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CPC – Based Reactive Balancing of Linear Loads in Four-Wire Supply Systems with Nonsinusoidal Voltage

Abstract. *The paper provides the Currents' Physical Components (CPC)-based fundamentals for reactive balancing of unbalanced loads supplied by a nonsinusoidal voltage in four-wire systems, meaning with a neutral conductor. The whole compensation of the unbalanced and reactive currents in such conditions requires reactive compensators of high complexity, i.e., built of a high number of reactive devices. This complexity can be reduced on the condition that the objective of a whole compensation is abandoned for a goal of only minimization of the supply current three-phase rms value. The paper presents a method of minimization of this three-phase rms value of the supply current by a compensator composed of branches, that have no more than two reactive devices, meaning an inductor and a capacitor.*

Streszczenie. *Artykuł przedstawia podstawy, oparte na teorii Składowych Fizycznych Prądów (Ang. CPC), reaktancyjnego równoważenia odbiorników niezrównoważonych, zasilanych napięciem niesinusoidalnym w układach czteroprzewodowych, to jest z przewodnikiem zerowym. Całkowita kompensacja prądu niezrównoważenia i prądu biernego wymaga w takich warunkach kompensatorów o wysokiej złożoności, to jest o dużej liczbie elementów reaktancyjnych. Złożoność tę można obniżyć przez rezygnację z całkowitej kompensacji na rzecz jedynie minimalizacji trójfazowej wartości skutecznej prądu zasilania. Artykuł przedstawia metodę takiej minimalizacji z pomocą kompensatora zbudowanego z gałęzi mających nie więcej niż dwa elementy reaktancyjne, to jest cewkę indukcyjną i kondensator.* ***(Reaktancyjne równoważenie liniowych odbiorników trójfazowych zasilanych czteroprzewodowo napięciem niesinusoidalnym, oparte na teorii mocy Składowych Fizycznych Prądów (Ang.- CPC)).***

Keywords: Current decomposition, unbalanced loads, asymmetrical systems, power definitions, power theory.
Słowa kluczowe: Rozkład prądu, odbiorniki niezrównoważone, systemy niesymetryczne, definicje mocy, teoria mocy.

Introduction

Distribution systems in manufacturing plants supply not only balanced three-phase loads, but also aggregates of single-phase loads such as lightning, instrumentation and control systems, and electrical transportation. Distribution systems that supply such unbalanced loads are built as three-phase systems with neutral, i.e., as four-wire systems.

Such unbalanced loads cause asymmetry of voltages and currents in the distribution system. This asymmetry, along with the reactive power, contributes to an increase in energy loss, to equipment overloading and to the reduction of the supply quality in such a system. Consequently, a reduction of the reactive power and asymmetry in such systems is often needed.

Because of the power level, compensation of the reactive and unbalanced currents in large manufacturing plants could be above the capability of switching compensators, known as "active power filters", however. Switching compensators are built of power transistors, used for shaping the compensating currents, and such transistors have relatively low switching power. Only reactive compensators can have a sufficient power for that.

Balancing compensators for three-wire systems with sinusoidal voltage were first developed by Steinmetz in 1917 [1] and there is a substantial amount of reported research on such compensators design [3-4, 9]. Although some results on methods of design of compensators for four-wire systems were published [8, 10, 11, 14, 15], the development of such methods is substantially retarded. A controversy [12, 13] on how to describe the power properties of four-wire systems is the main reason for that.

In the lack of such description, only optimization methods [11] could provide parameters of a compensator. Optimi-zation methods might not be appropriate for control of adap-tive reactive compensators, when the speed of control is crucial, however. Formulae that would enable direct calcula-tion of the compensator LC parameters are needed.

Such formulae for three-wire systems, at the assumption that the load is linear and time-variant (LTI), while the supply voltage can be nonsinusoidal, were developed in [6]. They were developed using the Currents' Physical Components (CPC) – based power theory.

A method of a reactive balancing compensator synthesis for four-wire systems at the assumption that the supply voltage is sinusoidal was developed in [17]. At the same time, the CPC-based power theory was generalized in [16] to four-wire systems with nonsinusoidal supply voltage. The results obtained in [16] and [17] provide a starting point for the development of a method of balancing four-wire systems with nonsinusoidal supply voltage, presented in this paper.

The presented method is confined to balancing linear loads with fixed parameters, while usually parameters are not fixed, and a compensator should have an adaptive property. Unfortunately, the space available in a single paper does not allow for studies on adaptive compensation.

This paper is a continuation of [16] and [17] and conse-quently, most symbols have the same meanings. Therefore, it is recommended that the reader is acquainted with these two papers. Nonetheless, an introduction to the CPC-based power theory of four-wire systems with nonsinusoidal supply voltage can be beneficial for the reader. A draft of it is presen-ted in the next Section.

A draft of CPC of four-wire systems

This section summarizes power properties of unbalanced linear loads, composed of aggregates of single-phase linear loads and three-phase similar loads, supplied with a symmet-rical but nonsinusoidal voltage, by a four-wire line, as des-cribed in details in terms of the Currents' Physical Compo-nents (CPC) – based power theory in [16].

A reader of this paper should be aware, since this is crucial for power properties of systems with nonsinusoidal voltages and currents and their compensation, that the association of a harmonic of specific order with symmetrical component of a specific sequence, is valid only on the condition that this harmonic is symmetrical. In particular, the 3rd order current harmonic, commonly regarded as the zero sequence component, when it is asymmetrical, i.e., when it has different values in different supply lines, can have also the positive and the negative sequence symmetrical compo-nents. The same applies to all harmonics.

Any linear time-invariant load (LTI) load in a four-wire system has an equivalent load, composed of three fictitious single-phase loads connected to the neutral conductor as shown in Fig. 1.

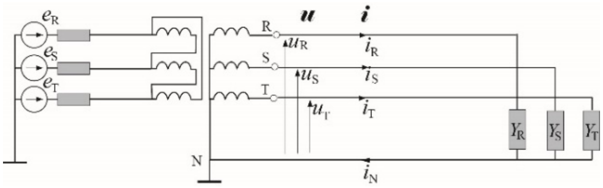

Fig. 1. A four-wire system with fictitious single-phase loads.

Let us assume that the supply voltage is nonsinusoidal but symmetrical. It can be presented in the form

$$\text{(1)}\quad \mathrm{u}(t)=\sum_{n\in N}\mathrm{u}_n(t)=\sqrt{2}\mathrm{Re}\sum_{n\in N}\begin{bmatrix}\boldsymbol{U}_{\mathrm{R}n}\\ \boldsymbol{U}_{\mathrm{S}n}\\ \boldsymbol{U}_{\mathrm{T}n}\end{bmatrix}e^{jn\omega_1 t}=\sqrt{2}\mathrm{Re}\sum_{n\in N}U_n e^{jn\omega_1 t}.$$

The supply currents can be presented as follows

$$\text{(2)}\quad \boldsymbol{i}(t)=\sum_{n\in N}\boldsymbol{i}_n(t)=\sqrt{2}\mathrm{Re}\sum_{n\in N}\begin{bmatrix}\boldsymbol{I}_{\mathrm{R}n}\\ \boldsymbol{I}_{\mathrm{S}n}\\ \boldsymbol{I}_{\mathrm{T}n}\end{bmatrix}e^{jn\omega_1 t}=\sqrt{2}\mathrm{Re}\sum_{n\in N}I_n e^{jn\omega_1 t}.$$

Symbol N in these formulae denotes the set of orders n of the supply voltage dominating harmonics.

