or another on the shape. The shape functional can be written as an integral over the domain Ω or on the boundary ∂Ω.

Suppose that the function u(Ω;x) ∈ H1(Ω), x ∈ Ω is a solution:

$$\begin{cases} -\Delta u(x) = f(x), & x \in \Omega \\ u(x) = g(x), & x \in \Gamma_D \subset \partial\Omega \\ \partial_n u(x) = h(x), & x \in \Gamma_N \subset \partial\Omega \end{cases} \tag{1}$$

where $f \in L^2(\Omega)$, $g \in H^{1/2}(\Gamma_D)$, $h \in L^2(\Gamma_N)$ and $\partial\Omega = \Gamma_N \cup \Gamma_D$ boundary of Ω. Consider the shape functional

$$\Im(\Omega) = \int_\Omega J(x, u(\Omega; x))\, dx \tag{2}$$

where $J : \Omega \times \mathbf{R} \to \mathbf{R}$ is a function of class C^1 defined in $\Omega \times \mathbf{R}$ and depending on $u(\Omega;x)$ to the boundary value problem (1). The shape optimization is following:

$$\text{Find } \Omega^* \in U_{od} \text{ such, that } \Im(\Omega^*) = \inf_{\Omega \in U_{od}} \Im(\Omega) \tag{3}$$

In order to decrease the values of the shape functional $\Im(\Omega)$ wit can possibilities to change the shape of the domain Ω, changing of the boundary are governed by shape derivative and topological derivative.

Numerical algorithms

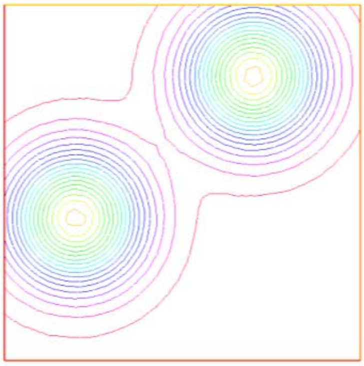
The forward problem was solved by the finite element method. The objective function is defined as the difference between the potential due to the current applied and the potential measured. The numerical iterative algorithm is a combination of level determination methods to track the evolution of stepped edges and finite element methods for speed calculation. Representation of the shape of the border and its evolution during the iterative process of reconstruction is achieved with the use of a fixed level method. In addition, various zero-level setting functions have been selected.

Numerical experiments

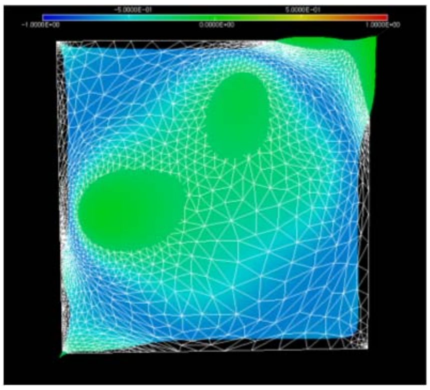
In the numerical part we consider a rectangular initial domain with a given applied current with 16 electrodes.

The point where the value of the topological derivatives is minimal stands for the location of a hidden object inside the domain. In the first example we consider the domain with one hidden inclusion placed at different points in the domain.

a)



b)



c)

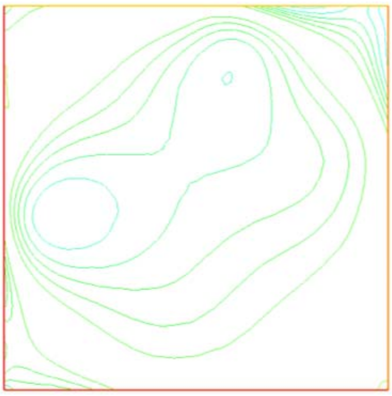


Fig.1. Two inclusions at points (-0.6,-0.2) and (0.2,0.6). (a) Reference domain, (b) Values of the topological derivative, (c) Resulting domain.

