Łukasz KĘDZIERSKI[1,2]

Lab Maintenance and Development, Institute of Aviation, Al. Krakowska 110/114, 02-256 Warsaw (1)
Faculty of Electrical Engineering, Warsaw University of Technology, Pl. Politechniki 1, 00-661 Warsaw (2)

doi:10.15199/48.2019.05.43

High speed imaging method for rotational speed calibration

Abstract. *The paper introduces a method of rotational speed measurement calibration based on the image acquired by a high speed camera. Presented method proves to be superior to the commonly used strobe light method in both the accuracy and immunity to aliasing. Detailed calculations of the uncertainty of the method are going to be presented followed by the description and results of the experiment conducted in order to prove the high speed imaging calibration method accuracy. The method could be used for applications where very high precision of rotational speed measurement is critical. The development of the presented method has been performed in the Institute of Aviation in Warsaw in order to fulfill the very stringent requirements of the turbo-fan engines certification tests.*

Streszczenie*. Artykuł ten opisuje metodę kalibracji prędkości obrotowej bazującą na obrazie z kamery szybkoklatkowej. Zaprezentowana metoda udowadnia swoją wyższość nad powszechnie stosowaną metodą stroboskopową, zarówno pod względem dokładności, jak i odporności na zjawisko aliasingu. W artykule opisano dokładne wyliczenia niepewności pomiaru oraz pokazano wyniki testu przeprowadzonego w celu wykazania dokładności kalibracji prędkości obrotowej. Opisana metoda znajduje zastosowanie w systemach, w których precyzja pomiaru prędkości obrotowej jest krytyczna. Rozwój prezentowanej metody został przeprowadzony w Instytucie Lotnictwa na potrzeby testów certyfikacyjnych silników turbowentylatorowych.* ***(Kalibracja pomiaru prędkości obrotowej na podstawie obrazu z kamery szybkoklatkowej)***

Keywords: high speed imaging, rpm, rotational speed, calibration, uncertainty, accuracy.
Słowa kluczowe: kamery szybkoklatkowe, rpm, prędkość obrotowa, kalibracja, niepewność, dokładność.

Introduction

Rotational speed calibration poses a general problem to users working with motors as there is no direct measurement standard (in a sense of a true embodiment of the unit [1]) one can rely upon like in the case of weight, length, time and so many other measurements. In such a case an indirect standard is needed. It is natural to use time as a measurement standard for rotational speed sensor calibration as the SI unit for rotational speed is radian per second [2]. However, it is common to use revolution per minute (rpm) as the unit for rotational speed (the calibration method proposed in this paper uses rpm as the preferred unit). It is an industry standard to use strobe light for the purpose of calibrating the sensors measuring rotational speed. This method is based upon the fact that when periodically moving object (such as a rotating shaft of the motor) is illuminated by a strobe light of frequency equal to the frequency of the object rotation, then the object appears to be stationary [3]. If the frequency of the strobe light can be accurately measured then the speed of rotation can be calibrated with the equal accuracy. The strobe light method suffers from two major drawbacks though. First is the aliasing - object appears to be stationary at a given frequency but it will look exactly the same when the frequency of the strobe light is doubled, tripled and so on. Second drawback is the accuracy of stroboscopes - even the top quality products when operating in rpm mode will introduce an inaccuracy of 0.02% ±1 rpm when measuring speeds above 1000 rpm [4]. High speed imaging method proposed in this paper doesn't suffer from the aliasing and is more accurate thanks to the very precise internal clock.

Test stand

The test stand consists of VFD (Variable Frequency Drive) controlled asynchronous electric motor driving a shaft through a gearbox. The shaft of which the speed is measured can rotate within 500÷12000 rpm range. The rotational speed is measured by a speed encoder based on VRS (Variable Reluctance Speed) sensor working on 16-notch geared wheel (Fig. 1). For the sake of accuracy the measurement of the rotational speed is realized by the PAC (Programmable Automation Controller) which can run highly advanced algorithms at rates much faster than traditional PLCs [5]. Through decades PLCs proved to be a reliable platform for both control of critical processes and acquisition of key measurements [6], however with growing capabilities of PACs the decision has been made to use such a controller for the rotational speed measurement.

