

# Space vector modulation techniques for improved stator flux trajectory in direct torque control of induction motor

**Abstract.** In direct torque vector control systems, voltage source inverter is described in discrete mode as a source of constant amplitude voltage sources with strictly controlled direction, time duration and limited sample frequency. The dynamic state of the inverter is discrete both in time and quantity. In this paper, therefore, the basic characteristics of stator flux space vector will be analysed in such a control concept for two different types of space vector modulation (SVM) techniques one is full block space vector modulation and second is called pulse edge space vector modulation.

**Streszczenie.** W układach bezpośredniego sterowania wektorami momentu obrotowego, falownik źródła napięcia jest opisany w trybie dyskretnym jako źródło stałych źródeł napięcia o amplitudzie ze ściśle kontrolowaną polaryzacją, czasem trwania i ograniczoną częstotliwością próbkowania. Stan dynamiczny falownika opisywany jest w sposób dyskretny zarówno pod względem czasu jak i wartości. W przedstawionym artykule została przeanalizowana podstawowa charakterystyka wektora przestrzennego strumienia stojana w koncepcji sterowania dla dwóch różnych sposobów modulacji wektora przestrzennego (SVM). Jedną jest modulacją wektora przestrzennego obszaru pracy blokowej, a druga jest nazywana dyskretną modulacją wektora przestrzennego. (Techniki modulacji wektora przestrzennego dla poprawy trajektorii strumienia stojana w bezpośrednim sterowaniu momentem obrotowym silnika indukcyjnego).

**Keywords:** direct torque vector control, induction machine, voltage source inverter, pulse edge space vector modulation.

**Słowa kluczowe:** bezpośrednie sterowanie momentem obrotowym, maszyna indukcyjna, falownik źródła napięcia, dyskretna modulacja wektora przestrzennego

## Introduction

In direct torque vector control (DTC) systems, voltage source inverter (VSI) is describe in discrete mode as a source of constant amplitude voltage sources with strictly controlled direction, time duration and limited sample frequency. The output vector voltage vector may take up to seven different states and is altered in precisely specified time only at the command of given controlling signal. Therefore, the VSI dynamic state is discrete both in time and quantity. The stator flux space vector is used as a fundamental value in designing the control circuits of induction machine (IM). In this paper, therefore, the basic characteristics of stator flux space vector will be analysed in such a control concept for two different types of space vector modulation (SVM) techniques one is full block space vector modulation and second is called pulse edge space vector modulation.

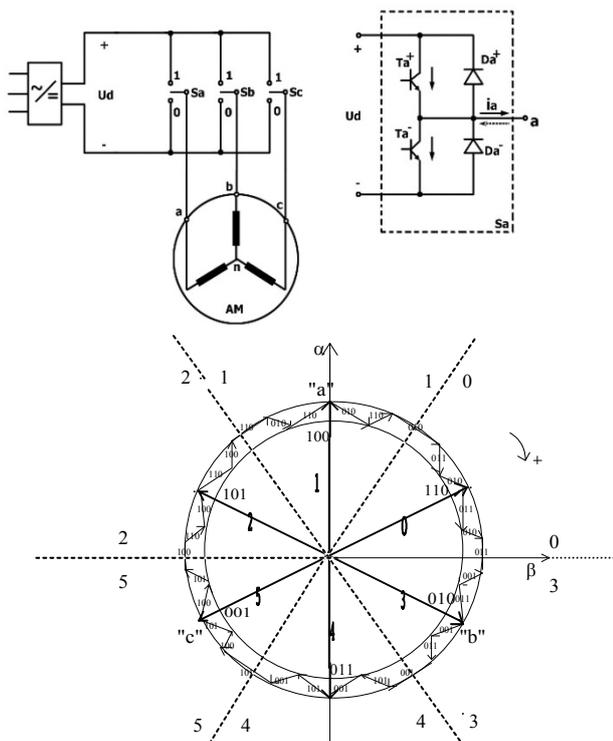


Fig. 1. VSI for DTC selecting the voltage vector for optimum stator flux trajectory

## Advantages of the $\vec{\psi}_1$ trajectory

The selection of the switch-on and switch-off states of the power switches  $S_a, S_b, S_c$  (Fig.1) is carried out so that the error between the real value of  $|\vec{\psi}_1|$  and the reference (assigned) value of  $|\vec{\psi}_1^*|$  moves within the limits of  $\Delta|\vec{\psi}_1|$  as defined by the hysteresis band of the flux controller.

