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Analysis of transient electromagnetic processes using the theory of variational approaches

Abstract. *A mathematical model of a local power line including a* distributed parameter transmission supply line *as its key element is presented in this paper. A mathematical model of this object is then developed on the basis of variational approaches.* Results of computer simulations are shown in drawings as time-spatial current and voltage waveforms in an electric power system.

Streszczenie. *W artykule, na podstawie interdyscyplinarnego modelowania przedstawiono model matematyczny lokalnej sieci elektrycznej, kluczowym elementem, której jest długa linia zasilania o parametrach rozłożonych. Na tej podstawie opracowano model matematyczny wymienionego obiektu na podstawie podejść wariacyjnych. Wyniki symulacji komputerowej przedstawiono na rysunkach w postaci czasowo-przestrzennych przebiegów prądu i napięcia w układzie elektroenergetycznym. (**Analiza nieustalonych procesów elektromagnetycznych wykorzystująca teorię podejść wariacyjnych**).*

Keywords: *mathematical modelling,* distributed parameter transmission supply line, telegraph equation, Hamilton-Ostrogradski principle, second-type boundary conditions.
Słowa kluczowe: modelowanie matematyczne, sieć elektroenergetyczna o parametrach rozłożonych, równanie telegrafistów, zasada Hamiltona-Ostrogradskiego, warunki brzegowe drugiego rodzaju.

Introduction

Highly adequate analysis of transient electromagnetic processes in power grids is a very important aspect at the stages of both design and operation. Determining electrical parameters in transient states allows for evaluation of grid operation and efficient use of power engineering system elements.

A local power supply line is analyzed here, presented as a transmission supply line that we treat as a dynamic system of distributed electric parameters. The theory of analysis of transient electromagnetic processes in such systems has virtually not changed recently. In order to simplify mathematical analysis of these processes, a range of assumptions is introduced, in particular, regarding high-voltage supply lines.

Determination of power supply line parameters in different operating conditions enforces compilation of effective mathematical models that can serve the purpose of a highly adequate analysis of dynamic processes in power engineering systems.

Experience shows very complicated field approaches should be applied to description of transient processes in transmission supply lines. Boundary conditions for electromagnetic field equations must be determined to develop a calculation model. In the present case, this applies to an equation of a transmission supply line known as a telegraph equation. Solving a telegraph equation in non-linear systems is a very complex undertaking and numerical methods are commonly utilized.

To define a discrete model of a dynamic system described with a partial derivative equation, it is necessary to find boundary conditions. Procedures of their determination are not always simple. On the contrary, these conditions are veiled in most power engineering tasks and solving highly complex equations is required to determine the conditions. This problem has been successfully solved in this study by determining second and third-type boundary conditions. A mathematical model of the facility under discussion is determined on the basis of variational approaches, including a modified variational Hamilton-Ostrogradski principle. This enables determination of a model of the system starting from only one energetic approach and thus avoiding a variety of electric power system decompositions.

Mathematical modelling of transient electromagnetic processes in a local power grid based on variational approaches and analysis of working and emergency operating states of such a system are **the objectives of this paper**.

Literature review

Power systems are key facilities in a national economy. Plenty of papers analysing these systems are available, therefore.

[1] introduces a phase coordinate mathematical model of a line in a distributed parameter system. Shortcomings of this approach are discussed below. The authors of [2] take advantage of their frequency method or d'Alembert's method. These methods consider wave processes in a system. EMTP-RV and MatLab are used in [3, 4]. It is not known if the authors consider boundary conditions in their line equations. This is of particular importance where a supply line works with other sub-systems. It is not always possible to account for transverse losses across a supply line in this type of software.

The review demonstrates modelling of transient processes in complicated electrical power systems including transmission supply lines is not developed to a sufficient degree.