The admittances of the fictitious loads for harmonic frequ-encies $\boldsymbol{Y}_{\mathrm{R}n}$, $\boldsymbol{Y}_{\mathrm{S}n}$ and $\boldsymbol{Y}_{\mathrm{T}n}$ can be calculated having the complex rms (crms) values of the line currents and the voltage harmo-nics, measured at the load terminals, namely

$$\text{(3)}\quad \boldsymbol{Y}_{Ln}=G_{Ln}+jB_{Ln}=\frac{\boldsymbol{I}_{Ln}}{\boldsymbol{U}_{Ln}},\qquad L=\mathrm{R,\ S\ or\ T}\ .$$

As it was demonstrated in [16] the load current can be decomposed into six Physical Components, such that

$$\text{(4)}\quad \boldsymbol{i}=\boldsymbol{i}_{\mathrm{a}}+\boldsymbol{i}_{\mathrm{s}}+\boldsymbol{i}_{\mathrm{r}}+\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}+\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}+\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}\ .$$

In this decomposition

$$\text{(5)}\quad \boldsymbol{i}_{\mathrm{a}}\overset{\mathrm{df}}{=}\begin{bmatrix}i_{\mathrm{Ra}}\\ i_{\mathrm{Sa}}\\ i_{\mathrm{Ta}}\end{bmatrix}=G_{\mathrm{e}}\boldsymbol{u}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}G_{\mathrm{e}}\,U_n\,e^{jn\omega_1 t}$$

is the active current, where

$$\text{(6)}\quad G_{\mathrm{e}}=\frac{P}{\|\boldsymbol{u}\|^2}$$

is referred to as an ***equivalent conductance*** of the load. The component

$$\text{(7)}\quad \boldsymbol{i}_{\mathrm{r}}=\sum_{n\in N}\boldsymbol{i}_{\mathrm{r}n}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}jB_{\mathrm{e}n}\,U_n\,e^{jn\omega_1 t}$$

is the reactive current, where

$$\text{(8)}\quad B_{\mathrm{e}n}=-\frac{Q_n}{\|\boldsymbol{u}_n\|^2}=\frac{1}{3}\mathrm{Im}\{\boldsymbol{Y}_{\mathrm{R}n}+\boldsymbol{Y}_{\mathrm{S}n}+\boldsymbol{Y}_{\mathrm{T}n}\}$$

is ***equivalent susceptance*** of the load for the n^{th} order harmonic.
The component

$$\text{(9)}\quad \boldsymbol{i}_{\mathrm{s}}=\sqrt{2}\mathrm{Re}\sum_{n\in N}(G_{\mathrm{e}n}-G_{\mathrm{e}})\,U_n e^{jn\omega_1 t}$$

is the scattered current, where

$$\text{(10)}\quad G_{\mathrm{e}n}=\frac{P_n}{\|\boldsymbol{u}_n\|^2}=\frac{1}{3}\mathrm{Re}\{\boldsymbol{Y}_{\mathrm{R}n}+\boldsymbol{Y}_{\mathrm{S}n}+\boldsymbol{Y}_{\mathrm{T}n}\}$$

is ***equivalent conductance*** of the load for the n^{th} order har-monic. The component

$$\text{(11)}\quad \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}=\sum_{n\in N}\boldsymbol{i}_{\mathrm{u}n}^{\mathrm{p}}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}}1^{\mathrm{p}}\boldsymbol{U}_{\mathrm{R}n}e^{jn\omega_1 t}$$

is the unbalanced current of the positive sequence, where

$$\text{(12)}\quad \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}}=\frac{1}{3}[(\boldsymbol{Y}_{\mathrm{R}n}+\alpha\beta\boldsymbol{Y}_{\mathrm{S}n}+\alpha^*\beta^*\boldsymbol{Y}_{\mathrm{T}n})-\boldsymbol{Y}_{\mathrm{e}n}(1+\alpha\beta+\alpha^*\beta^*)]$$

with

$$\text{(13)}\quad \beta\overset{\mathrm{df}}{=}(\alpha^*)^n=\begin{cases}1, & \text{for}\quad n=3k\\ \alpha^*, & \text{for}\quad n=3k+1,\\ \alpha, & \text{for}\quad n=3k-1\end{cases}\qquad \alpha=1e^{j2\pi/3}\ .$$

is ***the positive sequence unbalanced admittance*** of the load of the n^{th} order harmonic. Observe that for the voltage harmonics_of the positive sequence, as specified by (13), $\alpha\beta$ = $\alpha\alpha^*$ = 1, so that formula (12) results in the zero value of this admittance.

The component

$$\text{(14)}\quad \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}=\sum_{n\in N}\boldsymbol{i}_{\mathrm{u}n}^{\mathrm{n}}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}}1^{\mathrm{n}}\boldsymbol{U}_{\mathrm{R}n}e^{jn\omega_1 t}$$

is the unbalanced current of the negative sequence, where

$$\text{(15)}\quad \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}}=\frac{1}{3}[(\boldsymbol{Y}_{\mathrm{R}n}+\alpha^*\beta\boldsymbol{Y}_{\mathrm{S}n}+\alpha\beta^*\boldsymbol{Y}_{\mathrm{T}n})-\boldsymbol{Y}_{\mathrm{e}n}(1+\alpha^*\beta+\alpha\beta^*)]$$

is ***the negative sequence unbalanced admittance*** of the load of the n^{th} order harmonic. Observe that for the voltage harmonics of the negative sequence, $\alpha^*\beta=\alpha^*\alpha$ = 1, so that formula (15) results in the zero value of this admittance.