However, the selection of the states of the power switches  $S_a, S_b, S_c$  not only depends on the absolute error value but also on the rotation direction of vector  $\vec{\psi}_1$ . For this purpose, the stationary reference domain  $\alpha-\beta$  is divided into 6 (six) equal parts (zones). Each zone has its own carrier voltage space vector. Thus, for instance, the carrier vector of the '0' zone is vector  $\vec{U}_6(110)$ . For a positive rotation direction (clockwise) the subsequent two voltage space vectors,  $\vec{U}_6(110)$  and  $\vec{U}_3(011)$  switch on respectively, depending on whether the upper or lower limit of the hysteresis band has been reached. For a negative rotation direction (counter clockwise) the corresponding vectors  $\vec{U}_5(101)$  or  $\vec{U}_4(100)$  switch on depending on whether the upper or lower limit of the hysteresis control band has been reached. The stator flux space vector  $\vec{\psi}_1$  for IM supplied by in VSI is:

$$(1) \quad \frac{d\vec{\psi}_1}{dt} = \vec{u}_1 - R_1 \cdot \vec{i}_1$$

For VSI power greater than 10kV and angular frequencies of the fundamental harmonic  $\omega_g \geq 0,7$ , the stator resistor voltage drop can be neglected without changing the control accuracy. Below is shown that neglecting  $R_1$  is possible also in the area of low frequencies if only a short time interval  $0 \leq t \leq T$  is considered. By integrating equation (1) with stator resistance neglected the following is obtained:

$$(2) \quad d\vec{\psi}_1(t) = d\vec{\psi}_1(0) + \int_0^T \vec{u}_1(t) dt \quad 0 \leq t \leq T$$

Or with stator resistance integral has the form:

$$(3) \quad d\vec{\psi}_1(t) = d\vec{\psi}_1(0) - R_1 \cdot \int_0^T \vec{i}_1(t) dt + \int_0^T \vec{u}_1(t) dt \quad 0 \leq t \leq T$$

The starting stator flux vector in both equations, (2) and (3), is identical and so absolute error may be defined as:

$$(4) |\Delta\vec{\psi}_1(t)| = R_1 \cdot \left| \int_0^T \vec{i}_1(t) dt \right| \quad 0 \leq t \leq T$$

To estimate absolute error of the stator flux space vector, the fundamental harmonic of the stator current space vector can be used:

$$(5) \vec{i}_1(t) = \vec{i}_{1g}(t) = \vec{i}_{1g}(0) \cdot e^{j\omega_g t} \quad 0 \leq t \leq T$$

Combining the equations (4) and (5) gives the absolute stator flux vector error:

$$(6) |\Delta\vec{\psi}_1(t)| = R_1 \cdot |\vec{i}_{1g}(0)| \cdot \sqrt{2 \cdot \frac{1 - \cos(\omega_g t)}{\omega_g^2}} \quad 0 \leq t \leq T$$

The sampling period in this control concept may vary within the limits  $T = 0,4 \dots 0,8$ , which corresponds to several milliseconds. Considering the motor's drive state in frequency range  $0 \leq \omega_g \leq 2$ , can be assumed:

$$(7) \cos(\omega_g \cdot t) \approx 1 - \frac{(\omega_g \cdot t)^2}{2} \quad 0 \leq t \leq T$$

By means of Equation (7) absolute stator flux error in line with Equation (6) amounts to:

$$(8) |\Delta\vec{\psi}_1(t)| = R_1 \cdot |\vec{i}_{1g}(0)| \cdot t \quad 0 \leq t \leq T$$