Model of a power system

The power system analyzed in this paper is shown in Figure 1. A supply line is the key part of the system, described as a complex distributed parameter supply system. A step-up transformer T_1 and a step-down transformer T_2 are connected to either end of the line. T_1 jest is supplied by an infinite power e_1 source. T_2 works with a supply line of lumped parameters (R_L, L_L, C_L), connected to SEM e_2 on the loading side. The other line is constituted by Γ- equivalent circuit [6]. The power system includes three air-core coils: LR_1 and LR_2 are shunt reactors and LR_3 is a symmetrising coil. All the three are shown as branches including RL elements.

Since only symmetrical transition processes are analyzed, the power system is presented in its single-phase, single-line version, which greatly simplifies the analysis.

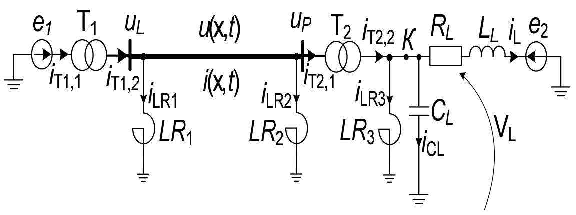

Fig.1. Diagram of the power system under analysis

The modified variational Hamilton-Ostrogradski principle as usual serves to develop a mathematical model of the system under analysis.

The expanded action functional S as per Hamilton-Ostrogradski is defined by [5]:

$$(1)\qquad S=\int_{t_1}^{t_2}\left(L^* + \int_l L_l\,dl\right)dt,\quad I=\int_l L_l\,dl\,,$$

where: L^* – modified Lagrange function and its linear density – L_l., I – energetic functional.

The expanded Lagrange function becomes [5]:

$$(2)\qquad L^* = \tilde{T}^* - P^* + \Phi^* - D^*,\qquad L_l = \tilde{T}_l - P_l + \Phi_l - D_l\,,$$

where: $\tilde{T}^*$ – kinetic co-energy, P^* – conservative energy, Φ^* – dissipation energy, D^* – energy of external, non-potential forces. The index l denotes linear density functions.

A supply line is analyzed as a system of distributed parameters; therefore, Lagrange functions for the line describe energetic functions and their appropriate densities [5]. Parts of the expanded Lagrange function for the power system in Figure 1 are as follows:

$$(3)\quad T^*=\sum_{m=1}^{2}\left(\int_0^{i_{Tm,1}}\Psi_{Tm,1}di_{Tm,1}+\int_0^{i_{Tm,2}}\Psi_{Tm,2}di_{Tm,2}\right)+\frac{L_L i_L^2}{2}+\sum_{n=1}^{3}\frac{L_{LR,n}i_{LR,n}^2}{2}\,;$$

$$(4)\quad \Phi^*=\frac{1}{2}\sum_{m=1}^{2}\int_0^t\left(r_{Tm,1}i_{Tm,1}^2+r_{Tm,2}i_{Tm,2}^2\right)d\tau+\frac{1}{2}\int_0^t\left(R_L i_L^2+\sum_{n=1}^{3}R_{LR,n}i_{LR,n}^2\right)d\tau\,;$$

$$(5)\qquad D^*=\int_0^t\left(e_1 i_{T1,1}+e_2 i_L\right)d\tau\,,\qquad P^*=\frac{Q_{CL}^2}{2C_L}\,;$$

$$(6)\qquad \frac{\partial T^*}{\partial x}\equiv T_l=\frac{L_0 Q_t^2(x,t)}{2}\,,\qquad \frac{\partial P^*}{\partial x}\equiv P_l=\frac{1}{2C_0}Q_x^2(x,t)\,;$$

$$(7)\quad \frac{\partial\Phi^*}{\partial x}\equiv\Phi_l=\Phi_{l3}-\Phi_{lB}=\int_0^t\left(\frac{R_0}{2}Q_t^2(x,t)-\frac{g_0}{2C_0^2}Q_x^2(x,t)\right)\Bigg|_{t=\tau}d\tau\,,$$