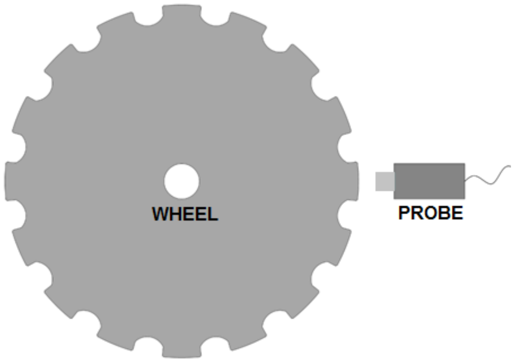

Fig. 1 - rotational speed measurement assembly

For the sake of the high speed imaging calibration method markers has been added on the rotor and on the stator. Camera itself was placed in such a way to look axially on the end of the shaft on which the marker has been drawn. High speed imaging requires intense lighting – LED lights providing total light intensity of 50 000 lumen has been used that enable working with frame rates up to 100 000 frames per second. High speed cameras used are capable of acquiring the image with frame rates up to 1 000 000 frames per second (which could further increase the accuracy of the method) however the higher the shutter speed is the more light needs to be provided.

Method description

The high speed imaging calibration method consist of five simple steps. First, the rotating object of which the rotational speed is measured needs to be accelerated to the desired speed. Second, after the speed has stabilized, the reading is taken using the speed sensor that is being calibrated. Third, during the time the reading is taken, the high speed camera looking at the markers on rotor and stator has to be triggered. Fourth, the image acquired by the high speed camera needs to be analyzed. For each revolution frame number at which the markers on the rotor and stator align should be established. Fifth, based on the frame numbers, series of rotational speed readings has to be calculated. The formula for speed calculation is the following:

$$(1)\quad \omega_i = \frac{60 \cdot N}{t \cdot (f_{N+i} - f_i)}\ [rpm], i = 0,1,\cdots,(n-N)$$

where: ω_i - i-th calculation of the rotational speed, N - number of revolutions taken for calculation of each rotational speed measurement, t – duration of a single frame in seconds f_i - number of the frame at which the markers on the rotor and stator aligned for i-th revolution, n - quantity of the frame readings taken.

Let us consider an example when the high speed camera has acquired twenty full revolutions. Through calculation we have established that we would get the smallest uncertainty (more on calculating the uncertainty in the next chapter) if we take the speed reading every fifteen revolutions. Then we can calculate six readings of rotational speed (ω_0 based on frames f_{15} and f_0, ω_1 based on frames f_{16} and f_1, ω_2 based on frames f_{17} and f_2, ω_3 based on frames f_{18} and f_3, ω_4 based on frames f_{19} and f_4, ω_5 based on frames f_{20} and f_5). Then the final reading of the rotational speed is the mean of the six calculated readings.

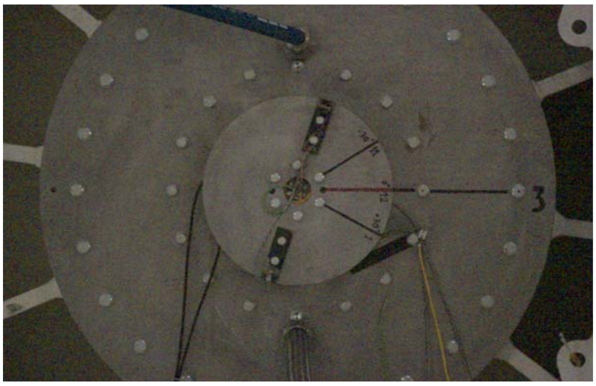

Figure 2 - image acquired by the high speed camera at the moment when markers on rotor and stator align

Uncertainty calculation

In the proposed calibration method the measurement is not taken directly but it's a function of four variables:

$$(2)\quad y = f(x_1, x_2, x_3, x_4) \rightarrow \omega_i = f(N, t, f_{N+i}, f_i)$$

Input values x_1, x_2, x_3, x_4 (which respectively correspond to N, t, f_{N+i} and f_i) are in this case uncorrelated, so the uncertainty formula is given by the following equation [7]:

$$(3)\quad u_c^2(y) = \sum_{j=1}^{4} \left(\frac{\partial f}{\partial x_j}\right)^2 u^2(x_j)$$