Following from (8) is the obvious conclusion that absolute error in determining the stator flux space vector is independent of the angular frequency  $\omega_g$ . Hence, just as neglecting the voltage drop of the stator resistance when calculating the stator flux space vector  $\vec{\psi}_1$  in the circular high frequency area is permitted, the same neglect is also allowed at low frequencies. The only restriction is that the IM drive state should be considered only in a short sampling interval of  $0 \leq t \leq T$ , which in practical microprocessor implementation of vector control is always fulfilled. Estimating phase error, i.e., the spatial position of the error vector  $\Delta\vec{\psi}_1$  can similarly be carried out. The space vector  $\Delta\vec{\psi}_1$  has the approximate direction that the space vector of the main stator current harmonic  $\vec{i}_{1g}$  takes in the middle of the observed interval  $0 \leq t \leq T$ :

$$(9) \arg\{\Delta\vec{\psi}_1(t)\} = \arg\{\vec{i}_{1g}(t/2)\} \quad 0 \leq t \leq T$$

From the above analysis it can be concluded that there is a very simple functional dependence between the stator voltage space vectors and the stator flux space vectors. The stator flux space vector  $\vec{\psi}_1$  does not depend either on the stator current space vector  $\vec{i}_1$ , or on the developed electromagnetic torque  $M$ , and consequently on the motor's drive state. By introducing the space vector time derivative in polar coordinates it is possible to interpret its "v<sub>1</sub>" module (10) as linear velocity at which the vector peak  $\vec{\psi}_1$  moves on a given trajectory (Fig.1). The movement direction is defined by the angle "ζ" (11).

$$(10) \frac{d\vec{\psi}_1}{dt} = \vec{u}_1 = v_1 \cdot e^{j\zeta}$$

$$(11) v_1 = |\vec{u}_1|, \quad \zeta_1 = \arg\{\vec{u}_1\}$$

IM has 6 (six) active non-zero, and two zero stator voltage vectors available, mutually displaced in space by 60° all of them with the length of  $\frac{2}{3} \cdot U_d$ . The stator flux space vector can move only in one of the six possible directions at the

speed of  $\frac{2}{3} \cdot U_d$ . When either zero voltage, or space vector

$\vec{u}_1(000)$  or  $\vec{u}_7(111)$  is switched on, the peak of the space vector  $\vec{\psi}_1$  stands still. From equations (10) and (11) it can be concluded that a sequence of voltage pulses determined by space vector modulation, irrespective of the corresponding IM drive state, can be represented as a corresponding  $\vec{\psi}_1$ -trajectory.

### Full block space vector modulation

In a three-phase six-step inverter drive, or 120-degree-mode VSI drive, Figure 2 shows switch on/off voltages, i.e., control signals of the phase „a“ of the three-phase bridge VSI inverter.

