# Automated method for formation and solving the instantaneous power components balances for the analysis of nonlinear electric circuits 


#### Abstract

The method of nonlinear electric circuit's analysis using the method of forming and solving instantaneous power orthogonal components balance equations is developed. The advantages of the developed method are that it allows to analyze the mechanism of formation of instantaneous power orthogonal components and to evaluate the influence of parameters on the researched electric circuit operation. Developed method allows to increase the efficiency of the nonlinear electric circuits analysis theory. It was shown that the proposed method avoids complex intermediate analytical transformations due to the use of the developed automated method of forming instantaneous constituents based on the discrete convolution algorithm.

Streszczenie. W artykule przedstawiono wyniki opracowanej metody analizy nieliniowego obwodu elektrycznego za pomocą metody formowania i rozwiązywania równań równowagi ortogonalnej składowej mocy. Zalety opracowanej metody polegają na tym, że pozwala ona analizować mechanizm powstawania składowych ortogonalnych mocy chwilowej i oceniać wpływ parametrów na działanie obwodu elektrycznego. Opracowana metoda pozwala zwiększyć efektywność teorii analizy nieliniowych obwodów elektrycznych. Wykazano, że proponowana metoda pozwala uniknąć skomplikowanych pośrednich przekształceń analitycznych dzięki zastosowaniu opracowanej zautomatyzowanej metody formowania natychmiastowych składników w oparciu o algorytm dyskretnego splotu. Automatyczna metoda do obliczanie składowych mocy chwilowej w zastosowaniu do analizy obwodówq nieliniowych


Key words: nonlinear electric circuit, automated analysis, instantaneous power, harmonic balance, discrete convolution
Słowa kluczowe: nieliniowy obwód elektryczny, zautomatyzowana analiza, moc chwilowa, równowaga harmoniczna, splot dyskretny.

## Introduction

Calculation of nonlinear electric circuits underlies the analysis of various electric devices [1]. An efficient and easily applied calculation method is necessary for attainment of sufficient accuracy of determining the parameters of electric circuits of electrical devices containing various nonlinear elements [2-4].

Calculation of nonlinear electric circuits is known to be reduced to calculation of nonlinear equations describing physical phenomena in electric circuits [5].

All methods for nonlinear circuit's analysis can be divided into three large groups: analytical [2, 6], graphic and numeral [2-5, 7-9]. There are certain shortcomings for each of these groups, such as: graphic methods do not give availability of general dependences, analytical and numeral are less demonstrable and cumbersome. The using of these methods is limited by their dependence of calculation accuracy on the nonlinear approximation accuracy.

Currently it has been offered to use instantaneous power (IP) method based on calculation of balance equations of IP harmonic components for power supply and equivalent circuit elements [6]. It makes it possible to calculate the parameters of nonlinear electric circuits when it is incorrect to use superposition principle for current harmonics [10-11]. However, formation of balance equations of IP harmonic components involves such negative aspects as calculation complexity and awkwardness. So an automated method for the formation and solving of the IP components balance equations developing is proposed to solve above noted problem and in order to develop the theory of analysis of nonlinear electric circuits.

## Prm sobletatement

Development of the method for the formation and solving of the IP components balance equations for the increasing the efficiency of the nonlinear electric circuits analysis theory.

## Material and results of the research

In order to develop the nonlinear electric circuits analysis theory an automated method of forming and solving the IP
orthogonal components balance equations systems has been developed.To implement the proposed software requires the following steps.

1. The output data are sated.
1.1. The number of voltage harmonics (by default «1») are set.
1.2. Supply voltage $u$ is set either by odd or even and odd harmonics.
1.3. The number of researched current harmonics is chosen not less than the number of voltage harmonics for the linear circuit:
(1)

$$
M=N
$$

where $M, N$ - number of researched current and voltage harmonics, respectively.

In the presence of a nonlinear element in the circuit - the number of current harmonics is chosen more than the number of voltage harmonics:
$M>N ;$
1.4. A coordinate system is selected: orthogonal or polar (orthogonal by default). There voltage supply is set:
a) orthogonal
b) polar

$n \quad U_{n} \quad \varphi$

where $n$ - voltage harmonic number; $\varphi$ - shift angle.

