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Determining the dwell time constraint for switched $\mathcal{H}_\infty$ filters

Abstract. *This paper presents an algorithm for determining the minimum dwell time constraint for switched linear $\mathcal{H}_\infty$ fault detection filters. When applying switched systems, ensuring the stability is a crucial part, which can be guaranteed, when we switch slowly enough between the subsystems, more precisely the intervals between two consecutive switching, called dwell time, are large enough. The problem formulation is based on multiple Lyapunov functions and is expressed through a special form of linear matrix inequities (LMIs), which include a nonlinear term with the dwell time. This represents a multivariable time dependent optimization problem. To solve this special formulated LMIs, we propose an algorithm, called T_d-iteration, which is a combination of the procedure of interval halving with an LMI solver. The results of the illustrative example suggest further benefits.*

Streszczenie. *W artykule zaprezentowano algorytm do określania minium czasowego miedzy kolejnymi przełączeniami liniowego filtru do detekcji błędu $\mathcal{H}_\infty$. Sformułowanie problemu bazuje na wielokrotnej funkcji Lapunva. Do rozwiązania tego problmu zaproponowano algorytm T_d**-iteration** który jest kombinacją procedury połówkowania intwerwału z wykorzystanie solvera LMI.* ***Agorytm do określania minium czasowego miedzy kolejnymi przełączeniami liniowego filtru do detekcji błędu $\mathcal{H}_\infty$***

Keywords: Switched linear system, Dwell time constraint, Switched linear fault detection filter, MFARE
Słowa kluczowe: filtr przełączający, funkcja dwell, minimum czasu miedzy przełączeniami

Introduction

Switched systems for purpose of nonlinear control have been studied extensively in the two past decades and useful results are now available, see e.g. [1], [2], [3], [4] and [5]. As it was stated by several authors e.g. (Liberzon and Morse in 1999, Hespana in 2004, Chen and Saif in 2004, Colaneri in 2008,) the asymptotic stability can be ensured when we switch slow enough between the subsystems, more precisely the intervals between two consecutive switchings -called dwell time-, are large enough. This problem has been specially addressed in the synthesis of switched state estimator of Luenberger type, e.g. (Prandini in 2003, Chen and Saif in 2004) and it is also a crucial part in our objective of the designing a switched linear $\mathcal{H}_\infty$ fault detection filter. In earlier researches different methods have been proposed for determining the minimum dwell time, see [4], [6], [7], [8], [9] and [10]. The most commonly used and powerful algorithms, like e.g. the representation based on Kronecker products (Geromel and Colaneri, 2006) or Logic-Based Switching Algorithms (Hespana, 1998), are based on multiple Lyapunov functions and expressed in form of linear matrix inequalities (LMIs), see in [6], [7], [9], [10] and [11].

Since we deal with $\mathcal{H}_\infty$ filtering, the basic Lyapunov theorem needs to be extended to cope with performance requirements such as the root mean square (RMS) property of a switched system, which corresponds normally to determining an upper bound of the minimum dwell time. To this aim, in our research we consider a method used by (Geromel and Colaneri, 2008) for $\mathcal{H}_\infty$ nonlinear control and we have adopted it to the classical $\mathcal{H}_\infty$ detection filtering problem, see in [12], [13], [14], [15] and [16]. More exactly, the concept of the switched $\mathcal{H}_\infty$ control in [7] can be associated to the switched $\mathcal{H}_\infty$ filtering problem by duality and sufficient stability conditions can be derived.

LMIs are nowadays widely used powerful tools for solving complex optimization problems in field of control engineering, see e.g. [17], [18], [19] and [20]. The common used advanced methods, however, refer to a LMI solver only accepts formulation where the decision variables are included in linear terms. On the contrary, our problem is formulated as a LMIs which include the term of matrix-exponential function with the dwell time, is consequently nonlinear. As a result, the task cannot be treated as a simple feasibility problem, see e.g. [17], [21] and [22]. Despite the widespread referring to this special LMI formulation, however, there cannot be found any solution algorithm about it in the control literature. To this aim, in this paper we present an algorithm to calculate the common minimum dwell time, within a pre-specified accuracy, assuring each specified $\mathcal{H}_\infty$ level calculated separately for each single filter.