The component

$$\text{(16)}\quad \boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}=\sum_{n\in N}\boldsymbol{i}_{\mathrm{u}n}^{\mathrm{z}}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}}1^{\mathrm{z}}\boldsymbol{U}_{\mathrm{R}n}e^{jn\omega_1 t}$$

is the unbalanced current of the zero sequence, where

$$\text{(17)}\quad \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}}=\frac{1}{3}[(\boldsymbol{Y}_{\mathrm{R}n}+\beta\boldsymbol{Y}_{\mathrm{S}n}+\beta^*\boldsymbol{Y}_{\mathrm{T}n})-\boldsymbol{Y}_{\mathrm{e}n}(1+\beta+\beta^*)]$$

is ***the zero sequence unbalanced admittance*** of the load of the n^{th} order harmonic. Observe that for the voltage harmo-nics of the zero sequence, $\beta=1$, so that (17) results in the zero value of this admittance. Symbols 1^{p}, 1^{n} and 1^{z} denote three-phase unite symmetrical vectors

$$\text{(18)}\quad \begin{bmatrix}1\\ \alpha^*\\ \alpha\end{bmatrix}\overset{\mathrm{df}}{=}1^{\mathrm{p}},\quad \begin{bmatrix}1\\ \alpha\\ \alpha^*\end{bmatrix}\overset{\mathrm{df}}{=}1^{\mathrm{n}},\quad \begin{bmatrix}1\\ 1\\ 1\end{bmatrix}\overset{\mathrm{df}}{=}1^{\mathrm{z}}.$$

The current components in decomposition (4) are mutu-ally orthogonal so that their three-phase rms values satisfy the relationship

$$\text{(19)}\quad \|\boldsymbol{i}\|^2=\|\boldsymbol{i}_{\mathrm{a}}\|^2+\|\boldsymbol{i}_{\mathrm{s}}\|^2+\|\boldsymbol{i}_{\mathrm{r}}\|^2+\|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}\|^2+\|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}\|^2+\|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}\|^2$$

and consequently, the power factor can be expressed in the form

$$\lambda = \frac{P}{S} = \frac{\|\boldsymbol{i}_{\mathrm{a}}\|}{\sqrt{\|\boldsymbol{i}_{\mathrm{a}}\|^2 + \|\boldsymbol{i}_{\mathrm{s}}\|^2 + \|\boldsymbol{i}_{\mathrm{r}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}\|^2}} \tag{20}$$

which shows how the scattered, reactive and unbalanced currents contribute to the power factor decline.

Compensation

The power factor can be improved by redaction of three-phase rms value of the useless currents $\boldsymbol{i}_{\mathrm{s}}, \boldsymbol{i}_{\mathrm{r}}, \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}, \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}$ or $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}$. The main question in this paper is whether this can be done by a shunt reactive compensator or not and if yes, how to calculate the compensator LC parameters?

An ideal, lossless shunt compensator cannot affect the active power and conductances G_{e} and $G_{\mathrm{e}n}$. It means that the scattered current $\boldsymbol{i}_{\mathrm{s}}$ is not affected by such a compensator. It means that in the presence of the supply voltage harmonics compensation to unity power factor is not possible.

As it was discussed in [17], a balancing reactive compensator of a load supplied by a four-wire line has to be built of two compensators, one of a Y-structure, and the second of the Δ-structure, as it is shown in Fig. 2.

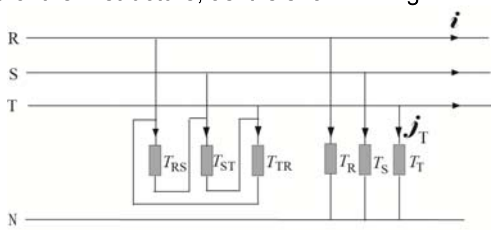

Fig. 2. A structure of reactive compensator.

The compensator properties at a sinusoidal supply voltage, are specified in terms of six susceptances, T_{R}, T_{S}, T_{T}, T_{RS}, T_{ST}, and T_{TR}, for the fundamental frequency ω_1. Symbol $\boldsymbol{j}_{\mathrm{T}}$ in Fig. 2 denotes a vector of six compensator's currents. Such a reactive compensator can compensate entirely the unbalanced current of the negative sequence, $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}$; the unba-lanced current of the zero sequence $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}$, and the reactive current $\boldsymbol{i}_{\mathrm{r}}$. Unfortunately, with now omnipresent voltage har-monics, this is no longer true.

When the supply voltage is nonsinusoidal, while the load, as assumed, is LTI, then the supply current of the load is a sum of current responses to individual voltage harmonics $\boldsymbol{u}_n(t)$. Consequently, compensation of such a load can be studied with a harmonic-by-harmonic approach. Branches of such a compensator should have susceptances $T_{\mathrm{R}n}$, $T_{\mathrm{S}n}$, $T_{\mathrm{T}n}$, $T_{\mathrm{RS}n}$, $T_{\mathrm{ST}n}$, and $T_{\mathrm{TR}n}$, as shown in Fig. 3, i.e., be specified not only for the fundamental frequency but for each harmonic of the order n from the set of dominating harmonic orders N.

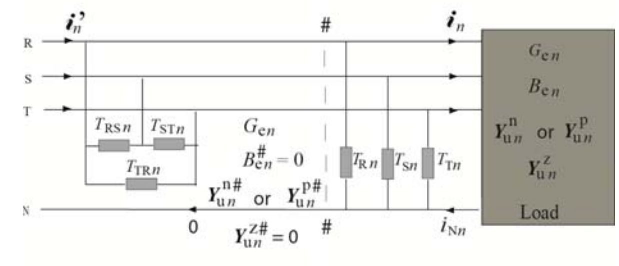

Fig. 3. Load with reactive compensator for the n^{th} order harmonic of the positive or negative sequence.

These susceptances for the voltage harmonics of the positive order, $n = 3k+1$, can be calculated exactly in the same way as for the fundamental harmonic, which usually have the positive sequence.

Let us assume, as it was done in [17] with respect to the fundamental harmonic, that the Y-structure compensator should compensate entirely the unbalanced current of the zero sequence of the n^{th} order harmonic, along with the reac-tive current. At such an assumption, its susceptances for the positive sequence harmonics should satisfy conditions

$$\frac{1}{3}\, j\,(T_{\mathrm{R}n} + \alpha^* T_{\mathrm{S}n} + \alpha T_{\mathrm{T}n}) + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} = 0 \tag{21}$$

$$\frac{1}{3}(T_{\mathrm{R}n} + T_{\mathrm{S}n} + T_{\mathrm{T}n}) + B_{\mathrm{e}n} = 0\,. \tag{22}$$

These equations result in the Y-structure compensator sus-ceptances

$$\begin{aligned} T_{\mathrm{R}n} &= -2\,\mathrm{Im}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} - B_{\mathrm{e}n} \\ T_{\mathrm{S}n} &= -\sqrt{3}\,\mathrm{Re}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} + \mathrm{Im}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} - B_{\mathrm{e}n}\,. \\ T_{\mathrm{T}n} &= \sqrt{3}\,\mathrm{Re}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} + \mathrm{Im}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} - B_{\mathrm{e}n} \end{aligned} \tag{23}$$