For the case for orthogonal coordinate system there is obtained the voltage harmonic composition:
(3)

$$
U_{a n}=\left[\begin{array}{c}
U_{a 0} \\
U_{a 1} \\
U_{a 2} \\
U_{a 3} \\
U_{a 4} \\
\vdots \\
U_{a n}
\end{array}\right] ; \quad U_{b n}=\left[\begin{array}{c}
0 \\
U_{b 1} \\
U_{b 2} \\
U_{b 3} \\
U_{b 4} \\
\vdots \\
U_{b n}
\end{array}\right],
$$

for the case of polar coordinate system the voltage harmonic composition is:

$$
U_{n}=\left[\begin{array}{c}
U_{0}  \tag{4}\\
U_{1} \\
U_{2} \\
U_{3} \\
U_{4} \\
\vdots \\
U_{n}
\end{array}\right]\left[\begin{array}{c}
0 \\
\varphi_{1} \\
\varphi_{2} \\
\varphi_{3} \\
\varphi_{4} \\
\vdots \\
\varphi_{n}
\end{array}\right] .
$$

2. The parameters of the equivalent circuits of the researched systems are set.
2.1. The output branch connected to the power supply is indicated by the index «1». Branches outgoing from the branching node and connected to the branch «1» are indicated by the indices «1.1», «1.2», respectively (fig. 1).


Fig. 1. Researched circuit example 1
Accordingly, if the circuit branching out from the branch «1.2», then the output from it branches will be indexed as «1.2.1», «1.2.2», «1.2.n» (fig.2). Further, the numbering continues regarding the branching branch.


Fig. 2. Researched circuit example 2
2.2. Currents of the equivalent scheme have indices corresponding to the indices of the branches in which they flow. The circuit elements $R, L, C$ have indices corresponding to the currents indices flowing through them. If an element is nonlinear, an «n» is added to its index.
2.3. The numerical parameters values are set. If the element is nonlinear, then a separate window specifies the approximation polynomial and its coefficients.
3. An equivalent circuit is formed in accordance with the indexation of branches and elements. Note that all branches are reduced to grounding. The formed circuit is displayed on the screen.
4. The indexes of the currents of the equivalent circuit are analyzed and the equations are formed according to the Kirchhoff's I-th law. Equations formation is performed according to the rule: current $\ln$ is equal to the currents sum, which are separated from it, that is:

$$
\begin{equation*}
I_{n}=\sum_{x}^{X} I_{n x} \tag{5}
\end{equation*}
$$

where $x$ - branch number, where the branched current $I_{n}$ flows; $X$ - branches number, where the branched current $I_{n}$ flows.

Let's consider the equations formation for presented in Fig. 1 and 2 schemes. For the scheme 1a current $I_{1}$ is equal to the currents $\mathrm{I}_{11}$ and $\mathrm{I}_{12}$ sum:

$$
\left\{\begin{array}{l}
I_{a 1}=I_{a 11}+I_{a 12}  \tag{6}\\
I_{b 1}=I_{b 11}+I_{b 12}
\end{array}\right.
$$

For the scheme 2 current $I_{1}$ is equal to the currents $I_{11}$ and $I_{12}$ sum, and the current $I_{12}$ - currents $I_{121}$ and $I_{122}$ sum:

$$
\left\{\begin{array}{l}
I_{a 1}=I_{a 11}+I_{a 12}  \tag{7}\\
I_{b 1}=I_{b 11}+I_{b 12} \\
I_{a 12}=I_{a 121}+I_{a 122} \\
I_{b 12}=I_{b 121}+I_{b 122}
\end{array}\right.
$$

5. Parameters of the formed equivalent circuit are analyzed.

The program analyzes the equivalent circuit parameters, determining which subprogram will be used for the obtaining the IP component for each scheme element. Such subprogram are realized using an automated method of IP components formation.
5.1. Here are the subprograms for the IP components formation realized in the program product. On electric circuit different elements the IP expression corresponds to the different well known dependencies in the time domain. The expressions for obtaining the IP components in the frequency domain are presented below:
a) on the power source:
(8)

$$
P_{S}=\left[\begin{array}{ll}
I_{a m} & I_{b m}
\end{array}\right] *\left[\begin{array}{ll}
U_{a n} & U_{b n}
\end{array}\right]
$$

