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Application of Gaussian Kernel with Regard to Correlations for Image Reconstruction in Electrical Tomography

***Abstract**. The article presents the application of support methods Vector Machine for Regression and Support Vector Machine for Regression with a modified correlation kernel in electrical impedance tomography. Statistical methods have been used to reconstruct imaging. In addition, a model was created that analyses similar objects of different sizes. It learns about a smaller object, but we recognize a larger object. The paper shows how to make an analysis for such cases.*

***Streszczenie.** W artykule przedstawiono aplikację opartą na metodach maszyna wektorów nośnych do regresji i maszyna wektorów nośnych do regresji z zmodyfikowanym jądrem korelacji w elektrycznej tomografii impedancyjnej. Metody statystyczne zostały wykorzystana do rekonstrukcji obrazuj. Dodatkowo stworzono model, który analizuje podobne obiekty o różnych rozmiarach. Uczy się na obiekcie o mniejszych gabarytach, natomiast rozpoznajemy obiekt o większym rozmiarze. W pracy pokazano w jaki sposób dokonywać analizę dla takich przypadków.* (**Zastosowanie jądra gaussowskiego z uwzględnieniem korelacji do rekonstrukcji obrazu w tomografii elektrycznej**).

Keywords: Electrical Tomography, Inverse Problem, SVR, Gaussian Kernel
Słowa kluczowe: tomografia elektryczna, zagadnienie odwrotne, SVR, jądro Gaussa

Introduction

This article proposes a new solution based on the analysed methods that enable the proper reproduction of the image. This work gives promising results as a new horizon to solve practical problems. The Support Vector Machine for Regression with Gaussian kernel was implemented. A regression method gives more accurate and stable reconstruction results in solving the inverse problem in electrical tomography. There are many ways to solve the optimization problem [1-10]. The statistical methods [11] were used to reconstruct the image in electrical impedance tomography. The main objective of the tomography is to perform image reconstruction. During the measurements, we can see that the measured values from some electrodes are strongly correlated (due to the way of measurement). In this case, we have a multicollinearity problem.

Electrical impedance tomography (EIT) is an ill-posed inverse problem. In the EIT, the electrical voltages are injected into the object using a set of electrodes attached to the object's surface, and the potentials are measured. The object's conductivity is reconstructed on the basis of known voltages and measured potentials. Reconstruction of electrical impedance tomography requires accurate modelling. EIT is a method of imaging in which the conductivity distribution of the tested object is estimated on the basis of measurements of electrical voltages and potentials of electrodes at the boundary. To obtain quantitative information on the change in conductivity, it would be better to use a non-linear model in the differential imaging solution [12-14].

In the case when the objects are different (in the sense of size), the grid is made, then the model parameters are estimated, and only in the final phase the reconstruction is a labour-intensive process. The approach used below is an attempt to create a model that would analyse similar objects of different sizes - we learn on a smaller object, but we recognize a larger object. The work shows how to make an analysis for the tested object. As we will see below, even the number of finite elements for grids varies.

Support Vector Machine

Support Vector Machine (SVM) is in a way an abstract concept of a learning machine whose main task is to classify objects or regression analysis. In the case of object classification, the SVM algorithm builds a model by means of which new objects are classified into one of two categories (it consists in constructing a non-stable binary classifier). Classification of objects takes place by designating a hyperplane that separates objects belonging to different classes (including the margin of error). SVM is quite often used in the process of recognizing images, text, handwriting. SVM is widely used in various fields of science.

The classic SVM algorithm with which the linear classifier was determined was proposed by Vladimir Vapnik and Alexey Chervonenkis in 1963. A method using nonlinear classification of objects was proposed by Bernhard Boser, Isabelle Guyon and Vladimir Vapnik in 1992. It involves using nuclei to determine the hyper-planes, by means of which objects are classified. The construction of the classifier taking into account the margin of error was proposed by Corinna Cortes and Vapnik in 1993. SVM version including regression analysis was proposed in 1996 by Vladimir Vapnik, Harris Drucker, Christopher Burges, Linda Kaufman and Alexander Smola. The method in the literature is called Support Vector Regression (SVR). The regression model is determined by the carrier vectors that belong to the training data set [15-17].