A compensator with such susceptances changes the unba-lanced component of the negative sequence of the n^{th} order harmonic of the current in the cross-section "# - #" to

$$\boldsymbol{i}_{\mathrm{u}n}^{\mathrm{n}\#} = \sqrt{2}\,\mathrm{Re}\{(\boldsymbol{Y}_{\mathrm{Cu}n}^{\mathrm{n}} + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}})\,\mathbf{1}^{\mathrm{n}}\boldsymbol{U}_{\mathrm{R}n}e^{jn\omega_1 t}\} \tag{24}$$

where $\boldsymbol{Y}_{\mathrm{Cu}n}^{\mathrm{n}}$ is the negative sequence unbalanced admittance of the Y - compensator for the n^{th} order harmonic, equal to

$$\begin{aligned} \boldsymbol{Y}_{\mathrm{Cu}n}^{\mathrm{n}} &= \frac{1}{3}\, j\,(T_{\mathrm{R}n} + \alpha T_{\mathrm{S}n} + \alpha^* T_{\mathrm{T}n}) = \frac{1}{3}\, j\,[(-2\,\mathrm{Im}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} - B_{\mathrm{e}n}) + \\ &+ \alpha(-\sqrt{3}\,\mathrm{Re}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} + \mathrm{Im}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} - B_{\mathrm{e}n}) + \alpha^*(\sqrt{3}\,\mathrm{Re}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} + \mathrm{Im}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} - B_{\mathrm{e}n})] = \\ &= \mathrm{Re}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} - j\,\mathrm{Im}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} = \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}*}. \end{aligned} \tag{25}$$

Hence

$$\boldsymbol{i}_{\mathrm{u}n}^{\mathrm{n}\#} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}\#}\,\mathbf{1}^{\mathrm{n}}\boldsymbol{U}_{\mathrm{R}n}e^{jn\omega_1 t}\} \tag{26}$$

where

$$\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}\#} = \boldsymbol{Y}_{\mathrm{Cu}n}^{\mathrm{n}} + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}} = \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}*} + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}}\,. \tag{27}$$

The unbalanced current of the negative sequence, of the value in the cross-section "# - #" specified by (26), can be compensated entirely by a Δ-structure compensator with line-to-line susceptances that satisfy the condition

$$j\,(T_{\mathrm{ST}n} + \alpha T_{\mathrm{TR}n} + \alpha^* T_{\mathrm{RS}n}) + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}\#} = 0\,. \tag{28}$$

Since the reactive component of the current n^{th} harmonic in this cross-section is compensated entirely by the Y-structure compensator, thus susceptances of the Δ-structure compen-sator should satisfy the condition

$$T_{\mathrm{ST}n} + T_{\mathrm{TR}n} + T_{\mathrm{RS}n} = 0\,. \tag{29}$$

Equations (28) and (29) result in the compensator suscep-tances for harmonic frequencies

$$\begin{aligned} T_{\mathrm{RS}n} &= \frac{1}{3}(\sqrt{3}\,\mathrm{Re}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}\#} - \mathrm{Im}\,\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}\#}) \\ T_{\mathrm{ST}n} &= \frac{1}{3}(2\,\mathrm{Im}\,\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}\#}) \\ T_{\mathrm{TR}n} &= \frac{1}{3}(-\sqrt{3}\,\mathrm{Re}\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}\#} - \mathrm{Im}\,\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}\#}) \end{aligned} \tag{30}$$

Compensation for the negative sequence voltage harmo-nics, i.e., of the order, $n = 3k–1$ does not differ substantially from that for the positive order. Only the condition (21) has now the form

$$\frac{1}{3}\,j\,(T_{\mathrm{R}n}+\alpha\, T_{\mathrm{S}n}+\alpha^{*}T_{\mathrm{T}n})+\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}=0 \tag{31}$$

while (22) remains unchanged. Both these equations result in a bit changed formulae for susceptances of the Y compen-sator, namely

$$\begin{aligned} T_{\mathrm{R}n} &= -2\,\mathrm{Im}\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}-B_{\mathrm{e}n}\\ T_{\mathrm{S}n} &= \sqrt{3}\,\mathrm{Re}\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}+\mathrm{Im}\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}-B_{\mathrm{e}n}\\ T_{\mathrm{T}n} &= -\sqrt{3}\,\mathrm{Re}\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}+\mathrm{Im}\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}-B_{\mathrm{e}n} \end{aligned} \tag{32}$$

A compensator with such susceptances changes the unba-lanced component of the positive sequence of the n^{th} order harmonic of the current in the cross-section "# - #" to

$$\boldsymbol{i}^{\mathrm{p}\#}_{\mathrm{u}n}=\sqrt{2}\,\mathrm{Re}\{(\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{Cu}n}+\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n})\,1^{\mathrm{p}}\boldsymbol{U}_{\mathrm{R}n}e^{jn\omega_1 t}\} \tag{33}$$

where $\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{Cu}n}$ is the positive sequence unbalanced admittance of the Y compensator for the n^{th} order harmonic, equal to

$$\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{Cu}n}=\frac{1}{3}j(T_{\mathrm{R}n}+\alpha^{*}T_{\mathrm{S}n}+\alpha T_{\mathrm{T}n})=\mathrm{Re}\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}-j\,\mathrm{Im}\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}=\boldsymbol{Y}^{\mathrm{z}*}_{\mathrm{u}n}\,. \tag{34}$$

Hence

$$\boldsymbol{i}^{\mathrm{p}\#}_{\mathrm{u}n}=\sqrt{2}\,\mathrm{Re}\{\boldsymbol{Y}^{\mathrm{p}\#}_{\mathrm{u}n}\,1^{\mathrm{p}}\boldsymbol{U}_{\mathrm{R}n}e^{jn\omega_1 t}\} \tag{35}$$

where

$$\boldsymbol{Y}^{\mathrm{p}\#}_{\mathrm{u}n}=\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{Cu}n}+\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}=\boldsymbol{Y}^{\mathrm{z}*}_{\mathrm{u}n}+\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}\,. \tag{36}$$

The n^{th} order harmonic of the positive sequence $\boldsymbol{i}^{\mathrm{p}\#}_{\mathrm{u}n}$ of the unbalanced current in the cross-section "# - #" can be compensated entirely, if susceptances of the Δ-structure compensator satisfy the condition

$$j(T_{\mathrm{ST}n}+\alpha^{*}T_{\mathrm{TR}n}+\alpha T_{\mathrm{RS}n})+\boldsymbol{Y}^{\mathrm{p}\#}_{\mathrm{u}n}=0\,. \tag{37}$$

Because the reactive current in the cross-section "# - #" is compensated by the Y compensator, these susceptances should satisfy moreover (29). Equations (37) and (29) result in formulae for susceptances of the Δ–structure compen-sator for the voltage harmonics of the negative sequence

$$\begin{aligned} T_{\mathrm{RS}n} &= \frac{1}{3}(-\sqrt{3}\,\mathrm{Re}\boldsymbol{Y}^{\mathrm{p}\#}_{\mathrm{u}n}-\mathrm{Im}\,\boldsymbol{Y}^{\mathrm{p}\#}_{\mathrm{u}n})\\ T_{\mathrm{ST}n} &= \frac{1}{3}(2\,\mathrm{Im}\,\boldsymbol{Y}^{\mathrm{p}\#}_{\mathrm{u}n})\\ T_{\mathrm{TR}n} &= \frac{1}{3}(\sqrt{3}\,\mathrm{Re}\boldsymbol{Y}^{\mathrm{p}\#}_{\mathrm{u}n}-\mathrm{Im}\,\boldsymbol{Y}^{\mathrm{p}\#}_{\mathrm{u}n}) \end{aligned} \tag{38}$$