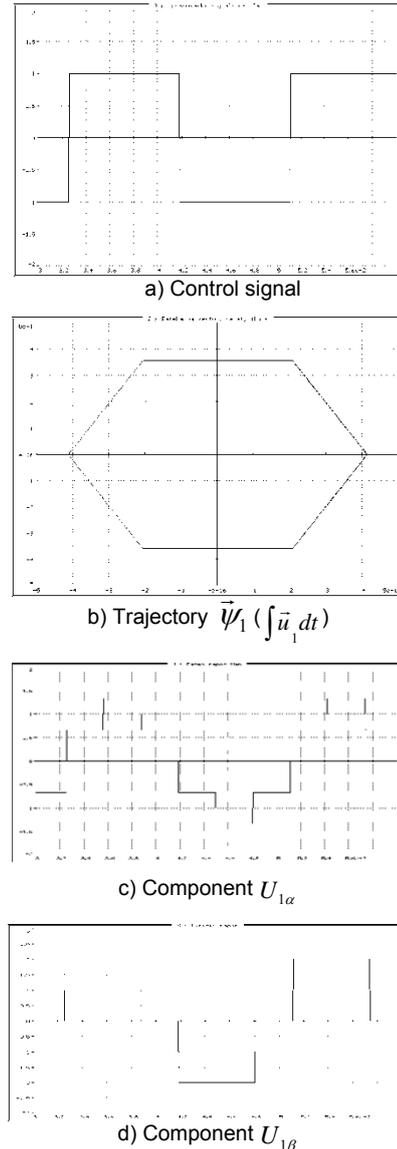


Fig. 2. Full block of space vector modulation  $V_p = \frac{f_{vM}}{f_g} = 1$

The on/off switching frequency equals to that of the fundamental harmonic, i.e.,  $V_p = f_{vd} / f_g = 1$ . Hence, the fundamental harmonic of the inverter output in this space vector modulation is at maximum amplitude. In this mode the power switch  $S_a$  is switched on entire 180°. Control signals of the power switches  $S_b$  and  $S_c$  are displaced by 120° and 240°, respectively. This inverter drive state, i.e.,

such vector modulation, is made possible by the subsequent switch-on of the six available active space vectors of the voltage (Fig.1) each  $60^\circ$ , i.e.,  $T_g/6$  ms, where  $T_g$  is the period of the fundamental harmonic.

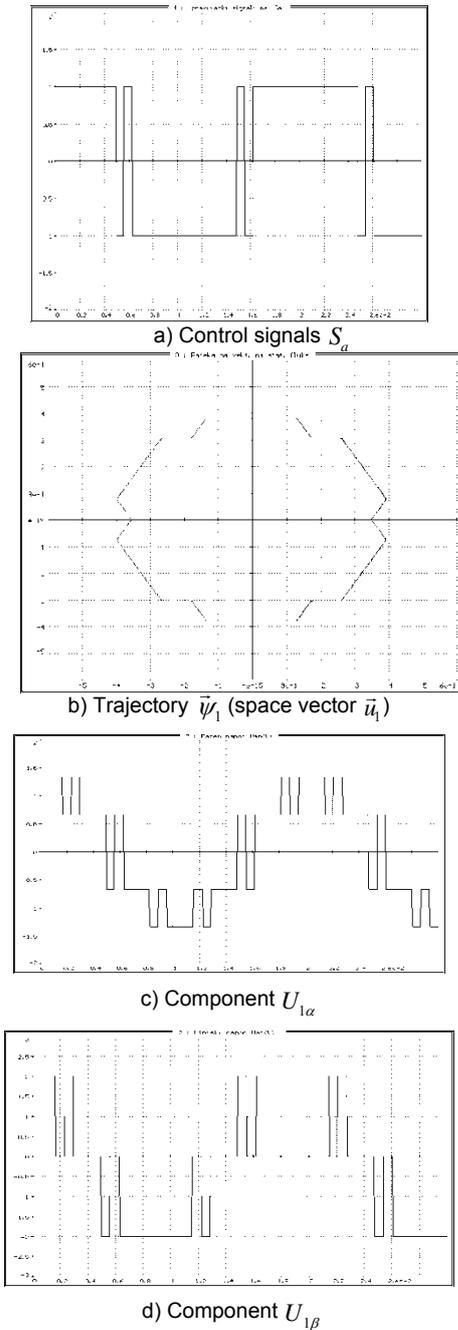


Fig. 3. Pulse edge space vector modulation  $V_p = \frac{f_{vkl}}{f_g} = 3$

### Pulse edge space vector modulation

In that case, for the  $\bar{\psi}_1$  trajectory an isosceles hexagon is obtained (Fig.3b). The peak of the  $\bar{\psi}_1$  space vector moves along the edges of the isosceles hexagon at constant speed  $v_1 = 2/3 U_d$ .

