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Further Results on Output Tracking for a Class of Uncertain High-Order Nonlinear Time-Delay Systems

***Abstract**. This paper investigates the problem of global practical output tracking by state feedback for a class of uncertain high-order nonlinear time-delay systems. Further, we design a homogeneous state feedback controller with an adjustable scaling gain, under mild conditions on the system nonlinearities involving time delay. Through the use of a homogeneous Lyapunov-Krasovskii functional method, the scaling gain is adjusted to dominate the time-delay nonlinearities bounded by homogeneous growth conditions and render the tracking error can be made arbitrarily small while all the states of the closed-loop system remain to be bounded.*

Streszczenie.** W artykule opisano problem globalnego praktycznego śledzenia wyjścia za pomocą sprzężenia zwrotnego od stanu dla klasy niepewnych nieliniowych układów opóźniających wysokiego rzędu. Ponadto zaprojektowany został jednorodny kontroler sprzężenia od stanu z regulowanym wzmocnieniem skali, przy łagodnej nieliniowości i opóźnieniu czasowym. Dzięki zastosowaniu jednorodnej metody funkcjonalnej Lapunowa-Krasowskiego, wzmocnienie skali jest przystosowane do dominacji nieliniowości opóźnienia czasowego ograniczonej przez jednorodne warunki wzrostu i powoduje, że błąd śledzenia może być dowolnie mały, podczas gdy wszystkie stany systemu ze sprzężeniem zwrotnym pozostają ograniczonye. **(Wyniki śledzenia wyjścia dla klasy nieliniowych systemów niepewnych wysokiego rzędu zawierających opóźnienia)

Keywords: global practical output tracking, nonlinear time delay systems, state feedback controller, Lyapunov-Krasovskii functional
Słowa kluczowe: globalne śledzenie wyjścia, nieliniowe systemy z opóźnieniem czasowym, kontroler sprzężenia zwrotnego od stanu, funkcjonał Lapunowa-Krasowskiego

Introduction

The control design is one of the most relevant topics in nonlinear system theory, so a number of researchers have paid particular attention to it, for example, it can be seen in references [1-11]. The fundamental problem is to construct a feedback control law making the controlled output track a given reference signal as much as possible.

The problem of global practical tracking for nonlinear state feedback systems was solved by the method of adding a power integrator [3,4] and using the idea of universal control [1,2]. However, the above results do not take into account time delays and their impact on the system as a whole. For example, in three-dimensional systems, delay is determined by the fact that the signals propagate at a finite speed and they need time to overcome distances [12]. Delay of the reaction to the signal and feedback with delay are inherent in many physical [13], chemical [14], climatic [15] and biological [16] objects and processes. In the study of systems with delay, it is important to know the values of time delays, the value of which largely determines the dynamics and properties of the system. Since time-delay exists widely in many practical systems such as electrical networks, microwave oscillator, and hydraulic systems, etc., and usually makes the considered system instable, to achieve some control objectives such as stabilization and trajectory tracking, the influence of time delay phenomenon should be considered. In view of these facts, it is meaningful and necessary to study control problems of accidental nonlinear systems with unknown parameters and time-delays.

In recent years, by employing the Lyapunov-Krasovskii method to deal with the time-delay, control theory, and techniques for stabilization problem of time-delay nonlinear systems were greatly developed and advanced methods have been made; see, for instance, [17-21] and reference therein. Compared with study the stabilization problem contain time-delay, the theory of output tracking control developed slower. In the case when the nonlinearities contain time-delay, for the output tracking problems, some interesting results have been obtained [22-24]. However, in [22-24, 28] only considered special case for the system (1), i.e., case. When the system under consideration is inherently time-delay non-linear, the problem becomes more complicated and difficult to solve. To the best of our knowledge, many interesting output tracking control problems for time delay inherently nonlinear systems unsolved yet. In this paper, we deal with such as the tracking problems via state feedback domination method in [25-26].