where: $m = 1, 2$ – autotransformer numbers; $n = 1, 2, 3$ – air-core coil number; $i(x, t)$ – line current; R_0, g_0, C_0, L_0 – line parameters; Φ_{l3} – external energy dissipation; Φ_{lB} – internal energy dissipation; $Q(x, t)$ – function of the line load; $r_{Tm,1}$, $r_{Tm,2}$ – resistances of the primary and secondary autotransformer windings of the m^{th} autotransformer; $L_{Tm,1}$, $L_{Tm,2}$ – winding inductances of the m^{th} primary and secondary autotransformer; L_L – inductance of the lumped parameter line; R_L – effective resistance of the lumped parameter line; $L_{LR,n}$ – inductance of the n^{th} air-core coil; $R_{LR,n}$ – effective resistance of the n^{th} air-core coil; i_L – current of the distributed parameter line; $i_{LR,n}$ – current of the n^{th} air-core coil; $i_{Tm,1}$, $i_{Tm,2}$ – currents across the primary and secondary winding of the m^{th} autotransformer; $\Psi_{Tm,1}$, $\Psi_{Tm,2}$ – flux linkages of the primary and secondary windings of the m^{th} autotransformer; C_L – capacitance of lumped parameter line; Q_{CL} – line load; i_C – leakage current across the lumped parameter line.

Development of a mathematical model for the system in Figure 1 requires the functional variation (1) to be computed and equated to zero once again [7, 8, 9]. The following results:

$$(8)\quad \frac{\partial v}{\partial t}=\left(C_0L_0\right)^{-1}\left(\frac{\partial^2 u}{\partial x^2}-\left(g_0L_0+C_0R_0\right)v-g_0R_0u\right),\ \frac{\partial u}{\partial t}=v\,;$$

$$(9)\qquad \frac{d\Psi_{T1,1}}{dt}=e_1-r_{T1,1}i_{T1,1}\,,\quad \frac{d\Psi_{T1,2}}{dt}=u_L-r_{T1,2}i_{T1,2}\,;$$

$$(10)\quad \frac{d\Psi_{T2,1}}{dt}=u_P-r_{T2,1}i_{T2,1}\,,\quad \frac{d\Psi_{T2,2}}{dt}=V_L-r_{T2,2}i_{T2,2}\,;$$

$$(11)\quad \frac{di_{LR1}}{dt}=\frac{1}{L_{LR1}}\left(u_L-R_{LR1}i_{LR1}\right),\ \frac{di_{LR2}}{dt}=\frac{1}{L_{LR2}}\left(u_P-R_{LR2}i_{LR2}\right);$$

$$(12)\quad \frac{di_{LR3}}{dt}=\frac{1}{L_{LR3}}\left(V_L-R_{LR3}i_{LR3}\right),\ \frac{di_L}{dt}=\frac{1}{L_L}\left(V_L-R_Li_L-e_2\right),\ \frac{du_{CL}}{dt}=\frac{i_{CL}}{C_L}\,,$$

where: $u_{T1,2}$ – voltage of the secondary autotransformer T_1 winding; u_P – voltage of the primary autotransformer T_2 winding; $V_L \equiv u_{CL}$ – voltage of the secondary autotransformer T_2 winding.

Ψ - type model (9), (10) is converted into A-type model [4] in the case of autotransformers [5]

$$(13)\qquad \frac{di_{T1,1}}{dt}=A_{1,11}\left(e_1-r_{T1,1}i_{T1,1}\right)+A_{1,12}\left(u_L-r_{T1,2}i_{T1,2}\right);$$

$$(14)\qquad \frac{di_{T1,2}}{dt}=A_{1,21}\left(e_1-r_{T1,1}i_{T1,1}\right)+A_{1,22}\left(u_L-r_{T1,2}i_{T1,2}\right);$$

$$(15)\qquad \frac{di_{T2,1}}{dt}=A_{2,11}\left(u_P-r_{T2,1}i_{T2,1}\right)+A_{2,12}\left(V_L-r_{T2,2}i_{T2,2}\right);$$

$$(16)\qquad \frac{di_{T2,2}}{dt}=A_{2,21}\left(u_P-r_{T2,1}i_{T2,1}\right)+A_{2,22}\left(V_L-r_{T2,2}i_{T2,2}\right),$$

where: $A_{m,kP}$ – coefficients dependent on reversed autotransformer inductances [5].