In the presence of the load imbalance, the zero sequence supply voltage harmonics $\boldsymbol{e}_n$, i.e., of the order $n = 3k$, can produce an unbalanced current harmonics of the positive and the negative sequence, $\boldsymbol{i}^{\mathrm{p}}_{\mathrm{u}n}$ and $\boldsymbol{i}^{\mathrm{n}}_{\mathrm{u}n}$. These harmonics can-not be compensated by any Δ–structure compensator, because at a symmetrical supply voltage, as assumed in this paper, such a compensator cannot have current harmonics of the zero sequence. Only a Y-structure compensator can be used for that.

To compensate currents $\boldsymbol{i}^{\mathrm{p}}_{\mathrm{u}n}$ and $\boldsymbol{i}^{\mathrm{n}}_{\mathrm{u}n}$, the susceptances of the Y-structure compensator have to satisfy, at the same time, equations

$$\frac{1}{3}\,j\,(T_{\mathrm{R}n}+\alpha T_{\mathrm{S}n}+\alpha^{*}T_{\mathrm{T}n})+\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}=0\,. \tag{39}$$

$$\frac{1}{3}\,j\,(T_{\mathrm{R}n}+\alpha^{*}T_{\mathrm{S}n}+\alpha T_{\mathrm{T}n})+\boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n}=0\,. \tag{40}$$

which is not possible, because three unknown real numbers cannot satisfy four equations i.e. for the real and imaginary parts of (39) and (40). Thus, only one of these two currents can be compensated. We have to select one of them. If we select the positive sequence unbalanced current $\boldsymbol{i}^{\mathrm{p}}_{\mathrm{u}n}$, i.e., condition (39), along with compensation of the reactive current $\boldsymbol{i}_{\mathrm{r}n}$, i.e., (22) has to be satisfied, then

$$\begin{aligned} T_{\mathrm{R}n} &= -2\,\mathrm{Im}\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}-B_{\mathrm{e}n}\\ T_{\mathrm{S}n} &= \sqrt{3}\,\mathrm{Re}\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}+\mathrm{Im}\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}-B_{\mathrm{e}n}\\ T_{\mathrm{T}n} &= -\sqrt{3}\,\mathrm{Re}\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}+\mathrm{Im}\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}-B_{\mathrm{e}n} \end{aligned} \tag{41}$$

Thus, in a presence of the zero sequence supply voltage harmonics, some component of the unbalanced current, called ***residual unbalanced current***, $\boldsymbol{i}_{\mathrm{ur}}$, cannot be compen-sated by a reactive compensator. Consequently, the maxi-mum value of the power factor, which can be achieved by reactive compensation cannot be higher than

$$\lambda_{\max}=\frac{\|\boldsymbol{i}_{\mathrm{a}}\|}{\sqrt{\|\boldsymbol{i}_{\mathrm{a}}\|^{2}+\|\boldsymbol{i}_{\mathrm{s}}\|^{2}+\|\boldsymbol{i}_{\mathrm{ur}}\|^{2}}}\,. \tag{42}$$

This limit has a cognitive rather than a practical merit, because this value is usually very close to unity.

This section shows that having complex rms values of voltage and current harmonics at the load terminals we can calculate susceptances for harmonic frequencies T_{Xn} of the compensator branches. Each branch has to have a specified susceptance at a number of harmonic frequencies, $n\omega_1$. For example, if apart from the fundamental, the dominating harmonics taken into account at the compensator design are of the order n = 3rd, 5th, and 7th, then susceptances of the Y-structure compensator should have specified values at har-monic of the order, n = 1, 3, 5, and 7, while susceptances of the Δ-structure compensator should have specified values at frequencies of harmonics of the order n = 1, 5, and 7.

Methods of synthesis of such reactive branches are well developed and can be found, for example, in [2]. At this point, we encounter, however, a major obstacle in the com-pensator design process, namely, the compensator comple-xity. Two reactive elements are needed on average to fix the branch susceptance at a required value T_{Xn}. If N = {1, 3, 5, 7} then, up to eight reactive elements might be needed for each branch of the Y-structure compensator and up to six of them for the Δ-structure compensator. Such a compensator would be too complex to have a technical value. Reduction of this complexity is necessary.

Reduction of the compensator complexity

Compensator complexity can be reduced if the goal of whole compensation of the unbalanced and reactive currents by sort of ***an ideal compensator***, is abandoned for the only reduction of these currents by a compensator built of a lower number of reactive devices. It should minimize the three-phase rms value of the supply current.

The most simple branches of a compensator are purely inductive or purely capacitive. Since a purely capacitive branch in the compensator could cause a resonance for harmonic frequencies with an inductive impedance of the supply source, such capacitive branches are not acceptable, however. Therefore, the compensator can be built exclusi-vely of L or LC branches shown in Fig. 4.

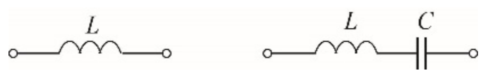

Fig. 4. Acceptable branches of a reduced complexity compensator.

To avoid confusion, susceptances of a compensator of a reduced complexity are denoted by D_n. These susceptances, depending on the branch structure, could have values

$$(43)\qquad D_n = -\frac{1}{n\omega_1 L} \quad \text{or} \quad D_n = \frac{n\omega_1 C}{1-n^2\omega_1^2 LC}$$

A compensator for the unbalanced and reactive currents rms value minimization can have the same structure as the ideal compensator, only its vector of branch currents will change, as shown in Fig. 5, from $\boldsymbol{j}_\mathrm{T}$ to $\boldsymbol{j}_\mathrm{D}$.

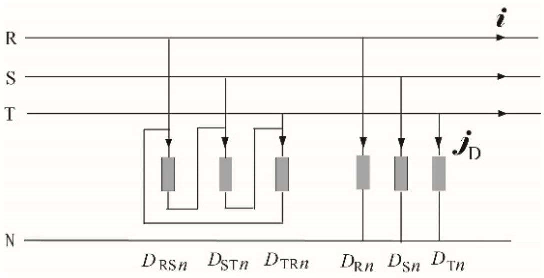

Fig. 5. A compensator with branches of reduced complexity and D_n susceptance.