The contents of this paper are as follows. After the introduction, in Section II the dwell time condition for assuring stability of the switched linear $\mathcal{H}_\infty$ filter is presented. The main outcome is a special form of LMIs including the nonlinear term with the dwell time, which represents a multivariable time dependent optimization problem. Section III presents the proposed numerical algorithm for the calculation of the common minimum dwell time assuring each specified $\mathcal{H}_\infty$ level. In Section IV the algorithm T_d-iteration is applied for an illustrative example in MATLAB. In Section V the main results are summarized and concluded the paper is concluded with some final remarks.

Stability of the State Estimation Error involving the dwell time constraint

The synthesis technique proposed below is originated from results (Geromel and Colaneri, 2008) with focus on the application to robust nonlinear control, see in [7] and [6]. We have adopted this concept to a $\mathcal{H}_\infty$ detection filtering problem and it will be introduced in this chapter.

However, in order to improve the detection's performance we form our concept slightly different from theirs. That means, instead of calculation of the minimum dwell time assuring a common specified $\mathcal{H}_\infty$ level for each controller, we determine the common minimum dwell time to each specified $\mathcal{H}_\infty$ level calculated separately for each single filter.

In the following we are referring to the concept in [12], which's system-description has been extended to a switched linear system.

Extending the switched linear system representation in [6] to the concept of perturbed system, see in [12], the extended switched linear system subjected to disturbance and faults, can be represented in state space form as follows:

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + \\ &+ B_{\kappa\sigma(t)}\kappa(t) + \sum_{i=1}^{k} L_{i\,\sigma(t)}\nu_i(t), \\ y(\mathrm{t}) &= C_{\sigma(t)}x(t), \end{aligned} \tag{1}$$

where for all $t \geq 0$, $x(t) \in \mathbb{R}^n$ is the state vector, $\xi \in \mathbb{R}^n$ is the arbitrarili fixed initial condition, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^p$ is the output vector, $\sigma(t): [0, \infty) \to \Theta$ is the piecewise constant switching function. $A_{\sigma(t)} \in \mathbb{R}^{nxn}$, $B_{\sigma(t)} \in \mathbb{R}^{nxm}$, $C_{\sigma(t)} \in \mathbb{R}^{pxn}$ are an appropriate matrices. Assume that the pairs $(A_{\sigma(t)}, C_{\sigma(t)})$ are observable for all $t \geq 0$. For further consideration denote n_e the number of subsystems, $\Theta = \{1, \dots, n_e\}$ an index set and $q = 1, \dots, n_e$ the sequence number of the switchings. $B_{\kappa\,\sigma(t)} = [B_{w,} L_{\Delta}]$ denotes the worst-case input direction and $\kappa(t) \in L_2\,[0, T]$ is the input function for all $t \in \mathbb{R}_+$ representing the worst–case effects of modelling uncertainties and external disturbances. The cumulative effect of a number of k faults appearing in known directions L_i of the state space and is modelled by an additive linear term $\sum L_{i\,\sigma(t)}\,\nu_i(t)$. $L_i \in \mathbb{R}^{nxs}$ and $\nu_i(t)$ are the fault signatures and failure modes respectively. $\nu_i(t)$ are arbitrary unknown time functions for $t \geq t_{ji}$, $0 \leq t \leq T$, where t_{ji} is the time instant when the i-th fault appears and $\nu_i = 0$, if $t < t_{ji}$. If $\nu_i(t) = 0$, for every i, then the plant is assumed to be fault free. Assume, however, that only one fault appears in the system at a time.

Denote t_ℓ and $t_{\ell+1}$ successive switching times satisfying $t_{\ell+1} - t_\ell \geq \tau_D$. Then the piecewise constant switching function between two consecutive switching as $\sigma(t): [0, \infty) \to \Theta$ for all $t(t_\ell, t_{\ell+1}]$ ensures, that the equilibrium point $x = 0$ of the system in (1) is globally asymptotically stable. The referred constant $\tau_D > 0$ is called the dwell time. Consequently, when designing a switched system one also has to make sure, that the time difference between two consecutive switching's is not smaller than τ_D. Then the asymptotical stability of the switched linear system is preserved, see e.g. in [1], [3], [4] and [7].

Generally interpreted the fault detection filtering is done by estimating the states of the subjected system. Of course, we consider a switched linear system approach, where the q-th sub-filter is selected whenever the q-th subsystem is active. The stability of the state estimation error dynamics may be a crucial part of such design, which can be ensured when we switch slowly enough between the subsystems, to allow the transient effects to dissipate (Chen and Saif, 2004), (Prandini, 2015).