Consider the following uncertain nonlinear time-delay system

$$
\begin{aligned}
&\dot{x}_1(t) = x_2^{p_1}(t) + \varphi_1(t, x(t), x(t-d_1), u(t)),\\
&\quad\vdots\\
&\dot{x}_{n-1}(t) = x_n^{p_{n-1}}(t) + \varphi_{n-1}(t, x(t), x(t-d_{n-1}), u(t)),\\
&\dot{x}_n(t) = u + \varphi_n(t, x(t), x(t-d_n), u(t)),\\
&y(t) = x_1(t),
\end{aligned}
\tag{1}
$$

where $x(t) := (x_1(t), \ldots, x_n(t))^{\mathrm{T}} \in R^n$, $u \in R$, and $y(t) \in R$ are the system state, control input and output, respectively. The constants $d_i \geq 0\ (i = 1, \ldots n)$ are a given time-delays of the system, and the system initial condition is $x(\theta) = \varphi_0(\theta),\ \theta \in [-d, 0],\ d \geq \max\{d_1, \ldots d_n\}$. The terms $\varphi_i(\cdot)$ represent nonlinear perturbations that are unknown continuous functions and $p_i \in R_{odd}^{\geq 1} := \{p/q \in [0, \infty) : p$ is a positive integer, and q is a positive odd integer, $p \geq q\}$ $(i = 1, \ldots, n-1)$ are said to be the high orders of the system.

Mathematical Preliminaries

We collect the definition of homogeneous function and several useful lemmas.

Definition 1 ([27]). For a set of coordinates $x = (x_1, \ldots, x_n) \in R^n$ and an n-tuple $r = (r_1, \ldots, r_n)$ of positive real numbers we introduce the following definitions.

(i) *A dilation* $\Delta_s(x)$ is a mapping defined by
$$\Delta_s^r(x) = \left(s^{r_1} x_1, \cdots, s^{r_n} x_n\right),\ \ \forall x = (x_1, \ldots, x_n) \in R^n,\ \forall s > 0,$$
where r_i are called *the weights of the coordinate*. For simplicity of notation, the dilation weight is denoted by

$\Delta = (r_1, \dots, r_n)$.

(ii) A function $V \in C(R^n, R)$ is said to be *homogeneous of degree* τ if there is a real number $\tau \in R$ such that $V(\Delta_s^r(x)) = s^\tau V(x_1, \cdots, x_n), \quad \forall x \in R^n - \{0\}$.

(iii) A vector field $f \in C(R^n, R^n)$ is said to be *homogeneous of degree* τ if the component f_i is *homogeneous of degree* $\tau + r_i$ for each *i*, that is, $f_i(\Delta_s^r(x)) = s^{\tau + r_i} f_i(x_1, \cdots, x_n), \quad \forall x \in R^n, \quad \forall s > 0$, for $i = 1, \dots, n$.

(iv) *A homogeneous* p*-norm* is defined as $\|x\|_{\Delta, p} = \left(\sum_{i=1}^n |x_i|^{p/r_i} \right)^{1/p}$, $\forall x \in \mathbb{R}^n$, $p \geq 1$.

For the simplicity, write $\|x\|_\Delta$ for $\|x\|_{\Delta, 2}$.

Next, we introduce several technical lemmas which will play an important role and be frequently used in the later control design.

Lemma 1 [27]. Denote $\Delta = (r_1, \dots, r_n)$ as dilation weight, and suppose $V_1(x)$ and $V_2(x)$ are homogeneous functions with degree τ_1 and τ_2, respectively. Then, $V_1(x)V_2(x)$ is also homogeneous function with degree of $\tau_1 + \tau_2$ with respect to the same dilation Δ.

Lemma 2[27]. Suppose $V : R^n \to R$ is a homogeneous function of degree τ with respect to the dilation weight Δ. Then, the following (i) and (ii) hold:

(i) $\partial V / \partial x_i$ is also homogeneous of degree $\tau - r_i$ with r_i being the homogeneous weight of x_i.

(ii) There is a constant $\sigma > 0$ such that $V(x) \leq \sigma \|x\|_\Delta^\tau$. Moreover, if $V(x)$ is positive definite, there is a constant $\rho > 0$ such that $\rho \|x\|_\Delta^\tau \leq V(x)$.