Considering the boundary conditions for the telegraph equation (8), the second Kirchhoff's law for electric circuits of distributed parameters is applied [10, 11]:

$$(17)\qquad -\frac{\partial u(x,t)}{\partial x}=R_0 i(x,t)+L_0\frac{\partial i(x,t)}{\partial t}\,.$$

(8) and (17) are then discretised by means of the lines method [5]:

$$(18)\quad \frac{dv_j}{dt}=\frac{1}{C_0L_0}\left(\frac{u_{j-1}-2u_j+u_{j+1}}{(\Delta x)^2}-\left(g_0L_0+C_0R_0\right)v_j-g_0R_0u_j\right);$$

$$(19)\quad \frac{di_j}{dt}=\frac{u_{j-1}-u_{j+1}}{2\Delta x L_0}-\frac{R_0}{L_0}i_j\,,\ \frac{du_j}{dt}=v_j\,,\quad j=1,\dots,N\,.$$

Analysis of (18) and (19) shows it is necessary to determine voltage across the fictitious discretisation nodes u_0 and u_{N+1} [5] In order to find voltage of the first and last discretisation nodes as well as current across the first and last branches (of the discretisation nodes), traditional equations of stationary conditions and equivalent circuit of the distributed parameter line in Figure 1 are employed [6, 8, 11]:

$$i_{T1,2} - i_1 - i_{LR1} - i_{1C} - i_{1g} = 0\,,\; i_{1g} = \Delta x g_0 u_1\,, \tag{20}$$
$$i_{1C} = \Delta x C_0 \frac{du_1}{dt} = \Delta x C_0 v_1\,,$$

where: i_{1C}, i_{1g} – leakage currents across No. 1 node of the spatial line discretisation [6].

Let the expressions from (20) be differentiated in time considering the initial conditions [5]:

$$\frac{di_{T1,2}}{dt} - \frac{di_1}{dt} - \frac{di_{LR1}}{dt} - \frac{di_{1C}}{dt} - \frac{di_{1g}}{dt} = 0\,,\; \frac{di_{1g}}{dt} = \Delta x g_0 v_1\,, \tag{21}$$
$$\frac{di_{1C}}{dt} = \Delta x C_0 \frac{dv_1}{dt}\,.$$

On the basis of the first equation in (19), an equation is then written to determine currents in the first and final discrete branches of the line (19).

$$\frac{di_1}{dt} = \frac{1}{L_0}\left(\frac{u_0 - u_2}{2\Delta x} - R_0 i_1\right),\; \frac{di_N}{dt} = \frac{1}{L_0}\left(\frac{u_{N-1} - u_{N+1}}{2\Delta x} - R_0 i_N\right). \tag{22}$$

Substituting (14), the first equation from (22), the first equation from (11), the second and third equations from (21) to the first expression in (21), and considering (18) written for the first discrete line node, voltage of a fictitious node u_0 is determined:

$$u_0 = \frac{2\Delta x L_0}{3}\Bigg[A_{21}\left(e_1 - r_{T1,1} i_{T1}\right) + A_{22}\left(u_L - r_{T1,2} i_{T1,2}\right) - \Delta x g_0 v_1 + \tag{23}$$
$$+ \frac{\Delta x\left(g_0 L_0 + C_0 R_0\right)}{L_0} v_1 + \frac{\Delta x g_0 R_0}{L_0} u_1 + \frac{\left(2u_1 - u_2\right)}{L_0 \Delta x} -$$
$$- \frac{1}{L_{LR1}}\left(u_L - R_{LR1} i_{LR1}\right) + \frac{u_2}{2L_0\Delta x} + \frac{R_0}{L_0} i_1 \Bigg],\; u_L \equiv u_1\,.$$