In the case of the high speed imaging calibration method the formula (3) can be rewritten as follows:

$$(4)\quad u_c^2(\omega_i) = \left(\frac{\partial \omega_i}{\partial N}\right)^2 u^2(N) + \left(\frac{\partial \omega_i}{\partial t}\right)^2 u^2(t) + \left(\frac{\partial \omega_i}{\partial f_{N+i}}\right)^2 u^2(f_{N+i}) + \left(\frac{\partial \omega_i}{\partial f_i}\right)^2 u^2(f_i)$$

Because the uncertainty u(N) of the number of revolutions taken for calculation of each rotational speed measurement is equal to zero (the user chooses the value, it isn't measured), the formula (4) can be simplified as follows:

$$(5)\quad u_c(\omega_i) = \sqrt{\left(\frac{\partial \omega_i}{\partial t}\right)^2 u^2(t) + \left(\frac{\partial \omega_i}{\partial f_{N+i}}\right)^2 u^2(f_{N+i}) + \left(\frac{\partial \omega_i}{\partial f_i}\right)^2 u^2(f_i)}$$

The partial derivatives and corresponding uncertainties of input values are the following:

$$(6)\quad \frac{\partial \omega_i}{\partial t} = -\frac{60 \cdot N}{t^2 \cdot (f_{N+i} - f_i)}$$

$$(7)\quad u(t) = y \cdot (t + \Delta_1)$$

where: $y = 1.7 \times 10^{-6}$ – parameter calculated by an accredited national laboratory during camera's annual calibration, Δ_1[s] - shutter speed (parameter set by the user).

$$(8)\quad \frac{\partial \omega_i}{\partial f_{N+i}} = -\frac{60 \cdot N}{t \cdot (f_{N+i} - f_i)^2}$$

$$(9)\quad \frac{\partial \omega_i}{\partial f_{N+i}} = -\frac{60 \cdot N}{t \cdot (f_{N+i} - f_i)^2}$$

$$(10)\quad u(f_{N+i}) = u(f_i) = 0.5$$

The uncertainty of the frames measurement (10) is given as a half of the reading resolution. Through experiment it has been proven time after time that the image provided by the camera has good enough quality to enable the user to tell the exact frame of the markers alignment each and every revolution. In the case of calibrating the sensor at low rotational speed using very high frames per second rate the uncertainties $u(f_{N+i})$ and $u(f_i)$ might need to be increased (for example at 600 rpm and 100 000 fps almost 28 frames correspond to 1 degree of rotation, therefore the user won't be able to tell the exact frame of the markers alignment each revolution). However, in such a case the major part of the total uncertainty $u_c(\omega_i)$ comes from the uncertainty u(t) so increasing the uncertainty of frames measurement won't affect $u_c(\omega_i)$ that much (the value of total uncertainty doubles only when both $u(f_{N+i})$ and $u(f_i)$ are increased from 0.5 to 12.5).

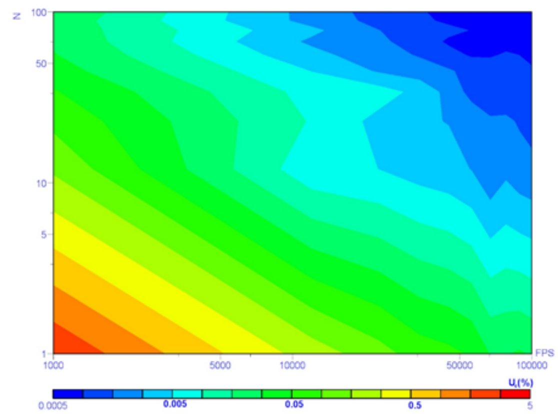

Figure 3 - uncertainty $U_c(\omega_i)$ of the method shown as the percentage of the value (confidence level P=95%, t-Student distribution value k=2.32) when calibrating the speed sensor at 1000 rpm

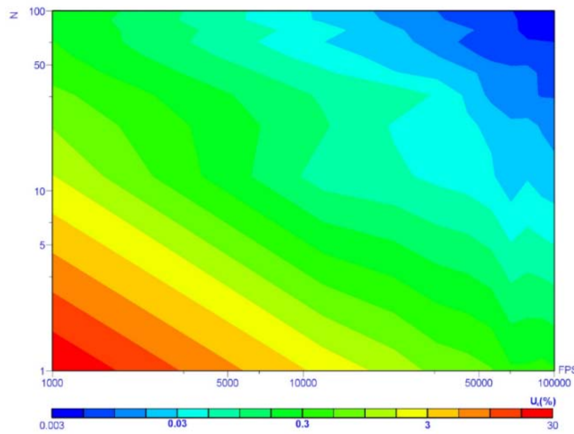

Figure 4 - uncertainty $U_c(\omega_i)$ of the method shown as the percentage of the value (confidence level P=95%, t-Student distribution value k=2.32) when calibrating the speed sensor at 12000 rpm