Let us denote the vector of crms values of the voltage harmonics on the compensator branches by

$$(44)\qquad \boldsymbol{U}_n = [\boldsymbol{U}_{\mathrm{RS}n}, \boldsymbol{U}_{\mathrm{ST}n}, \boldsymbol{U}_{\mathrm{TR}n}, \boldsymbol{U}_{\mathrm{R}n}, \boldsymbol{U}_{\mathrm{S}n}, \boldsymbol{U}_{\mathrm{T}n}]^\mathrm{T}.$$

The vector of currents of the ideal compensator branches can be presented in the form

$$(45)\qquad \boldsymbol{j}_\mathrm{T} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} j\,\boldsymbol{T}_n\,\boldsymbol{U}_n\, e^{jn\omega_1 t}$$

where

$$(46)\qquad \boldsymbol{T}_n = \begin{bmatrix} T_{\mathrm{RS}n} & 0 & 0 & 0 & 0 & 0\\ 0 & T_{\mathrm{ST}n} & 0 & 0 & 0 & 0\\ 0 & 0 & T_{\mathrm{TR}n} & 0 & 0 & 0\\ 0 & 0 & 0 & T_{\mathrm{R}n} & 0 & 0\\ 0 & 0 & 0 & 0 & T_{\mathrm{S}n} & 0\\ 0 & 0 & 0 & 0 & 0 & T_{\mathrm{T}n}\end{bmatrix}.$$

For the compensator with branches shown in Fig. 5, this vector changes to the form

$$(47)\qquad \boldsymbol{j}_\mathrm{D} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} j\,\boldsymbol{D}_n\,\boldsymbol{U}_n\, e^{jn\omega_1 t}$$

where

$$(48)\qquad \boldsymbol{D}_n = \begin{bmatrix} D_{\mathrm{RS}n} & 0 & 0 & 0 & 0 & 0\\ 0 & D_{\mathrm{ST}n} & 0 & 0 & 0 & 0\\ 0 & 0 & D_{\mathrm{TR}n} & 0 & 0 & 0\\ 0 & 0 & 0 & D_{\mathrm{R}n} & 0 & 0\\ 0 & 0 & 0 & 0 & D_{\mathrm{S}n} & 0\\ 0 & 0 & 0 & 0 & 0 & D_{\mathrm{T}n}\end{bmatrix}.$$

Vectors $\boldsymbol{j}_\mathrm{T}$ and $\boldsymbol{j}_\mathrm{D}$ can be regarded as elements in a six-dimensional space. If the matrix of branch susceptances is denoted by $\boldsymbol{B}$, then the length of the vector $\boldsymbol{j}_\mathrm{B}$ is defined as

$$(49)\qquad \begin{aligned}||\,\boldsymbol{j}_\mathrm{B}\,|| &= \sqrt{\frac{1}{T}\int_0^T \boldsymbol{j}_\mathrm{B}(t)\bullet \boldsymbol{j}_\mathrm{B}(t)\mathrm{d}t} = \\ &= \mathrm{Re}\sum_{n\in N}(j\,\boldsymbol{B}_n\,\boldsymbol{U}_n)(j\,\boldsymbol{B}_n\boldsymbol{U}_n)^* = \mathrm{Re}\sum_{n\in N}\boldsymbol{B}_n^2\,|\boldsymbol{U}_n|^2.\end{aligned}$$

In this formula

$$(50)\qquad |\boldsymbol{U}_n|^2 = [U_{\mathrm{RS}n}^2, U_{\mathrm{ST}n}^2, U_{\mathrm{TR}n}^2, U_{\mathrm{R}n}^2, U_{\mathrm{S}n}^2, U_{\mathrm{T}n}^2]^\mathrm{T}.$$

A distance between vectors $\boldsymbol{j}_\mathrm{T}$ and $\boldsymbol{j}_\mathrm{D}$ in this space can be defined as

$$(51)\qquad d = ||\,\boldsymbol{j}_\mathrm{T} - \boldsymbol{j}_\mathrm{D}||.$$

The LC parameters of a compensator which is to minimize the three-phase rms value of the supply current should be selected such that this distance is minimum.

The difference of branch currents vectors $\boldsymbol{j}_\mathrm{T}$ and $\boldsymbol{j}_\mathrm{D}$ can be expressed as

$$(52)\qquad \boldsymbol{j}_\mathrm{T} - \boldsymbol{j}_\mathrm{D} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} j[\boldsymbol{T}_n - \boldsymbol{D}_n]\,\boldsymbol{U}_n\, e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}\Delta\boldsymbol{J}_n\, e^{jn\omega_1 t}$$

where

$$(53)\qquad \Delta\boldsymbol{J}_n = j\begin{bmatrix}(T_{\mathrm{RS}n}-D_{\mathrm{RS}n})\boldsymbol{U}_{\mathrm{RS}n}\\ (T_{\mathrm{ST}n}-D_{\mathrm{ST}n})\boldsymbol{U}_{\mathrm{ST}n}\\ (T_{\mathrm{TR}n}-D_{\mathrm{TR}n})\boldsymbol{U}_{\mathrm{TR}n}\\ (T_{\mathrm{R}n}-D_{\mathrm{R}n})\boldsymbol{U}_{\mathrm{R}n}\\ (T_{\mathrm{S}n}-D_{\mathrm{S}n})\boldsymbol{U}_{\mathrm{S}n}\\ (T_{\mathrm{T}n}-D_{\mathrm{T}n})\boldsymbol{U}_{\mathrm{T}n}\end{bmatrix}$$

and consequently, the distance d of these vectors is

$$(54)\qquad d = ||\,\boldsymbol{j}_\mathrm{T} - \boldsymbol{j}_\mathrm{D}|| = \sqrt{\sum_{n\in N}\Delta\boldsymbol{J}_n^\mathrm{T}\Delta\boldsymbol{J}_n^*} = \sqrt{\sum_{n\in N}\sum_{k\in K}(T_{kn}-D_{kn})^2 U_{kn}^2}$$

where k is an index from the set $K = \{\mathrm{RS, ST, TR, R, S, T}\}$.

The sequence of summation over harmonic orders n and the branch index k in (54) can be switched so that the distance d can be expressed as

$$(55)\qquad d = \sqrt{\sum_{k\in K}\sum_{n\in N}(T_{kn}-D_{kn})^2 U_{nk}^2} = \sqrt{\sum_{k\in K} d_k^2}.$$

Since the terms under the root of (55) are non-negative numbers, so that mutual cancellation is not possible, the distance d is minimum on the condition that all distances d_k for individual branches

$$(56)\qquad d_k = \sqrt{\sum_{n\in N}(T_{kn}-D_{kn})^2 U_{kn}^2}$$

are minimum, i.e., for each branch of the compensator

$$(57)\qquad \sum_{n\in N}(T_{kn}-D_{kn})^2 U_{kn}^2 = \mathrm{Min.}$$

Susceptnces T_{kn} in this formula are known from formulae (23), (30), (32), (38) and (41). Susceptances D_{kn}, dependent on LC parameters of the compensator branches, shown in Fig. 5, should minimize (57).