By means of pulse edge vector modulation, symmetrical cropping of the hexagon legs in the edge-area is obtained, (Fig.3b). The length of these symmetrical deformations is determined by characteristic time  $T_F$  between two consecutive pulses of different width. The rotation speed of  $\bar{\psi}_1$  still remains constant,  $v_1 = 2/3 U_d$ . In this modulation, in each of the six possible space zones (each zone is a  $60^\circ$

angle) three space vectors are switched on: the carrier vector of the zone with its antecedent and subsequent vector (Fig. 3b). For this modulation method it holds that  $V_p = \frac{f_{vkl}}{f_g} = 3$ . If the antecedent and subsequent vector are

switched on at time  $T_F$  ms, (the crop length at the sides of the isosceles hexagon's edge-area amounts to  $T_F \cdot \frac{2}{3} \cdot U_d$ ),

then the carrier vector is switched on during  $\frac{T_g}{6} - 2 \cdot T_F$  ms,

while the side length of the hexagon is  $\left(\frac{T_g}{6} - 2 \cdot T_F\right) \cdot \frac{2}{3} \cdot U_d$ . The

form of the  $\alpha$  and  $\beta$  components of  $\bar{u}_1$  space vector is given in Fig.3c and Fig.3d, respectively.

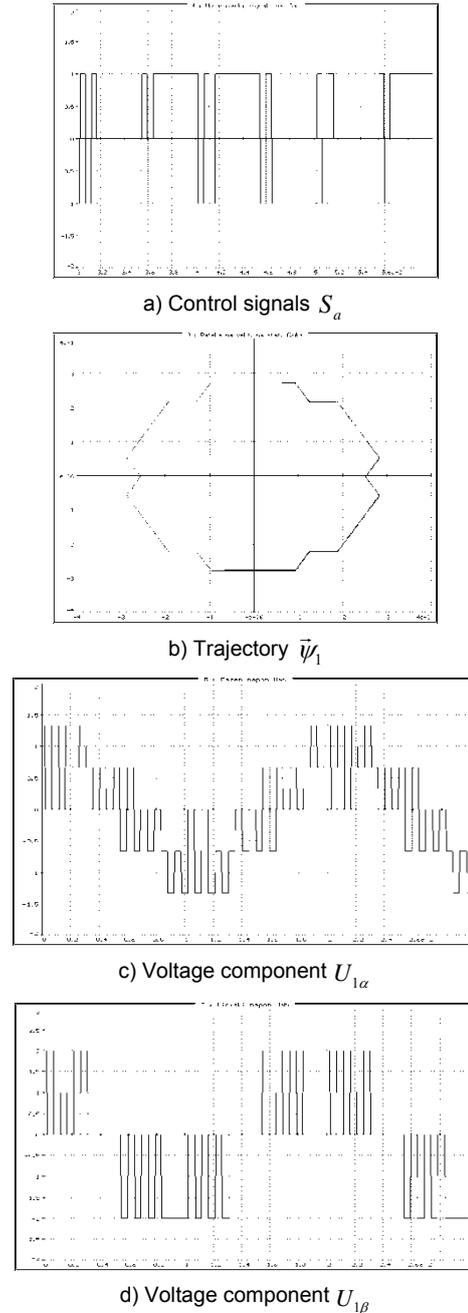


Fig. 4. Pulse edge space vector modulation  $V_p = f_{vkl} \cdot f_g = 7$

In the previous two models, space vector modulation was defined by 6 (six) active vectors  $\bar{U}_1(001)$ ,  $\bar{U}_2(010)$ ,

$\vec{U}_3(011)$ ,  $\vec{U}_4(100)$ ,  $\vec{U}_5(101)$ ,  $\vec{U}_6(110)$ . However, two remaining zero vectors  $\vec{U}_0(000)$  and  $\vec{U}_7(111)$  are still available to the inverter. When these two vectors are switched on, the space vector remains idle at locations marked by „. “. The speed of the  $\vec{\psi}_1$  space vector thereby still remains unchanged,  $v_1 = \frac{2}{3} \cdot U_d$ . In modulation procedure, presented

on Figure 4 the on/off switching frequency is seven times higher than the carrier wave frequency  $V_p = \frac{f_{vkl}}{f_g} = 7$ . To carry

out this modulation alternative in each  $60^\circ$  area, three active stator voltage space vectors are switched on (the zone carrier vector and its antecedent i.e., subsequent vector) plus the two zero vectors  $\vec{U}_0(000)$  and  $\vec{U}_7(111)$ .