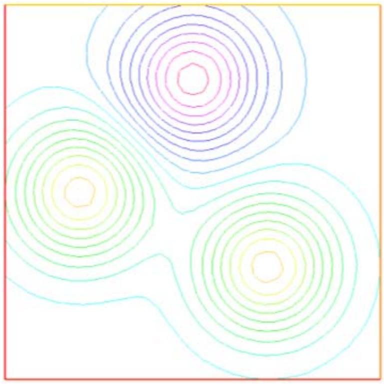
In the Fig.1(a) we have the reference domain (target), in (b) the values of the topological derivative and in (c) the result given by the algorithm. In this example we consider the domain with three hidden inclusion placed inside of the . In the Fig. 2(a) we have the reference domain (target), in (b) the values of the topological derivative and in (c) the result given by the algorithm.

In the second example we suppose that the current um is applied only on two opposite boundaries as in Fig. 3

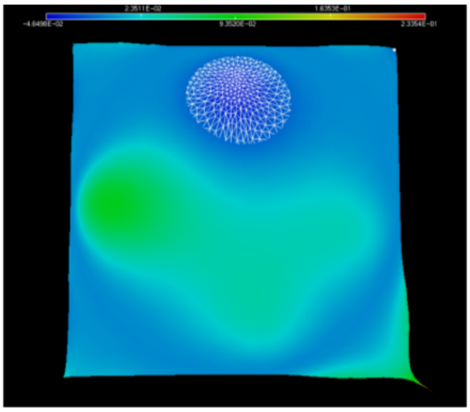
In this case we suppose that m = 8 and we reaped the procedure as previously. The result are presented for one, two and tree inclusions in the rectangular domain.

Example 1. In the first example we consider the domain with one hidden inclusion placed at point (0.4,0) in the domain . In the Fig. 4(a) we have the reference domain (target), in (b) the values of the topological derivative and in (c) the result given by the algorithm.

a)



b)



c)

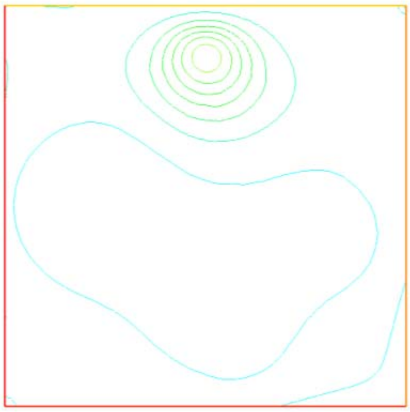


Fig. 2. Three inclusions at points (-0.6,0), (0,0.6) and (0.4,-0.4). (a) Reference domain, (b) Values of the topological derivative, (c) Resulting domain.

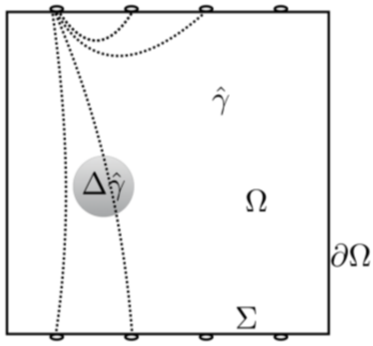
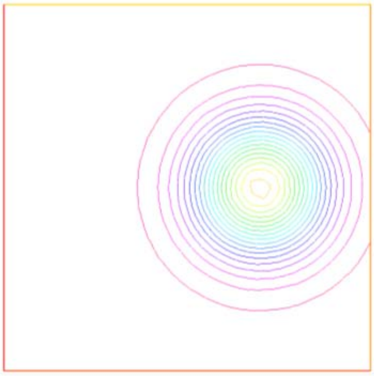
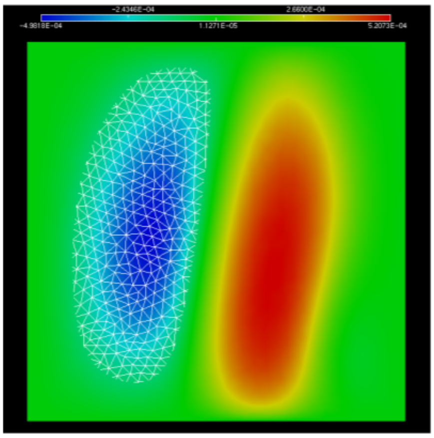


Fig. 3. Backgroud domain with an object inside. The electrodes are placed at opposite boundarys,

a)



b)



c)