The state estimator for the system description (1) can be represented by the switched system as follows. Let $z \in \mathbb{R}^p$ denote the output signal, then the state estimate can be obtained as

$$
\begin{aligned}
\dot{\hat{x}}(t) &= \left(A_{\sigma(t)} - Y_{\sigma(t)} C^T{}_{\sigma(t)} C_{\sigma(t)}\right)\hat{x}(t) + \\
&\quad + B_{\sigma(t)} u(t) + Y_{\sigma(t)} C^T{}_{\sigma(t)}\, y(t), \\
\hat{y}(t) &= C_{\sigma(t)} \hat{x}(t), \\
\hat{z}(t) &= C_{z\sigma(t)}\, \hat{x}(t),
\end{aligned} \tag{2}
$$

where $\hat{x} \in \mathbb{R}^n$ represents the observer state, $\hat{y} \in \mathbb{R}^p$ represents the output estimate, $\hat{z} \in \mathbb{R}^p$ is the weighted output estimate, $Y_{\sigma(t)}$ is a positive definite matrix the solution of the optimization problem in (5) and $C_{z\sigma(t)}$ is the estimation weighting.

The equation of the state estimation error for (2) is expressed as

$$
\begin{aligned}
\dot{\tilde{x}}(t) &= (A_{\sigma(t)} - Y_{\sigma(t)} C^T_{\sigma(t)} C_{\sigma(t)})\,\tilde{x}(t) + \\
&\quad + B_{\kappa\sigma(t)} w(t) + \sum_{i=1}^{k} L_{i\sigma(t)}\, \nu_i(t), \\
\varepsilon(t) &= C_{z\sigma(t)} \tilde{x}(t),
\end{aligned} \tag{3}
$$

where $\tilde{x}(t)$ and $\tilde{\varepsilon}(t)$ are defined as

$$
\begin{aligned}
\tilde{x}(t) &= x(t) - \hat{x}(t), \\
\varepsilon(t) &= z(t) - \hat{z}(t).
\end{aligned} \tag{4}
$$

As the switching occurs within the finite set of $q \in \Theta = \{1, \dots, n_e\}$ subsystems, the system description in (1) and consequently in (2) and (3) can be simply represented by the matrices $(A_q,\ B_q,\ B_{\kappa q},\ C_q,\ C_{zq},\ L_{iq},\ Y_q)$, for each $q \in \Theta$. Assume that all matrices A_q, $q \in \Theta$ are Hurwitz.

By duality we can associate the $\mathcal{H}_\infty$ control problem of switched linear system described in [12] to our switched $\mathcal{H}_\infty$ filtering task, which's synthesis is based on the Modified Riccati Equation (MFARE), that can be formulated for switched linear system as

$$
\begin{aligned}
& A_q Y_q + Y_q A_q^T + \\
& + Y_q \left(\frac{1}{\gamma_q^2} C_{z\,q}^T C_{zq} - C_q^T C_q \right) Y_q + \\
& + B_{\kappa q} B_{\kappa\,q}^T = 0,
\end{aligned} \tag{5}
$$

for all $q \in \Theta$. In (5) the $\gamma_q > 0$ are positive rational constants and $Y_q \in R^{nxn}$ denote the decision variables which are positive definite matrices.

Following the steps of the synthesis procedure in [7], the MFARE can be factorized in form of Riccati Equation as

$$
H_q Y_q + Y_q H_q^T + Q_q = 0, \ \forall q \in \Theta, \tag{6}
$$

where the associated matrices are

$$
W_q = Y_q \left[C_{z\,q}^T C_q^T \right] \begin{bmatrix} -\frac{1}{\gamma_q^2} I & 0 \\ 0 & I \end{bmatrix}, \tag{7}
$$

$$
H_q = \left(A_q - W_q \begin{bmatrix} C_{zq} \\ C_q \end{bmatrix} \right), \tag{8}
$$

$$
\begin{aligned}
Q_q &= \left(W_q \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} W_q^T + B_{\kappa q} B_{\kappa\,q}^T \right) - \\
&\quad - \gamma_q^2 W_q \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} W_q^T .
\end{aligned} \tag{9}
$$

We have note that the optimal gain W_q is determined from the unique stabilizing solution to MAFARE and the matrix H_q is Hurwitz for each $q \in \Theta$. Since Q_q depends on the γ_{minq} value, $Q_q \geq 0$ is not guaranteed for any $q \in \Theta$. However, (6) admit a positive definite solution, since that was created by factorizing the MFARE. It is to note that for solving the LMIs in (11) is the condition $Q_q \geq 0$ necessary, hence, if $Q_q \geq 0$ does not hold, $\gamma_q > \gamma_{minq}$ can always be chosen such, that $Q_q \geq 0$ holds.