Let $D = \{(x_i, y_i): x_i \in R^m, y_i \in \{-1,1\}, 1 \le i \le n\}$ means a set of training data. Elements of the sequence $\{x_i\}_{1\le i\le n}$ belong to two classes, where we belong to the class as $y_i \in \{-1,1\}$ for $1 \le i \le n$. Observing the element $x \in R^m$ should be classified belonging to the appropriate class.

So the task is to find such a classifier (Support Vector Classifier), which would allow the best to classify objects $x \in R^m$ for different classes. In the case under consideration, a hyperplane should be found that would allow to divide these two classes. Classifier should be constructed $f : R^m \to \{-1,1\}$.

Definition 1. Two classes are linearly separable if there is such a hyperplane $f(x) = 0$ of the form

(1) $$f(x) = \beta^T x + \beta_0 ,$$

where for $(x, y) \in D$, $\beta \in R^n$, $\beta_0 \in R$ the condition is met

(2) $$\begin{cases} f(x) > 0 & \text{for } y = 1 \\ f(x) < 0 & \text{for } y = -1 \end{cases}$$

The condition (2) can be presented in the form

(3) $f(x)y > 0$,

Setting a hyperplane $f(x) = 0$ consists in solving the task

(4) $\max_{\beta,\beta_0,M} M$,

with limitations

(5) $\begin{cases} \beta^T\beta = 1 \\ f(x_i)y_i \geq M \text{ dla } 1 \leq i \leq n. \end{cases}$,

The size M is the class separating margin. In case when not all objects can be delimited, then we consider the task taking into account the slack variables $\varepsilon_i > 0$, $1 \leq i \leq n$ and, additionally, we are considering incurring

(6) $\max_{\beta,\beta_0,\varepsilon_1,\ldots,\varepsilon_n,M} M$,

with limitations

(7) $\begin{cases} \beta^T\beta = 1 \\ f(x_i)y_i \geq M(1-\varepsilon_i) \text{ dla } 1 \leq i \leq n \\ \varepsilon_i > 0, 1 \leq i \leq n \text{ oraz } \sum_{i=1}^{n}\varepsilon_i < C \end{cases}$,

Let the points x_i and x_j lie on the boundaries of the margin of delimitation, except that

(8) $\begin{aligned} \beta^T x_i + \beta_0 = M, \\ \beta^T x_j + \beta_0 = -M. \end{aligned}$,

then $\beta^T(x_i - x_j) = 2M$

or $\frac{\beta^T}{\|\beta\|}(x_i - x_j) = 2\frac{M}{\|\beta\|}$

Task (7) with conditions (8) is reduced to the task

(9) $\min_{\beta,\beta_0,\varepsilon_1,\ldots,\varepsilon_n} \frac{\|\beta\|}{2} + \frac{C}{n}\sum_{i=1}^{n}\varepsilon_i$,

with limitations

(10) $\begin{cases} f(x_i)y_i \geq M(1-\varepsilon_i) \text{ dla } 1 \leq i \leq n \\ \varepsilon_i > 0, 1 \leq i \leq n. \end{cases}$,

In case the class separation boundary is not linear, we use the SVM extension. Consider a hyperplane

(11) $f_h(x) = \sum_{i=1}^{n}\beta_i h_i(x) + \beta_0$, ,

stretched on n basis functions $h_i : R^m \to R$ (linear or nonlinear) for $1 \leq i \leq n$, $\beta = (\beta_1,\ldots,\beta_n)$, and $h : R^m \to R^n$.