The supply voltage rms value of the fundamental harmo-nic U_{k1} in common distribution systems is much higher than the rms value of other harmonics. Because of that, the com-ponent of (57) with U_{k1} is the dominating one. Therefore, the term $(T_{k1}-D_{k1})$ should be as close to zero as possible. Thus, when $T_{k1} < 0$, the compensator branch should be chosen such that $D_{k1} < 0$, i.e., the inductive branch. When $T_{k1} > 0$, the branch should be chosen such that $D_{k1} > 0$, i.e., the LC branch. Consequently, for purely inductive branches, the inductance L_k should be chosen such that

$$(58)\qquad \sum_{n\in N}(T_{kn}+\frac{1}{n\omega_1 L_k})^2 U_{kn}^2 = \mathrm{Min.}$$

For LC branches, the inductance L_k and capacitance C_k should be chosen such that

$$\sum_{n\in N}(T_{kn}-\frac{n\omega_1 C_k}{1-n^2\omega_1^2 L_k C_k})^2 U_{kn}^2 = \text{Min.} \tag{59}$$

Condition (57) is satisfied when its derivative with respect to L_k is zero, i.e.,

$$\frac{d}{dL_k}\{\sum_{n\in N}(T_{kn}+\frac{1}{n\omega_1 L_k})^2 U_{kn}^2\} = 0 \tag{60}$$

and this condition results in the optimum value of the branch inductance

$$L_{k,\text{opt}} = -\frac{1}{\omega_1}\frac{\sum_{n\in N}\frac{1}{n^2}U_{kn}^2}{\sum_{n\in N}T_{kn}\frac{1}{n}U_{kn}^2}. \tag{61}$$

The form on the left side of (59) is a function of two varia-bles, the inductance L_k and capacitance C_k. With respect to L_k it is continuously declining function, meaning it does not have a minimum for any finite value of inductance L_k. It has to be selected at a designer discretion [5, 7]. When it is selec-ted, the capacitance C_k can be calculated such that (59) is minimum. When it is minimum, derivative of (59) with respect to C_k is zero, i.e.,

$$\frac{d}{dC_k}\{\sum_{n\in N}(T_{kn}-\frac{n\omega_1 C_k}{1-n^2\omega_1^2 L_k C_k})^2 U_{kn}^2\} = 0\,. \tag{62}$$

It results in the equation

$$\sum_{n\in N}\frac{T_{kn}\, nU_{kn}^2}{1-n^2\omega_1^2 L_k C_k} - \sum_{n\in N}\frac{n^2\omega_1 C_k U_{kn}^2}{(1-n^2\omega_1^2 L_k C_k)^2} = 0 \tag{63}$$

which cannot be solved directly with respect to the optimum value of the capacitance C_k. Numerical methods are needed for that. In particular, it can be solved in an iterative process

$$C_{k,s+1} = \frac{\sum_{n\in N}\frac{T_{kn}\, nU_{kn}^2}{1-n^2\omega_1^2 L_k C_{k,s}}}{\omega_1\sum_{n\in N}\frac{n^2 U_{kn}^2}{(1-n^2\omega_1^2 L_k C_{k,s})^2}} \tag{64}$$

which results in a sequence of capacitances usually convergent to the optimum capacitance $C_{k,\text{opt}}$ which minimizes (59).

Numerical illustration Let us assume that the load shown in Fig. 6, with $\omega_1 L = R = 0.5\ \Omega$, is supplied with a symmetrical voltage of the fundamental harmonic rms value $U_1 = 240$ V, distorted by the 3rd, 5th, and 7th order harmonics of relative rms value $U_3 = 2\% U_1$, $U_5 = 3\% U_1$ and $U_7 = 1.5\% U_1$.

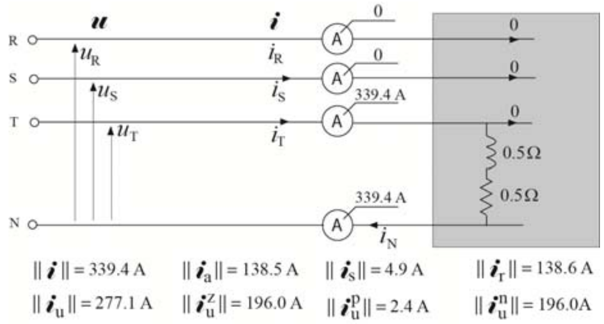

Fig. 6. Example of an unbalanced load and its analysis.

The rms values of line-to-neutral voltage harmonics and the load admittance for harmonics are compiled in Table1.

The active power of the load is

$$P = \sum_{n\in N} G_{\text{T}n} U_{\text{T}n}^2 = 57.609\ \text{kW}\,.$$

The supply voltage three rms value

$$\|\boldsymbol{u}\| = \sqrt{\sum_{n\in N}\|\boldsymbol{u}_n\|^2} = \sqrt{3\sum_{n\in N}U_n^2} = 416.01\ \text{V}\,.$$

Thus, the equivalent conductance of the load is

$$G_\text{e} = \frac{P}{\|\boldsymbol{u}\|^2} = 0.3329\ \text{S}$$

and the three-phase rms value of the active current

$$\|\boldsymbol{i}_\text{a}\| = G_\text{e}\|\boldsymbol{u}\| = \frac{P}{\|\boldsymbol{u}\|} = 138.5\ \text{A}\,.$$

The values of equivalent conductance G_{en}, susceptance B_{en}, and magnitude of unbalanced admittances $\boldsymbol{Y}_{\text{u}n}^\text{p}$, $\boldsymbol{Y}_{\text{u}n}^\text{n}$ and $\boldsymbol{Y}_{\text{u}n}^\text{z}$ for harmonic frequencies, calculated according to (8), (10), (12), (15) and (17), are compiled in Table 2.

The three-phase rms value of the scattered current is

$$\|\boldsymbol{i}_\text{s}\| = \sqrt{3\sum_{n\in N}[(G_{\text{e}n}-G_\text{e})U_n]^2} = 4.9\ \text{A}$$

and the reactive current

$$\|\boldsymbol{i}_\text{r}\| = \sqrt{3\sum_{n\in N}(B_{\text{e}n}U_n)^2} = 138.6\ \text{A.}$$

Table 1. Rms values of voltage harmonics and the load admittance

n	U_n [V]	$\boldsymbol{Y}_{\text{T}n} = G_{\text{T}n}+jB_{\text{T}n}$ [S]
1	240	$1.000 - j1.000$
3	4.8	$0.200 - j0.600$
5	7.2	$0.0769 - j0.385$
7	3.6	$0.0400 - j0.280$

Table 2. Equivalent parameters of the load for harmonics

n	G_{en}	B_{en}	$Y_{\text{u}n}^\text{z}$	$Y_{\text{u}n}^\text{p}$	$Y_{\text{u}n}^\text{n}$
-	mS	mS	mS	mS	mS
1	333	–333	471	0	471
3	66.6	–200	0	211	211
5	25.6	–128	131	131	0
7	13.3	–93.3	94.3	0	94.3