The procedure of establishing voltage across the fictitious node u_{N+1} is not presented for space considerations. The voltage can be computed using a method similar to that used to determine u_0, cf. (23). u_{N+1} is then:

$$u_{N+1} = u_{N-1} - 2\left(u_N - u_P\right). \tag{24}$$

u_P across the primary autotransformer T_2 winding is derived from:

$$u_P = \left(\frac{1}{\Delta x L_0} + \frac{1}{L_{LR2}} + A_{2,11}\right)^{-1} \Bigg[\frac{u_N}{\Delta x L_0} - \frac{R_0}{L_0} i_N + \tag{25}$$
$$+ A_{2,11} r_{T2,1} i_{T2,1} - A_{2,12}\left(V_L - r_{T2,2} i_{T2,2}\right) - \frac{R_{LR2}}{L_{LR2}} i_{LR2} \Bigg]\,.$$

i_{CL} in the third equation of (12) can be calculated by means of the first Kirchhoff's law:

$$i_{CL} = i_{T2,2} - i_R - i_L\,. \tag{26}$$

The following system of differential equations: (11) – (16), (18), (19) is integrated jointly, in consideration of: (23) – (26).

Results of computer simulation

The model of a local electric power grid in Figure 1 serves as the starting point for developing a computer algorithm for calculating the grid parameters using Visual Fortran. A computer simulation of transient electromagnetic processes in the system under analysis is prepared on this basis.

At a moment in time $t = 0$ s, the system is started by simultaneous switching on of two electromotive forces in zero initial conditions: $e_1 = 215.9\cdot\sin(\omega t)$ kV and $e_2 = 210.1\cdot\sin(\omega t - 5{,}9^{\circ})$ kV. Once the system enters its steady state for $t = 0.32$ s, a symmetrical three-phase short-circuiting occurs at point *K* of Figure 1.

The system under analysis has the following parameters: $R_0 = 1.9\cdot 10^{-5}$ Ω/m, $L_0 = 9.24\cdot 10^{-7}$ H/m, $C_0 = 1.3166\cdot 10^{-11}$ F/m, $g_0 = 3.25\cdot 10^{-11}$ Sm/m; two 750/300 kV autotransformers have a total resistance $R_{T1} = R_{T2} = 0.98$ Ω and total inductance $L_{T1} = L_{T2} = 0.188$ H; parameters of the air-core coils: $R_{LR1} = R_{LR2} = 3.415$ Ω, $L_{LR1} = L_{LR2} = 5.974$ H; symmetrising parameters of the air-core coil: $R_{LR3} = 5.58$ Ω, $L_{LR3} = 4.72$ H; parameters of the lumped parameter line: $R_L = 1.9$ Ω, $L_L = 0.1$ H, $C_L = 3.27\cdot 10^{-7}$ F.

Length of the distributed parameter line: $l = 600$ km. A discretisation step: $\Delta x = 600/20 = 30$ km. The system of discretised line equations and ordinary equations of the remaining elements are integrated with the implicit Euler method at a discretisation step $\Delta t = 0.02$ ms and the improved Seidel method [6].

Figure 2 illustrates spatial voltage and current waveforms along the line for the time $t = 0.002$ s. It is clear both voltage and current have maximum values in the initial section of the line: 240 kV and 900 A, respectively. It is also clear the voltage function has a non-linear spatial waveform, whereas the current function displays a near-linear waveform. Time waveforms of both the functions are obviously linear in steady states. An electromagnetic wave shows complex motions in space and the voltage function is directly dependent on the electric field [6].

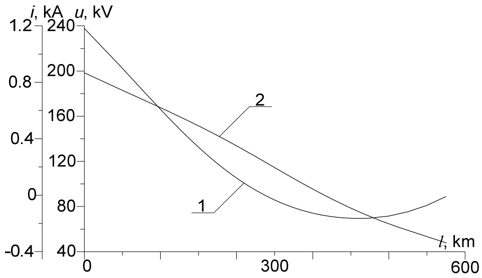

Fig.2. Spatial voltage (1) and current (2) waveforms along the line for the time $t = 0.002$ s.