Figure 3 and Figure 4 show the total uncertainty $U_c(\omega_i)$ of the method as calculated for two different rotational speed. Both charts show the remarkably low error introduced by this method and its superiority as compared to the stroboscope light method.

Results

The test stand is capable of achieving rotational speed within 500÷12000 rpm range. However, during the time of the calibration it was being prepared for a test at which maximum speed has been set at 2025 rpm. Therefore the calibration has been performed within 500÷2025 rpm range. Five calibration points were chosen: 510 rpm, 1002 rpm, 1456 rpm, 1968 rpm and 2025 rpm. For each calibration point 10 full revolutions have been recorded (the cameras are capable of recording many more revolutions, however the accuracy of the method was good enough at 10 revolutions for the purpose of the test) with 18002.5718 fps frame rate and 4.94×10^{-5} s shutter speed. Table 1 shows frame numbers of markers alignment for each revolution at every calibration point.

Table 1 - frame numbers of markers alignment for each calibration point

Revolution	510rpm	1002rpm	1456rpm	1968rpm	2025rpm
0	89	854	380	36	88
1	2200	1933	1121	585	621
2	4314	3011	1862	1134	1155
3	6425	4090	2605	1683	1688
4	8539	5168	3347	2232	2221
5	10652	6247	4089	2781	2755
6	12767	7326	4831	3330	3288
7	14883	8404	5573	3879	3822
8	16999	9483	6314	4428	4355
9	19114	10561	7055	4977	4888
10	21230	11639	7797	5526	5422

Having all the data, the last unknown is the value N at which the uncertainty will be the smallest. The higher the number of revolutions considered for each speed measurement the higher the accuracy. However increasing the N value means to decrease the number of measurements of speed and therefore increasing the t-Student distribution value. In the case of recording 10 full revolutions and including the t-Student distribution the lowest uncertainty of the speed measurement happens when N equals 7. Figure 5 shows a chart of the total uncertainty $U_c(\omega)$ versus N value after including the t-Student distribution for 1002 rpm speed measurement (the shape of the chart will be the same for all calibration points).

Table 2 shows the calculated values of rotational speed measurement and corresponding total uncertainty and standard deviation for all instances of N value. The row where the value of N equals 7 has been highlighted as the best case scenario. It is worth noting that the standard deviation in almost all the cases is smaller than the value of the total uncertainty of the method, therefore the final error of the calibration won't be increased much beyond the uncertainty U_c of the method. Other thing worth noting is the standard deviation equal to zero of the measurement for the 1968 rpm calibration point. It can be easily explained as the frame rate (18002.5718 fps) corresponds perfectly to the rotational speed of the shaft (549 frames per each revolution), therefore image is taken at exactly the same spot every revolution. The accuracy of the method can be then further increased if we chose calibration points that correspond to the camera frame rate.

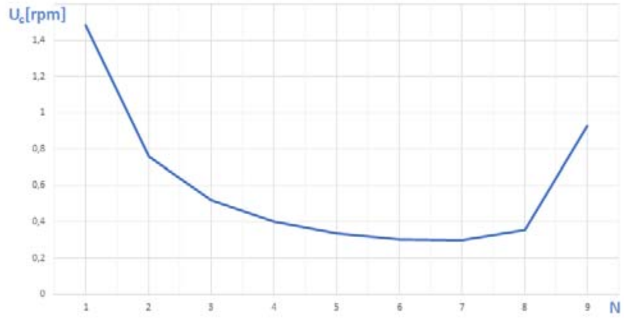

Figure 5 - total uncertainty $U_c(\omega)$ versus N value after including the t-Student distribution for 1002 rpm speed measurement

Different methods of vision analysis can be used to further increase the accuracy of the measurement and to increase the reliability. For example the linear speed of a known point on the rotor can be calculated by tracking the position of this point frame by frame. To calculate the linear speed two parameters has to be known: the position of the point relative to the axis of rotation and the distance between the camera and the plane of the rotor. These parameters can be either set by the user or calculated in real time by the computer vision algorithms [8]. Linear speed measurement can be used as a cross-reference proving the correct calculation of the rotational speed.