where * - discreet convulsion operation; $m$ - current harmonic number;
b) on linear inductance $L$ :
(9) $\quad P_{L}=\left[\begin{array}{ll}I_{a m} & I_{b m}\end{array}\right] *\left[\begin{array}{ll}I_{b m} L \omega m & -I_{a m} L \omega m\end{array}\right]$,
where $\omega$ - angular frequency;
c) the following expression is used to get the derivative of current in frequency domain:

$$
\frac{d I}{d t} \sim\left[\begin{array}{ll}
I_{b m} \omega m & -I_{a m} \omega m \tag{10}
\end{array}\right] ;
$$

d) on linear active resistance $R$ :

$$
P_{R}=\left[\begin{array}{ll}
I_{a m} & I_{b m}
\end{array}\right] *\left[\begin{array}{ll}
I_{a m} R & I_{b m} R \tag{11}
\end{array}\right] ;
$$

e) on linear capacitor $C$ :

$$
P_{C}=\left[\begin{array}{ll}
\frac{1}{C} I_{a m} & \frac{1}{C} I_{b m}
\end{array}\right] *\left[\begin{array}{ll}
\frac{-I_{b m}}{m \omega} & \frac{I_{a m}}{m \omega} \tag{12}
\end{array}\right]
$$

f) the following equation is used to obtain the integral:

$$
\int I d t \sim\left[\begin{array}{cc}
-\frac{I_{b m}}{m \omega} & \frac{I_{a m}}{m \omega} \tag{13}
\end{array}\right]
$$

g) to obtain the $n$-th power polynomial dependence, the following sequence is performed. The dependence of the inductance on the current is presented as an example:

- polynomial power is set:
(14) $L(i)=a_{0}+a_{1}(i)+a_{2} i^{2}(t)+a_{3} i^{3}(t)+\ldots+a_{n} i^{n}(t)$,
- squared current components $\left(I_{m}\right)^{2}$ are formed:

$$
\left(I_{m}\right)^{2}=\left[\begin{array}{ll}
I_{a m} & I_{b m}
\end{array}\right] *\left[\begin{array}{ll}
I_{a m} & I_{b m} \tag{15}
\end{array}\right],
$$

$$
\begin{align*}
L d I_{m} & =\left[\begin{array}{ll}
a_{0}+a_{2}\left(I_{a m}\right)^{2} & a_{2}\left(I_{b m}\right)^{2}
\end{array}\right] *  \tag{23}\\
& *\left[\begin{array}{ll}
I_{a m} \omega m & -I_{b m} \omega m
\end{array}\right]
\end{align*}
$$

- the instantaneous power on the nonlinear inductance $P_{k L}$ are formed:

$$
\begin{align*}
P_{k L} & =\left[\begin{array}{ll}
\left(I_{a m} d L+L d I_{a m}\right) & \left(I_{b m} d L+L d I_{b m}\right)
\end{array}\right] *  \tag{24}\\
& *\left[\begin{array}{ll}
I_{a m} & I_{b m}
\end{array}\right]
\end{align*}
$$

6. The equations of IP components balance are formed in an analytical form.

$$
\left\{\begin{array}{l}
P_{0}=\sum_{j}^{J} P_{o R j}  \tag{25}\\
P_{a k}=\sum_{j}^{J} P_{a k R j}+\sum_{i}^{I} P_{a k L i}+\sum_{g}^{G} P_{a k \tilde{̃} g} \\
P_{b k}=\sum_{j}^{J} P_{b k R j}+\sum_{i}^{I} P_{b k L i}+\sum_{g}^{G} P_{b k \tilde{N} g}
\end{array}\right.
$$

where $P_{0}, P_{a k}, P_{b k}$ - active power, cosine and sine IP components on the power source, respectively; $\sum_{j}^{J} P_{O R j}$, $\sum_{j}^{J} P_{a k R j}, \quad \sum_{j}^{J} P_{b k R j}$ - sum of active, cosine and sine IP components on $j$-rth active element of equivalent circuit, respectively; $\sum_{i}^{I} P_{\text {okLi }}, \sum_{i}^{I} P_{b L L i}$ - sum of active, cosine and sine IP components on $i$-rth inductive element of equivalent circuit, respectively; $\sum_{g}^{G} P_{a k G g}, \sum_{g}^{G} P_{b K C_{g}}$-sum of active, cosine and sine IP components on $g$-th capacitive element of equivalent circuit, respectively.
7. A numerical calculation of the formed equations system is realized. The solution of it is to obtained the ;amplitudes values of the finding current harmonics $i(t)$.
8. The current curve is constructed $i(t)$.