For any $\sigma(t): [0, \infty) \to \Theta$ and for all $t \in (t_\ell, t_{\ell+1}]$, where $t_{\ell+1} = t_\ell + T_\ell$ with $T_\ell \geq T_d > 0$ and at $t = t_{\ell+1}$ the switching jumps to $\sigma(t) = j \in \Theta$, where the corresponding solution of the Lyapunov function along a trajectory of the switched filter state estimation error (2) is expressed by

$$
\begin{aligned}
V\left(\tilde{x}\left(t_\ell\right)\right) &= \tilde{x}\left(t_{\ell+1}\right)^T Z_j\, \tilde{x}\left(t_{\ell+1}\right) = \\
&= \tilde{x}\left(t_\ell\right)^T e^{H_q{}^T T_\ell} Z_j e^{H_q T_\ell}\, \tilde{x}\left(t_\ell\right),
\end{aligned} \tag{10}
$$

where $Z_j \in R^{nxn}$ is positive definite matrix.

The $\mathcal{H}_\infty$ control problem described in [7] can be associated to the $\mathcal{H}_\infty$ filtering problem by duality. By using the time varying solution Riccati equation in finite time interval such that any $\sigma(t): [0, \infty) \to \Theta$ further satisfies the additional constraint $T_U \geq t_{\ell+1} - t_\ell$ for all $q > 0$, for some $T_U \geq T_d > 0$ is given [7]. Based on (6) and the Lyapunov function formulated along a trajectory of the filter error system in (10) one can derive a time varying LMIs, which can be used to obtain the common minimum dwell time for each specified $\mathcal{H}_\infty$ level of each particular filter.

Assume that for a given T_d there exists a collection of positive definite matrices $\{Z_1, \dots, Z_{ne}\}$ of compatible dimensions such that the LMI

$$\begin{aligned} & H_q Z_q + Z_q H_q^T + Q_q < 0, \\ & \forall q \in \Theta, \\ & e^{H_q T_d} Z_j e^{H_q^T T_d} - Z_q + Y_q < 0, \\ & \forall q \neq j \in \Theta, \end{aligned} \tag{11}$$

hold under the worst-case input assumption in (1) for any switching signal $\sigma(t): [0, \infty) \to \Theta$ satisfying the condition $T_d = t_{\ell+1} - t_\ell \geq T_{dmin}$. Then, the equilibrium solution of the state estimation error (3) is globally asymptotically stable.

Numerical method for finding the common minimal dwell time by means of T_d –iteration

As we have shown it in the previous chapter, the problem of determining the common minimum dwell time can be obtained by solving the set of LMIs (11). According to this idea, by means of combining of an algorithm interval halving for fixed scalar T_d the LMIs can be treated as an a feasibility problem and the common minimum dwell time calculated.

After solving the MFARE in (5) and doing the factorization, the Y_q, H_q and Q_q matrices previously obtained for each $q \in \Theta$. In [16] it is explained how the MFARE as a LMI can be formulated and solved. Then (11) can be represented via the following optimization problem:

$$\begin{cases} \min T_d \\ s.t.\ Z_q > 0 \\ \quad Z_j > 0 \\ \quad H_q Z_q + Z_q H_q^T + Q_q < 0 \\ \quad e^{H_q T_d} Z_j e^{H_q^T T_d} - Z_q + Y_q < 0. \\ \forall q \neq j \in \Theta \end{cases} \tag{12}$$

The main benefit of the LMI formulation is, that it defines a convex constraint with respect to the variable vector. For that reason, it has a convex feasible set which can be found guaranteed by means of convex optimization procedure. When using an LMI solver, however, it usually only accepts formulation where the decision variables are included in linear terms. Unfortunately the LMIs in (12) include the term of matrix-exponential with the design scalar variable T_d are nonlinear, consequently the task cannot be treated as a feasibility problem, see in [18], [19], [20] and [22], so the T_d obtained. To overcome this difficultness we implemented an algorithm called T_d-iteration, in which an interval halving method is used iteratively. The algorithm reduces gradually the value of the T_d scalar variable until the constraints of the LMIs in (12) are no longer feasible, consequently any of Z_q matrices, has no longer a positive definite solutions. The T_{dmin} which is so reached, is within the limits given by an arbitrarily small tolerance $\varepsilon > 0$ and is the common minimum dwell time, such holds that $T_{dmin} \leq \tau_D$.