Definition 2. Two classes are non-linearly separable if there is such a hyperplane $f_h(x) = 0$ of the form

(12) $f_h(x) = \beta^T h(x) + \beta_0$,

where for $(x, y) \in D$, $\beta \in R^n$, $\beta_0 \in R$ the condition is met

$\begin{cases} f_h(x) > 0 & \text{for } y = 1 \\ f_h(x) < 0 & \text{for } y = -1 \end{cases}$

Solving the task (9) with constraints (10), we determine the coefficients in equation (12). We are looking for a solution where coefficients w (12) are given a formula

(13) $\beta = \sum_{i=1}^{n}\alpha_i y_i h(x_i)$.,

From the formula (13), the hyperplane (12) can be represented in the form

(14) $f_h(x) = \sum_{i=1}^{n}\alpha_i y_i \langle h(x_i), h(x)\rangle + \beta_0$,

Function $k : R^m \times R^m \to R$ form

$k(x, x') = \langle h(x), h(x')\rangle$

we call it kernel. The classic kernels used in SVM:

- linear function

$$k(x, x') = \langle x, x'\rangle$$

- polynomial function

$$k(x, x') = (\theta\langle x, x'\rangle + \eta)^\kappa,$$

- Gaussian function

$$k(x, x') = \exp\left(-\sigma\|x - x'\|^2\right),$$

- Laplace function

$$k(x, x') = \exp\left(-\sigma\|x - x'\|\right),$$

- hyperbolic tangent

$$k(x, x') = \tanh(\theta\langle x, x'\rangle + \eta),$$

- ANNOVA function

$$k(x, x') = \left(\sum_{i=1}^{m}\exp\left(-\sigma(x_i - x_i')^2\right)\right)^d$$

Instead of task (9) with constraints (10), we solve the dual task

(15) $\max_{\alpha_1,\ldots,\alpha_n} \sum_{i=1}^{n}\alpha_i - \frac{1}{2}\sum_{i,j=1}^{n}\alpha_i\alpha_j y_i y_j k(x_i, x_j)$,

with limitations

(16) $\begin{cases} 0 \leq \alpha_i \leq \frac{C}{n} \text{ dla } 1 \leq i \leq n \\ \sum_{i=1}^{n}\alpha_i y_i = 0. \end{cases}$,

Support Vector Machine for Regression

Let $D = \{(x_i, y_i) : x_i \in R^m, y_i \in R, 1 \leq i \leq n\}$ means a set of training data. The regression spread on support vectors (Support Vector Machine for Regression) consists in determining the regression coefficients defined by the formula (1). Let

$\|y - f_h(x)\|_\varepsilon = \max\{0, \|y - f_h(x)\| - \varepsilon\}$

means the loss intensity function above $\varepsilon > 0$, we omit losses below ε. To determine the regression, we solve the task

(17) $\min_{\beta,\beta_0,\varepsilon_1,\ldots,\varepsilon_n} \frac{\|\beta\|^2}{2} + \frac{C}{n}\sum_{i=1}^{n}(\xi_i + \xi_i^*)$,

with limitations

(18) $\begin{cases} f_h(x_i) - y_i \leq \varepsilon - \xi_i \text{ dla } 1 \leq i \leq n, \\ y_i - f_h(x_i) \leq \varepsilon - \xi_i^* \text{ dla } 1 \leq i \leq n, \\ \xi_i, \xi_i^* > 0 \text{ dla } 1 \leq i \leq n. \end{cases}$,

We are looking for a solution where the coefficients in (12) are in form

(19) $$\beta = \sum_{i=1}^{n} \left(\alpha_i^* - \alpha_i\right) h\left(x_i\right),$$

From the formula (19), the hyperplane (12) can be represented in the form

(20) $$f_h(x) = \sum_{i=1}^{n} \left(\alpha_i^* - \alpha_i\right) \left\langle h\left(x_i\right), h(x) \right\rangle + \beta_0 ,$$