Lemma 3[25]. For all $x, y \in R$ and a constant $p \geq 1$ the following inequalities hold:

(i) $|x + y|^p \leq 2^{p-1} |x^p + y^p|$,

$$\left(|x| + |y| \right)^{1/p} \leq |x|^{1/p} + |y|^{1/p} \leq 2^{(p-1)/p} \left(|x| + |y| \right)^{1/p}$$

If $p \in R_{odd}^{\geq 1}$, then

(ii) $|x - y|^p \leq 2^{p-1} |x^p - y^p|$, $|x^{1/p} - y^{1/p}| \leq 2^{(p-1)/p} |x - y|^{1/p}$.

Lemma 4[26]. Let c, d be positive constants. For any real-valued function $\gamma(x, y) > 0$, the following inequality holds:

$$|x|^c |y|^d \leq \frac{c}{c+d} \gamma(x, y) |x|^{c+d} + \frac{d}{c+d} \gamma^{-c/d}(x, y) |y|^{c+d}.$$

This paper deals with the practical output tracking problem by state feedback for time-delay high-order nonlinear systems (1). Here, we give a precise definition of the problem.

The problem of global practical tracking by a state feedback

Consider system (1) and assume that the reference signal $y_r(t)$ is a time-varying C^1-bounded function on $[0, \infty)$. For any given $\varepsilon > 0$, design a state feedback controller having the following structure

(2) $\quad u(t) = g(x(t), y_r(t)),$

such that

(i) All the state of the closed-loop system (1) with state controller (2) is well-defined and globally bounded on $[0, \infty)$.

(ii) For any initial condition, there is a finite time $T > 0$, such that

(3) $\quad |y(t) - y_r(t)| < \varepsilon, \quad \forall t \geq T > 0 \quad (3)$

In order to solve the global practical output tracking problem, we made the following two assumptions:

Assumption 1. There are constants C_1, C_2 and $\tau \geq 0$ such that

(4)
$$\begin{aligned} &|\varphi_i(t, x(t), \bar{x}(t - d_i), u(t))| \\ &\leq C_1 \Big(|x_1(t)|^{(r_i + \tau)/r_1} + \cdots + |x_i(t)|^{(r_i + \tau)/r_i} \\ &+ |x_1(t - d_1)|^{(r_i + \tau)/r_1} + \cdots + |x_i(t - d_i)|^{(r_i + \tau)/r_i} \Big) + C_2 \end{aligned}$$

where $\bar{x}(t - d_i) = x(t - d_1), x(t - d_2), \dots, x(t - d_n)$, $r_1 = 1, \quad r_{i+1} p_i = r_i + \tau > 0, \quad i = 1, \dots, n$ and $p_n = 1$.

Assumption 2. The reference signal $y_r(t)$ is continuously differentiable. Moreover, there is a known constant $D > 0$, such that

(5) $\quad |y_r(t)| + |\dot{y}_r(t)| \leq D, \quad \forall t \in [0, \infty)$

State Feedback Tracking Control Design

In this paper, we deal with the practical output tracking problem by delay-independent state feedback for high-order time-delay nonlinear systems (1) under Assumptions 1-2. To this end, we first introduce the following coordinate transformation:

(6) $\quad z_1 := x_1 - y_r, \quad z_i := \dfrac{x_i}{L^{\kappa_i}}, \quad i = 2, \dots, n, \quad v := \dfrac{u}{L^{\kappa_n + 1}}$

where $\kappa_1 = 0, \kappa_i = (\kappa_{i-1} + 1)/p_{i-1}$, $i = 2, \dots, n$ and $L \geq 1$ is a scaling gain to be determined later. Then, the system (1) can be described in the new coordinates z_i as

(7)
$$\begin{aligned} &\dot{z}_i = L z_{i+1}^{p_i} + \psi_i(t, z(t), z(t - d_i), v), \quad i = 1, \dots, n-1, \\ &\dot{z}_n = L v + \psi_n(t, z(t), z(t - d_n), v), \\ &y = z_1 \end{aligned}$$

where

(8)
$$\begin{aligned} &\psi_1(t, z(t), z(t - d_1), v) = \varphi_1(t, z(t), z(t - d_1), v) - \dot{y}_r, \\ &\psi_i(t, z(t), z(t - d_i), v) = \varphi_i(t, z(t), z(t - d_i), v) / L^{\kappa_i}, \\ &\qquad i = 2, \dots, n. \end{aligned}$$