The interface of the developed software is presented on fig. 3.


Fig. 3. The interface of the developed software

## Conclusions

The method of nonlinear electric circuit's analysis using the method of forming and solving IP orthogonal components balance equations is developed. The advantages of the developed method are that it allows to analyze the mechanism of formation of IP orthogonal components and to evaluate the influence of parameters on the researched electric circuit operation.

The usage of the developed automated method allows to improve the efficiency of the analysis of nonlinear electric circuits of alternating current in the frequency-domain.

The usage of the developed automated method for forming and solving of IP orthogonal components balance equations systems made it possible to increase the efficiency of the nonlinear AC circuits in the frequency domain analysis. It is shown that the proposed method avoids complex intermediate analytical transformations due to the use of the developed automated method of forming instantaneous constituents based on the discrete convolution algorithm.

Authors: Rector of Kremenchuk Mykhailo Ostrohradskyi National University and the Chairman and the Professor of Electric Machines Department Mykhaylo Zagirnyak, Pershotravneva str. 20, Kremenchuk, Ukraine, 39600, e-mail: mzagirn@gmail.com; Associate Professor of Electric Machines Department of Kremenchuk Mykhaylo Ostrohradskyi National University Mariia Maliakova, Pershotravneva str. 20, Kremenchuk, Ukraine, 39600, e-mail: mariia.maliakova@gmail.com; Associate Professor of Electric Drive and Control Systems Department of Kremenchuk Mykhaylo Ostrohradskyi National University Andrii Kalinov, Pershotravneva str. 20, Kremenchuk, Ukraine, 39600, e-mail: andrii.kalinov@gmail.com

## REFERENCES

[1] Zagirnyak, M., Kalinov, A., Chumachova, A. (2013) Correction of operating condition of a variable-frequency electric drive with a non-linear and asymmetric induction motor, IEEE Conference transactions. EUROCON 2013, Croatia, pp. 1033-1037.
[2] Aprille, T. J., Trick, T. N. (1972) Steady state analysis of nonlinear circuits with periodic inputs, Proceeding of the IEEE, vol. 60, no. 1, pp. 108-114.
[3] Chao-Ren Chang (1990) Computer-aided analysis of nonlinear microwave analog circuits using frequency-domain spectral balance, North Carolina State University, 184 p.
[4] Chua Leon, O., Paul Lin, Y. (1975) Computer-Aided Analysis of Electronic Circuits: Algorithms and Computational Techniques, Prentice Hall PTR, 737 p
[5] Luchetta, A., Manetti Reatti, S. (2001) SAPWIN-A symbolic simulator as a support in electrical engineering education, IEEE Transactions on education, vol. 44, no. 2, pp. 9-15.
[6] Zagirnyak, M., Kalinov, A., Maliakova, M. (2011) An algorithm for electric circuits calculation based on instantaneous power component balance, Przegląd Elektrotechniczny (Electrical Review), vol. 87, no. 12b, pp. 212-215.
[7] Moura, L. (2005) Introduction to linear circuit analysis and modeling from DC to RF, MatLAB and SPICE, Burlington, 176 p.
[8] Li, Xin, Hu, Bo, Ling, Xieting, Zeng, Xuait. (2001) A wavelet balance approach for steady-state analysis of nonlinear circuits, IEEE Journal Circuits and Systems, vol. 3, pp. 73-76.
[9] Zhou, Xin, Zhou, Dian, Liu, Jin, Li, Ruiming, Zeng, Xuan, C. Chiang (2004) Steady-state analysis of nonlinear circuits using discrete singular convolution method, IEEE Proceedings of the Design, Automation and test in Europe Conference and Exhibition, vol. 2, pp. 1322-1326.
[10] Zagirnyak, M., Maliakova, M., Kalinov, A. (2015) Analysis of operation of power components compensation systems at harmonic distortions of mains supply voltage, Proceedings 2015 ACEMP-OPTIM-Electromotion 2015 Joint Conference. Side, Turkey, pp. 355-362.
[11] Zagirnyak, M., Maliakova, M., Kalinov, A. (2017) An analytical method for calculation of passive filter parameters with the assuring of the set factor of the voltage supply total harmonic distortion, Przeglad Elektrotechniczny, vol. 1(12), pp. 197-200.