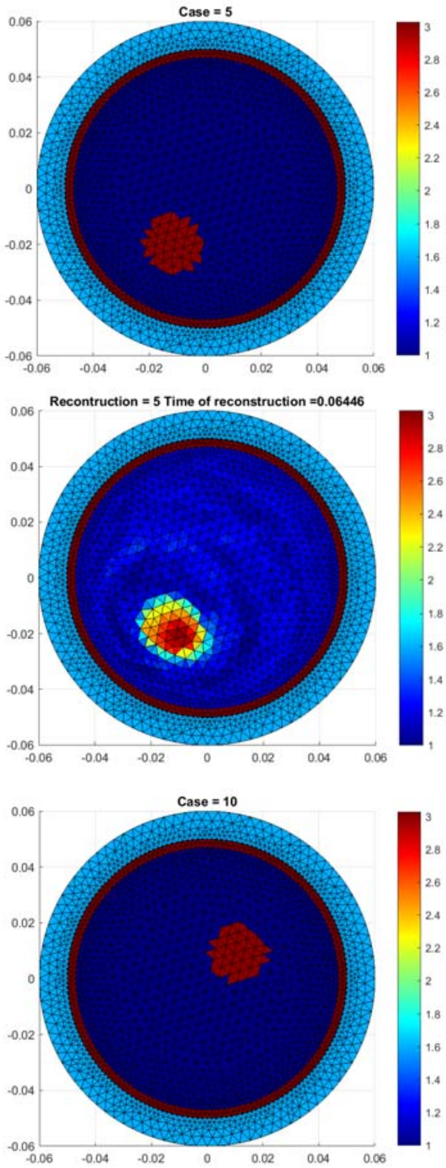
Dual task to task (17) with restrictions (18)

(21)
$$\max_{\alpha_1,\ldots,\alpha_n} \sum_{i=1}^{n} \left(\alpha_i^* - \alpha_i\right) y_i - \frac{1}{2} \sum_{i,j=1}^{n} \left(\alpha_i^* - \alpha_i\right)\left(\alpha_j^* - \alpha_j\right) k\left(x_i, x_j\right) - \varepsilon \sum_{i=1}^{n} \left(\alpha_i^* + \alpha_i\right) h\left(x_i\right)$$

with limitations

(22) $$\begin{cases} 0 \le \alpha_i \le \dfrac{C}{n} \text{ dla } 1 \le i \le n \\ \displaystyle\sum_{i=1}^{n} \left(\alpha_i^* - \alpha_i\right) = 0 . \end{cases},$$

The image reconstruction by SVR method in electrical tomography model was presented in Fig.1.



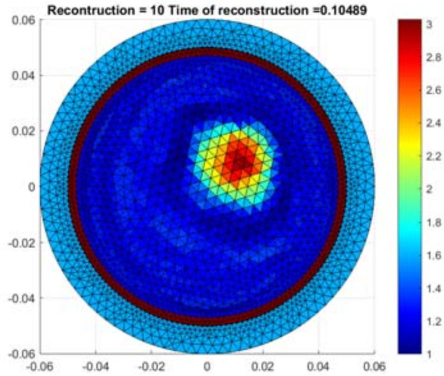


Fig. 1. Image reconstruction by SVR

SVR with a modified correlation kernel

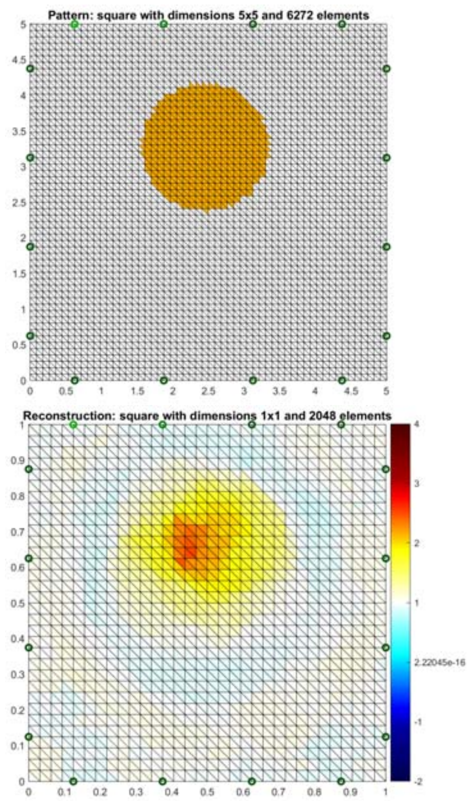
In the case when the objects are of various sizes, then the grid is made, then the model parameters are estimated, and only in the final stage the reconstruction is a labour-intensive process.