Summary

The experiment conducted proved that the high speed imaging calibration method possesses an extraordinary accuracy, especially considering the fact that the parameters used (18002 fps frame rate and total 10 revolutions recorded) were far below the method limitations. The biggest drawback of the presented method is the price of the high speed camera with precise internal clock and capability to acquire image with frame rates up to 100 000 fps. Possessing the camera for the sole purpose of rotational speed measurement calibration might not have strong enough economical substantiation. Second drawback is complexity of camera systems and lights operation which might require experienced engineering support. However, both the high speed camera system and onsite engineering service can be provided by Research Institutes and Companies specializing in high speed image recording for a reasonable price. With that approach camera systems introduce new opportunities for applications where the speed of rotating object has to be measured with superior accuracy. Opportunities which haven't been available so far with standard stroboscope methods.

Table 2 - calculation of the rotational speed value, total uncertainty and standard deviation (including t-Student distribution) for all instances of N value

N	510 rpm			1002 rpm			1456 rpm			1968 rpm			2025 rpm		
	ω_{mean}	U_c	σ	ω_{mean}	U_c	σ	ω_{mean}	U_c	σ	ω_{mean}	U_c	σ	ω_{mean}	U_c	σ
1	510,929	1,163	1,046	1001,534	1,484	1,107	1456,323	3,147	2,997	1967,494	5,732	0,000	2025,038	6,081	4,433
2	510,913	0,592	0,824	1001,534	0,757	0,535	1456,279	1,604	2,295	1967,494	2,922	0,000	2025,079	3,094	1,931
3	510,913	0,405	0,846	1001,495	0,518	0,379	1456,143	1,095	1,641	1967,494	1,998	0,000	2025,131	2,117	1,058
4	510,901	0,314	0,765	1001,501	0,402	0,392	1456,084	0,849	1,336	1967,494	1,550	0,000	2025,063	1,643	1,242
5	510,905	0,264	0,699	1001,472	0,338	0,195	1456,061	0,714	1,180	1967,494	1,303	0,000	2025,036	1,380	0,000
6	510,905	0,237	0,572	1001,472	0,304	0,235	1456,061	0,642	0,908	1967,494	1,173	0,000	2025,036	1,242	0,962
7	510,901	0,233	0,495	1001,501	0,299	0,211	1456,084	0,631	0,446	1967,494	1,152	0,000	2025,063	1,220	0,863
8	510,913	0,276	0,397	1001,495	0,353	0,288	1456,142	0,747	0,610	1967,494	1,363	0,000	2025,131	1,444	0,000
9	510,913	0,724	1,206	1001,534	0,928	0,927	1456,279	1,961	1,960	1967,494	3,578	0,000	2025,078	3,792	3,790

***Authors**: Łukasz Kędzierski*
Lab Maintenance and Development, Institute of Aviation, Al. Krakowska 110/114, 02-256 Warsaw
Institute of Theory of Electrical Engineering, Measurement and Information Systems, Faculty of Electrical Engineering, Warsaw University of Technology, Pl. Politechniki 1, 00-661 Warsaw
E-mail: lukasz.kedzierski@ee.pw.edu.pl

REFERENCES

[1] Dewayne B. Sharp, "Measurement Standards", CRC Press, 2000
[2] Ambler Thompson, Barry N. Taylor, "Guide for the Use of the International System of Units", National Institute of Standard and Technology, 2008
[3] Frederick Van Veen, "Handbook of Stroboscopy", GenRad Inc., Concord, 1977
[4] "Fluke 820-2 LED Stroboscope", Fluke Corporation, 2014
[5] "PACs for Industrial Control, the Future of Control", National Instruments, 2006
[6] Vanessa Romero Segovia, Alfred Theorin, "History of PLC and DCS", Lund University, 2013
[7] Kai Oliver Arras, "An Introduction To Error Propagation", Swiss Federal Institute of Technology Lausanne, 1998
[8] Marek Piszczek, "Finding the distance to the object using the method of spatial-temporal framing", Przegląd Elektrotechniczny, No. 11/2011