Now, using Assumption 1, Lemma 3, the fact that $L \geq 1$ and the boundedness of y_r and $\dot{y}_r$ guaranteed by Assumption 2, ensures the existence of constants $\bar{C}_i$, $i = 1, 2$ only depending on constants $C_1, C_2, \tau, \kappa_i, L$ under which (4) becomes

(9)
$$\begin{aligned} &|\psi_1(t, z(t), z(t - d_1), v)| \leq \\ &\bar{C}_1 \Big(|z_1(t)|^{(r_1 + \tau)/r_1} + |z_1(t - d_1)|^{(r_1 + \tau)/r_1} \Big) + \bar{C}_2 \\ &|\psi_i(t, z(t), z(t - d_i), v)| \leq \\ &\bar{C}_1 L^{1 - \nu_i} \sum_{j=1}^{i} \Big(|z_j(t)|^{(r_i + \tau)/r_j} + |z_j(t - d_i)|^{(r_i + \tau)/r_j} \Big) + \frac{\bar{C}_2}{L^{\kappa_i}}, \\ &i = 2, \dots, n \end{aligned}$$

where $\quad \nu_i := \min\{1 - \kappa_j (r_i + \tau)/r_j + \kappa_i, \ 2 \leq j \leq i, \ 1 \leq i \leq n\} > 0$

and $\bar{C}_1 > 0$, $\bar{C}_2 > 0$ are some constants.

In what follows, we will employ the homogeneous domination approach to construct a global state feedback controller for system (7).

Stability Analysis

First, we construct a homogeneous state feedback controller for the nominal nonlinear system without considering the non-linearity of $\psi_i(\cdot)$, $i = 1, \dots, n-1$ in (7), i.e.,

(10) $\dot z_i = L z_{i+1}^{p_i},\quad i=1,\dots,n-1,\quad \dot z_n = Lv,\quad y = z_1$

Using similar the approach in [19, 25-26], we can design a homogeneous state feedback stabilizer for (8), which can be described in the following Theorem 1.

Theorem 1. For a real given number $\tau \ge 0$, there is a homogeneous state feedback controller of degree τ such that the nonlinear systems (10) is globally asymptotically stable.

Proof. To prove the result, we use an inductive argument (recursive design method) to explicitly construct a homogeneous stabilizer for system (10).

Initial step 1. Let $\xi_1 = z_1^{\sigma/r_1} - z_1^{*\sigma/r_1}$, where $z_1^* = 0$ and $\sigma \ge \max_{1\le i\le n}\{1, \tau + r_i\}$ is a positive number. Choose the Lyapunov function

(11) $$V_1 = W_1 = \int_{z_1^*}^{z_1}\left(s^{\sigma/r_1} - z_1^{*\sigma/r_1}\right)^{(2\sigma-\tau-r_1)/\sigma} ds$$

From (10), it follows that

(12) $$\dot V_1 \le -nL\xi_1^2 + L\xi_1^{(2\sigma-\tau-r_1)/\sigma}\left(z_2^{p_1} - z_2^{*p_1}\right)$$

where z_2^* the virtual controller and it is chosen as

(13) $$z_2^* = -n^{1/p_1} z_1^{(r_1+\tau)/p_1} := -\beta_1^{r_2/\sigma}\xi_1^{r_2/\sigma},\quad \beta_1 = n^{\sigma/(r_2 p_1)}$$

Step $k\ (k=2,\cdots,n)$. Suppose at the ***step k-1***, there is a C^1, positive definite and proper Lyapunov function V_{k-1}, and a set of virtual controllers $z_1^*,\dots,z_k^*$ defined by

(14) $$\begin{aligned} &z_1^* = 0, && \xi_1 = z_1^{\sigma/r_1} - z_1^{*\sigma/r_1} \\ &z_i^* = -\beta_{i-1}^{r_i/\sigma}\xi_{i-1}^{r_i/\sigma}, && \xi_i = z_i^{\sigma/r_i} - z_i^{*\sigma/r_i},\ i=2,\cdots,k \end{aligned}$$

with $\beta_i > 0,\ 1 \le i \le k-1$ being constants, such that

(15) $$\dot V_{k-1} \le -(n-k+2)L\sum_{l=1}^{k-1}\xi_l^2 + L\xi_{k-1}^{(2\sigma-\tau-r_{k-1})/\sigma}\left(z_k^{p_{k-1}} - z_k^{*p_{k-1}}\right)$$