The approach used below is an attempt to create a model that would analyse similar objects of different sizes - we learn on a smaller object, but we recognize a larger object. The work shows how to make an analysis for the tested object.

Instead of the classical Gaussian kernels, a modified Gaussian kernels using a correlation coefficient was used

(23) $$k(x, x') = \exp\left(-\sigma \cdot cor(x, x')\right),$$

As we will see in Figures, even the number of finite elements for grids varies. SVM Regression was used for reconstruction. The image reconstruction by SVR with a modified correlation kernel method in electrical tomography model was presented in Fig.1.



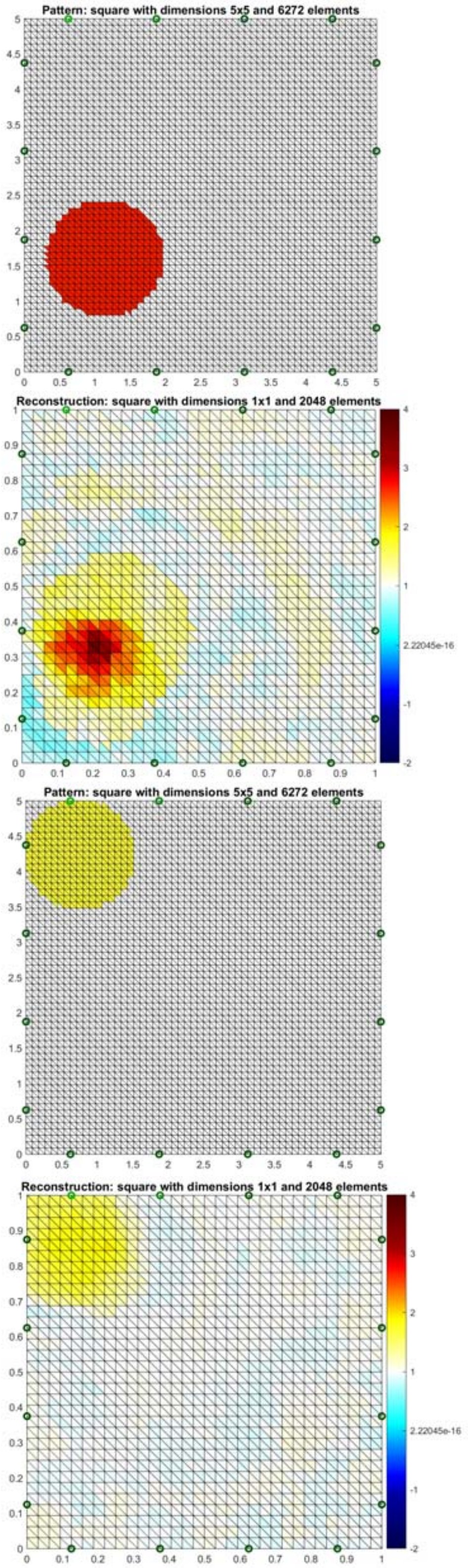


Fig. 2. SVR Image reconstruction with a modified correlation kernel

Summary

In this work, there was presented the solution using statistical method to solve the inverse problem in electrical impedance tomography. Proposed algorithms based on the Support Vector Machine were proposed such as: SVR and SVR with a modified correlation kernel in electrical impedance tomography. A model has been created that analyses similar objects of different sizes. The constructed algorithm and the scientist are on a smaller object, and then it attempts to recognize the object on another discretization of the grid. The analysed methods allow for proper image reproduction. This work gives promising results as a new horizon to solve practical problems.

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