The three-phase rms value of the unbalanced current components are

$$\|\boldsymbol{i}_\text{u}^\text{z}\| = \sqrt{3\sum_{n\in N}(Y_{\text{u}n}^\text{z}U_n)^2} = 196.0\ \text{A}$$

$$\|\boldsymbol{i}_\text{u}^\text{p}\| = \sqrt{3\sum_{n\in N}(Y_{\text{u}n}^\text{p}U_n)^2} = 2.4\ \text{A}$$

$$\|\boldsymbol{i}_\text{u}^\text{n}\| = \sqrt{3\sum_{n\in N}(Y_{\text{u}n}^\text{n}U_n)^2} = 196.0\ \text{A}$$

and consequently, the three-phase rms value of the unbalanced current is

$$\|\boldsymbol{i}_\text{u}\| = \sqrt{\|\boldsymbol{i}_\text{u}^\text{p}\|^2+\|\boldsymbol{i}_\text{u}^\text{n}\|^2+\|\boldsymbol{i}_\text{u}^\text{z}\|^2} = 277.1\ \text{A}\,.$$

The three-phase rms value of the load current, calculated as the root of the sum of squares of rms values of the line currents, is

$$\|\boldsymbol{i}\| = \sqrt{\|i_\text{R}\|^2+\|i_\text{S}\|^2+\|i_\text{T}\|^2} = \|i_\text{T}\| = \sqrt{\sum_{n\in N}(Y_{\text{T}n}U_n)^2} = 339.4\ \text{A}\,.$$

This value can be used for verification of the decomposition of the load current into physical components, since the root of the sum of squares their three-phase rms values should result in the same value of $||\boldsymbol{i}||$. Indeed

$$||\boldsymbol{i}|| = \sqrt{||\boldsymbol{i}_{\rm a}||^2 + ||\boldsymbol{i}_{\rm s}||^2 + ||\boldsymbol{i}_{\rm r}||^2 + ||\boldsymbol{i}_{\rm u}||^2} = 339.4\ {\rm A}$$

which confirms the numerical correctness of calculations of the current physical components three-phase rms values. The results of the load analysis are compiled in Fig. 6. The power factor of the load is $\lambda = P/S = 0.408$.

Unbalanced admittances of the zero, positive and negative sequence, for particular supply voltage harmonics, which are needed for balancing compensator design, calculated from formulae (12), (15), and (17), have values compiled in Table 3.

Table 3. Unbalanced admittances of the load

n	$\boldsymbol{Y}^{\rm z}_{{\rm u}n}$ [mS]	$\boldsymbol{Y}^{\rm p}_{{\rm u}n}$ [mS]	$\boldsymbol{Y}^{\rm n}_{{\rm u}n}$ [mS]
1	$122 + j455$	0	$-455 - j122$
3	0	$-206 + j42.3$	$140 + j158$
5	$-124 + j41.9$	$58.2 + j68.3$	0
7	$74.2 + j58.2$	0	$-87.5 + j35.1$

Susceptances of the ideal compensator branches, calcu-lated from formulae (23), (30), (32), (38), and (41), have the values compiled in Table 4.

Table 4. Susceptances of the ideal compensator branches

n	$T_{{\rm R}n}$	$T_{{\rm S}n}$	$T_{{\rm T}n}$	$T_{{\rm RS}n}$	$T_{{\rm ST}n}$	$T_{{\rm TR}n}$
	mS	mS	mS	mS	mS	mS
1	–577	577	1000	0	–385	385
3	115	–115	600	0	0	0
5	44.4	–44.4	385	0	29.6	–29.6
7	–23.1	23.1	280	0	–15.4	15.5

A compensator with such susceptances of its branches affects the supply current rms values as shown in Fig. 7. It does not reduce the scatter current and the unbalanced current of the positive sequence, but it improves the power factor to $\lambda = 0.999$.

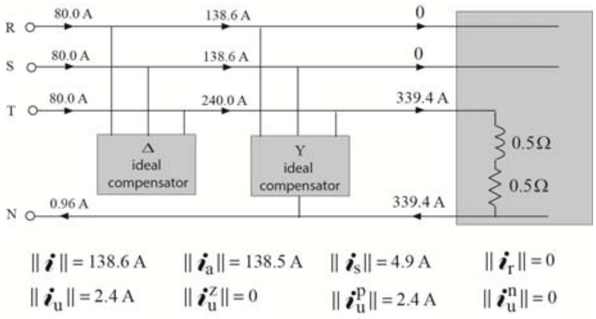

Fig. 7. Results of ideal compensation

As explained above, the Δ–structure compensator can compensate the positive sequence component of the unbalanced current or the negative sequence component, but not both of them. Since the former component is much lower than the last one, therefore, it is reasonable to select the negative sequence component as a goal for compensation, so that the positive sequence component has to remain in the supply lines. The current in the neutral conductor (0.96A) is compo-sed of the third order harmonic of the active current.

The reduced complexity compensator should have the LC parameters, calculated according to (61) and (64), as compiled in Table 5. It was assumed that the supply voltage frequency is normalized to ω_1 = 1 rad/s, and the resonance frequency of the compensator LC branches is approximately equal to $\omega_{\rm r}$ = 2.5 rad/s.

Table 5. LC parameters of a reduced complexity compensator

	Line:	R	S	T	RS	ST	TR
L	mH	1730	770	444	0	2600	1155
C	mF	0	399	691	0	0	266

The results of compensation are shown in Fig. 8. The power factor is improved by the compensator of the reduced complexity to $\lambda = 0.994$.

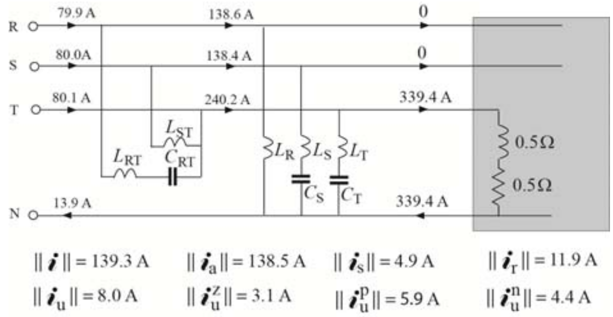

Fig. 8. Results of compensation with a reduced complexity compensator.

The presented approached enables reduction of the number of reactive devices needed for the compensator construction from approximately 40 to only eight of them, without any substantial degradation of the compensation effectiveness.

Conclusions

The paper demonstrates that unbalanced LTI loads supplied with a nonsinusoidal voltage by a four-wire line can be almost ideally balanced, and their power factor can be impro-ved to almost unity value, using reactive compensator with substantially reduced complexity.

The results presented in this paper can provide a starting platform for the development of an adaptive reactive com-pensator of varying loads.

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