We claim that (15) also holds at *Step k,* i.e., there is a C^1, proper, positive definite Lyapunov function defined by

(16) $$\begin{aligned} &V_k(\bar z_k) = V_{k-1}(\bar z_{k-1}) + W_k(\bar z_k), \\ &W_k(\bar z_k) = \int_{z_k^*}^{z_k}\left(s^{\sigma/r_k} - z_k^{*\sigma/r_k}\right)^{(2\sigma-\tau-r_k)/\sigma} ds \end{aligned}$$

and virtual controller $z_{k+1}^* = -\beta_k^{r_{k+1}/\sigma}\xi_k^{r_{k+1}/\sigma}$ such that

(17) $$\dot V_k \le -(n-k+1)L\sum_{j=1}^{k}\xi_j^2 + L\xi_k^{(2\sigma-\tau-r_k)/\sigma}\left(z_{k+1}^{p_k} - z_{k+1}^{*p_k}\right)$$

Since the prove of the claim (17) is very similar [7-8, 22], so omitted here.

Using the inductive argument above, we can conclude that at the n-th step, there exists a state feedback controller of the form

(18) $$v = -\beta_n^{r_{n+1}/\sigma}\xi_n^{r_{n+1}/\sigma} = -\left(\sum_{i=1}^{n}\bar\beta_i z_i^{\sigma/r_i}\right)^{r_{n+1}/\sigma}$$

with the C^1, proper and positive definite Lyapunov function,

(19) $$V_n = \sum_{i=1}^{n}\int_{z_i^*}^{z_i}\left(s^{\sigma/r_i} - z_i^{*\sigma/r_i}\right)^{(2\sigma-\tau-r_i)/\sigma} ds$$

we arrive at

(20) $$\dot V_n \le -L\sum_{j=1}^{n}\xi_j^2,$$

where $\xi_i = z_i^{\sigma/r_i} - z_i^{*\sigma/r_i}$ and $\bar\beta_i = \beta_n\cdots\beta_i,\ i=1,\dots,n$ are positive constants. Thus, the closed-loop system (10) and (18) is globally asymptotically stable.

Tracking control design for the time-delay nonlinear system (1)

Now, we are ready to use the homogeneous domination approach to design a global tracking controller for the system (1), i.e., state the following main result in this paper.

Theorem 2. For the time-delay nonlinear system (1) under Assumptions 1-2, the global practical output tracking problem is solvable by the state feedback controller $u = L^{\kappa_n+1}v$ in (7) and (18).

Proof. From (18), we have

(21) $$v = -\beta_n^{r_{n+1}/\sigma}\xi_n^{r_{n+1}/\sigma} = -\left(\sum_{i=1}^{n}\bar\beta_i z_i^{\sigma/r_i}\right)^{r_{n+1}/\sigma}$$

Now, we define the compact notations

(22) $$\begin{aligned} &z = \left(z_1,\dots,z_n\right)^T, \\ &E(z) = \left(z_2^{p_1},\dots,z_n^{p_{n-1}}, v\right)^T, \\ &F(z) = \left(\varphi_1, \varphi_2/L^{\kappa_2},\dots,\varphi_n/L^{\kappa_n}\right)^T. \end{aligned}$$

Using the same notation (7) and (22), the closed-loop system (7) - (18) can be written as the following compact form:

(23) $$\dot z = LE(z) + F(z)$$

Moreover, by introducing the dilation weight $\Delta = (r_1,\dots,r_n)$, from Definition 1, it can be shown that V_n is homogeneous of degree $2\sigma - \tau$ with respect to Δ.

Hence, adopting the same Lyapunov function (19) and by Lemma 2 and Lemma 3, it can be concluded that

(24) $$\begin{aligned} &\dot V_n(z) = L\frac{\partial V_n}{\partial Z}E(z) + \frac{\partial V_n}{\partial Z} \\ &\dot F(z) \le -m_1 L\|z\|_\Delta^{2\sigma} + \sum_{i=1}^{n}\frac{\partial V_n}{\partial z_i}\psi_i \end{aligned}$$

where $m_1 > 0$ is constant.