Figure 3 shows space waveforms of voltage (1) and current (2) along the line for the time $t = 0.007$ s. It becomes clear a similar situation holds for the instant $t = 0.002$ s. Voltage is 830 kV 400 km away from the start of the line and current is 1 kA at the line's end.

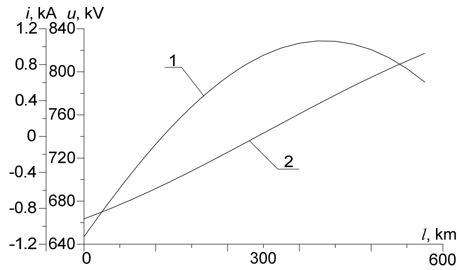

Fig.3. Spatial voltage (1) and current (2) waveforms along the line for the time $t = 0.007$ s.

Figures 4 and 5 illustrate a temporary current waveform at the line's start and voltage waveform mid-line. It is clear it takes approximately 0.3 s for the system to enter its steady state, which depends on parameters of the three air-core coils.

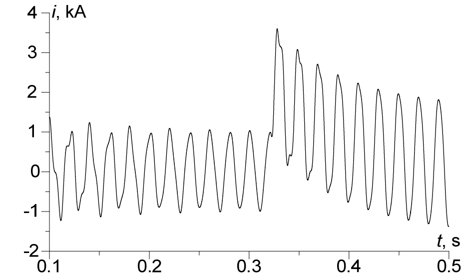

Fig.4. Transient current at the start of the line

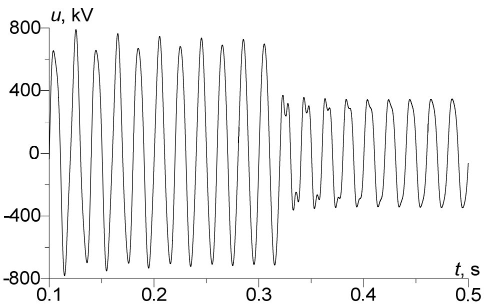

Fig.5. Transient voltage mid-line

Once the system has entered its steady state, the current's amplitude at the start of the line is 698 kV. The steady state voltage mid-line is $1.08U_{MW}$, which points to the presence of Ferranti effect in the system. After a short-circuiting at point *K,* the surge current at the line's start reaches 4 kA while mid-line voltage declines to 380 kV.

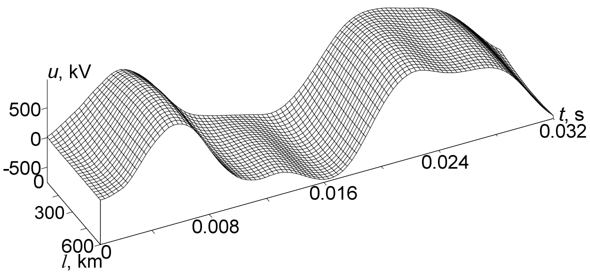

Fig.6. Temporal-spatial voltage waveform across the line for $t \in [0;0{,}032]$ s.

Figure 6 contains a three-dimensional temporal-spatial voltage waveform in the time interval: $t \in [0;0{,}032]$ s. Voltage fluctuations are virtually identical all along the line at the rate of the electromagnetic wave motion corresponding to 50 Hz. Figure 6 should be analysed together with Figures 2 – 5.

Conclusion

1. Application of variational approaches to modelling of transient electromagnetic processes in local power grids helps to develop mathematical models of these grids using a single energetic approach without decomposing the single integrated dynamic system. A local power grid model is developed solely by means of determining an expanded functional according to Hamiltona-Ostrogradski principle. This is a highly effective approach to analysis of power systems.
2. Transient electromagnetic processes in local power lines including transmission supply lines are normally analysed by finding equivalent distributed parameters of straight lines in circuital equivalent circuits. Such an approach greatly simplifies real physical processes described in a line. A very important factor – electromagnetic wave velocity – cannot be taken into account in the circumstances.
3. Application of numerical methods allows for describing a mathematical model of a complex physical facility, namely, a distributed parameter local electric power grid including transmission supply lines.

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