When the two zero vectors are switched on, the peak of  $\vec{\psi}_1$  space vector stands still (Fig.4b), and the distance between the switching on of the two zero vectors is determined by time  $T_N$  and the line-to-line rotation speed  $v_1 = \frac{2}{3} \cdot U_d$ . If in this modulation the antecedent and

subsequent vector of each  $60^\circ$  ( $T_g/6$  ms) active zone is switched on  $T_F$  ms, then the length of the cropped hexagon sides in the edge-area amounts equal to  $T_F \cdot \frac{2}{3} \cdot U_d$ . In each

zone, the voltage carrier space vector is switched on/off three times (it is separated by two zero vectors) each with duration of  $\frac{1}{2} \cdot \left( \frac{T_g}{6} - 2 \cdot T_F - T_N \right)$ ,  $T_N$ ,  $\frac{1}{2} \cdot \left( \frac{T_g}{6} - 2 \cdot T_F \right)$

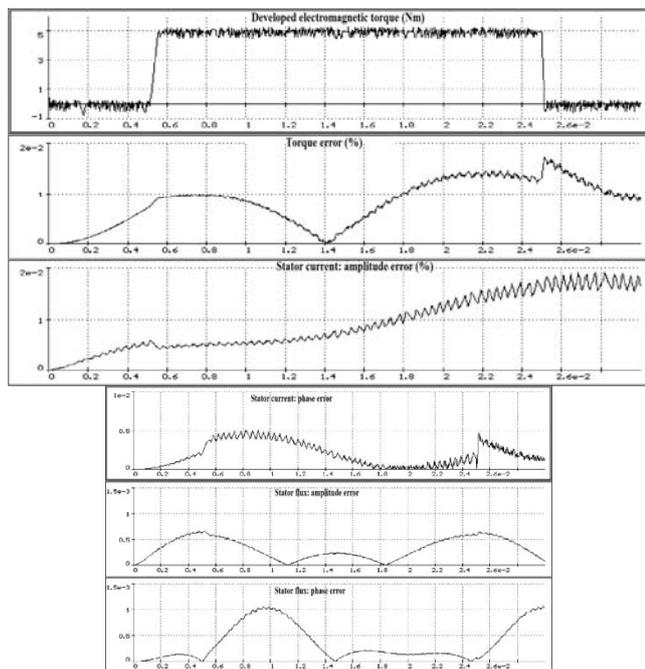


Fig. 5 Error assessment in direct vector control system (assumed:  $R_1$  change by 15%)

## Vector error assessment due to uncertain stator resistance

For the purpose of quantitative and qualitative assessment of the developed electromagnetic torque, current and stator flux space vector errors, due to variation in the winding resistances, the simulation of DTC drive is obtained. Figure 5. shows time diagrams of the developed torque in 30 milliseconds, the torque error, as well as the amplitude and phase errors of the  $\vec{i}_1$  и  $\vec{\psi}_1$  space vectors when parameters  $R_1$  и  $R_2$  change their values by 15%. The amplitude error of  $\vec{i}_1$  is limited to 2%, while its angular error to only 0,28°. Amplitude error of  $\vec{\psi}_1$  is even lower, 0,6%, and its angular error is 0,05°. The maximum developed torque error is 1,5%. This control structure demonstrably maintains its excellent dynamics and shows excellent stability when the parameters  $R_1$  and  $R_2$  change.

## Conclusion

There is a very simple functional dependence between the stator voltage space vectors and the stator flux space vectors. The stator flux space vector  $\vec{\psi}_1$  does not depend either on the stator current space vector  $\vec{i}_1$ , or on the developed electromagnetic torque  $M$ , and consequently on the motor's drive state. Furthermore the absolute error in determining the stator flux space vector is independent of the angular frequency  $\omega_g$ . Hence, just as neglecting the voltage drop of the stator resistance when calculating the stator flux space vector  $\vec{\psi}_1$  in the circular high frequency area is permitted, the same neglect is also allowed at low frequencies. The only restriction is that the IM drive state should be considered only in a short sampling interval of  $0 \leq t \leq T$ , which in practical microprocessor implementation of vector control is always fulfilled.

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