By (9), Assumption 1 and L > 1, we can find constants $\delta_i > 0$ and $0 < \gamma_i \le 1$ such that

(25) $$|\psi_i| \le \delta_i L^{1-\gamma_i}\left(\|z(t)\|_\Delta^{r_i+\tau} + \|z(t-d_j(t))\|_\Delta^{r_i+\tau}\right) + \bar C_2/L^{\kappa_i}$$

and noting that for $i=1,\dots,n$, by Lemma2, $\partial V_n/\partial z_i$ is homogeneous of degree $2\sigma-\tau-r_i$,

(26) $$\left|\frac{\partial V_n}{\partial z_i}\right| \le m_2\|z\|_\Delta^{2\sigma-\tau-r_i},\qquad m_2 > 0$$

We construct a Lyapunov-Krasovskii functional as follows:

(27) $$V(z(t)) = V_n(z(t)) + \int_{t-d}^{t}\|z(s)\|_\Delta^{2\sigma}\eta ds,$$

where η is a positive constant. Let $\eta = m_3\sum_{i=1}^{n}L^{1-\gamma_i}$ follows from (29) and (30) that

(28) $$\dot V \le -L\left(m_1 - (2 + m_2(1+\delta) + m_3)\sum_{i=1}^{n}L^{-\gamma_i}\right)\|z(t)\|_\Delta^{2\sigma} + \frac{\rho_1}{L^{1+\gamma}}.$$

Hence, by choosing a large enough *L* as $L > \max\left\{1, (((2+m_2(1+\delta)+m_3)/m_1))^{-\gamma}\right\}$, where $\gamma = \min_{1\le i\le n}\{\gamma_i\}$ and $\rho_1 = \sum_{i=1}^{n}\alpha^{2\sigma/(\tau+r_i)}$.

Then, there exists a constant $\rho_2 > 0$, such that

(29) $$\dot V\ (z(t)) \le -\rho_2\|z(t)\|_\Delta^{2\sigma} + 2\rho_1.$$

Moreover, $V_n(z)$ and $\int_{t-d}^{t}\|z(s)\|_\Delta^{2\sigma}\eta ds$ are homogeneous of degree $2\sigma-\tau$ and 2σ with respect to Δ, accordingly. Therefore, by Lemma 2, there are positive constants $\lambda_1,\ \lambda_2,\ \lambda_3$ and λ_4 such that

(30) $$\begin{aligned} &\lambda_1\|z(t)\|_\Delta^{2\sigma-\tau} \le V_n(z(t)) \le \lambda_2\|z(t)\|_\Delta^{2\sigma-\tau} \\ &\lambda_3\|z(t)\|_\Delta^{2\sigma} \le \int_{t-d}^{t}\|z(s)\|_\Delta^{2\sigma}\eta ds \le \lambda_4\|z(t)\|_\Delta^{2\sigma}, \end{aligned}$$

Therefore combining (29) and (30) yields

(31) $\dot{V}(z(t)) \le -\rho_2^{-1} V(z(t)) + \tilde{\rho}_1,$

where $\tilde{\rho}_1 = \tau \lambda_2^{(\tau-2\sigma)/\tau} \Big/ \left(2\sigma\rho_2 L^{(2\sigma-\tau)/\tau} \right) + \rho_1 \big/ L^{1+\gamma}$ and $\rho_2 = \left(\lambda_4 + (2\delta - \tau)/2\sigma \right)$.

From (31) it is not difficult to show that there is a finite time $T > 0$, such that

(32) $V(z) \le 3\tilde{\rho}_1, \quad \forall t \ge T$

from which it is clear that z_1 can be rendered smaller than any positive tolerance with a sufficiently large L.

Conclusion

In this paper, we have studied the practical output tracking problem for a type of undetermined high-order nonlinear time-delay systems under a homogeneous mild condition. We have constructed a homogeneous state feedback controller constructed with adjustable scaling gains. Then, we have redesigned the homogeneous domination approach to tune the scaling gain for the overall the closed loop systems with the help of a homogeneous Lyapunov-Krasovskii functional. It is shown that an appropriate choice of gain will enable us to globally track for a class of high-order uncertain nonlinear time-delay systems.

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