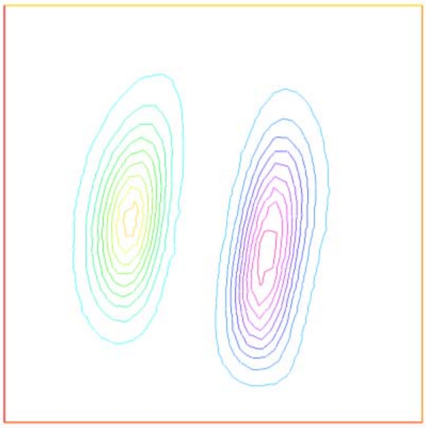
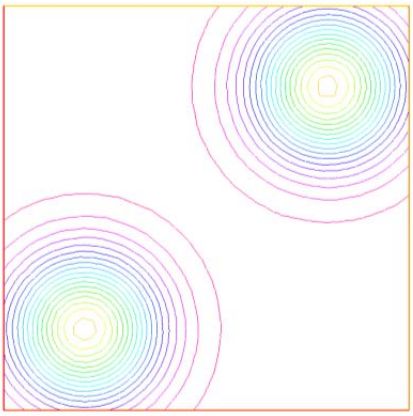
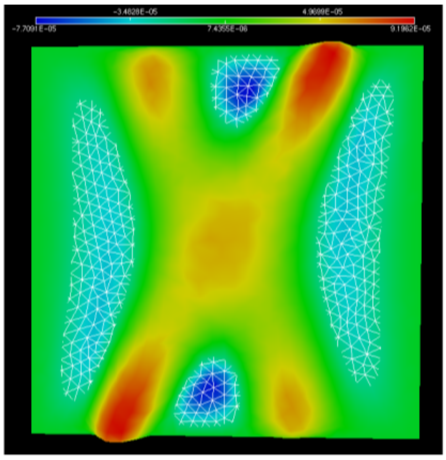


Fig. 4. One inclusion at point (0.4,0). (a) Reference domain, (b) Values of the topological derivative, (c) Resulting domain.

a)



b)



c)

Fig.5. Two inclusions at points (-0.6,-0.6), (0.6,0.6). (a) Reference domain, (b) Values of the topological derivative, (c) Resulting domain.

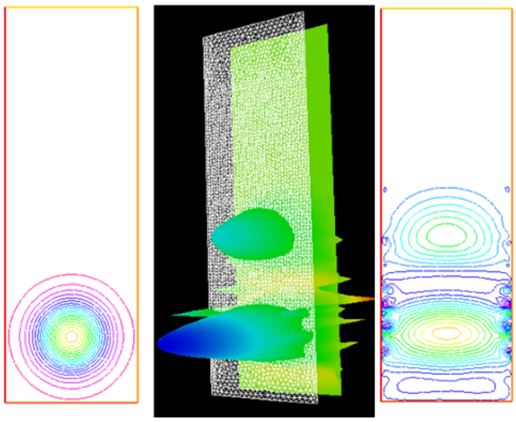


Fig.6. Inclusions in a rectangular model.

Example 2. In this example we consider the domain with two hidden inclusion placed inside of the . In the Fig.5(a) we have the reference domain (target), in (b) the values of the topological derivative and in (c) the result given by the algorithm. Figure 6 shows inclusions in a rectangular model.

Conclusion

This article proposes algorithms based on topological methods, improved methods of setting levels for more accurate and more stable results of reconstruction in solving the problem of reverse electrical impedance tomography. Numerical simulations and measurement experiments show that the proposed algorithm ensures stability and greater accuracy of reconstruction results compared to an algorithm based solely on deterministic methods. Topological algorithms were used to obtain images. It has been shown that the results depend on the initial values of the design variables. However, when certain properties of the plate being tested are known, the images are consistently better.

***Authors**: Tomasz Rymarczyk, Ph.D. Eng., University of Economics and Innovation, Projektowa 4, Lublin, Poland / Research & Development Centre Netrix S.A. E-mail: tomasz@rymarczyk.com; Katarzyna Szulc, Ph.D., Systems Research Institute Polish Academy of Sciences, Warsaw, Poland, kszulc@gmail.com.*

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