The algorithm for the feasibility problem of determing the minimum dwell time can be formulated as follows (remark: the algorithm has ben also implemented in MATLAB so some commands are referred to them):

The inputs for the method are: Y_q, H_q and Q_q matrices for each $q \in \Theta$ can be obtained from (5), as well as (8), (9), respectively. *eps* as the relative accuracy of the solution, T_{dmax} as the right limit of the interval (the left limit is zero).

The second variables are: *a, b* and *i* , they stand for assignation of interval and counting cycle respectively. The T_d as step size (midpoint), the T_{dm} variable, which contains the value of T_d at the end of the iteration

The outputs are: Z_q matrix $q \in \Theta$ is positive definite decision variable, the T_{dmin} contains the T_d value when the iteration is finished and it is also the minimum dwell time assuring each specified $\mathcal{H}_\infty$ level.

Each iteration performs the following steps:

1. Calculate T_d, the midpoint of the interval, which is assigned by a and b. That is $T_d = a + (b - a)/2$;
2. Calculate the matrix exponential function $e^{H_q T_d}$ for the fixed T_d value and substituting its values in (11);
3. Solve the LMIs in (11) as a feasibility problem by the MATLAB function *feasp* [22], which returns both the scalar value of t_{min} as a measure of the feasibility and the feasibility decision vector *xfeas*;
4. Call the MATLAB function *dec2mat* which returns the solutions for Z_q ;
5. If the feasibility criteria with fixed T_d are not satisfied, that is $t_{min} \geq 0$,

then the upper and lower bounds of interval are changed;
Otherwise the value of T_d is saved, that is $T_{dm} = T_d$ and the iteration is continued;

6. Examine whether the new interval assigned by b-a reached the relative accuracy of the solution - called *epsilon*:

– If not, the iteration is repeated;
– If yes, the iteration is finished and the Z_q matrices are calculated based on the previous value of T_d. Additionally $T_{dmin} = T_{dm}$.

An illustrative example

T_d – iteration algorithm

In this section the presented T_d –iteration algorithm will be implemented for synthesis of switched linear $\mathcal{H}_\infty$ filter based on 3 subsystems and in case study.

Consider the matrices Y_q, H_q and Q_q were previously calculated from (5), (8) and (9). The conditions of the definiteness for each matrix are satisfied according to conditions in (6). The input matrices of the 3 subsystems are,

for the subsystem 1:

$$H_1 = \begin{bmatrix} -107.4991 & 15.4019 & 27.8936 \\ 80.9958 & -564.3523 & 1.9649 \\ 0.1119 & 0.3748 & -8.6469 \end{bmatrix},$$

$$Q_1 = 10^5 * \begin{bmatrix} 0.1810 & -0.2182 & -0.0002 \\ -0.2182 & 2.7837 & 0.0006 \\ -0.0002 & 0.0006 & 0.0000 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 81.6978 & -17.0191 & -0.1127 \\ -17.0191 & 244.1844 & 0.2370 \\ -0.1127 & 0.2370 & 0.0103 \end{bmatrix},$$

for the subsystem 2:

$$H_2 = \begin{bmatrix} -91.9324 & 20.9809 & 28.0690 \\ 106.1269 & -661.2837 & 1.5668 \\ 0.1088 & 0.4258 & -8.6914 \end{bmatrix},$$

$$Q_2 = 10^5 * \begin{bmatrix} 0.1209 & -0.2449 & -0.0002 \\ -0.2449 & 3.0927 & 0.0006 \\ -0.0002 & 0.0006 & 0.0000 \end{bmatrix},$$

$$Y_2 = \begin{bmatrix} 61.5844 & -18.0647 & -0.1088 \\ -18.0647 & 249.8336 & 0.2452 \\ -0.1088 & 0.2452 & 0.0120 \end{bmatrix},$$

for the subsystem 3:

$$H_3 = \begin{bmatrix} -78.3760 & 25.7621 & 27.8720 \\ 132.6617 & -626.8514 & 1.9901 \\ 0.1106 & 0.4382 & -8.5647 \end{bmatrix},$$

$$Q_3 = 10^5 * \begin{bmatrix} 0.0854 & -0.2658 & -0.0001 \\ -0.2658 & 3.1959 & 0.0008 \\ -0.0001 & 0.0008 & 0.0000 \end{bmatrix},$$

$$Y_3 = \begin{bmatrix} 48.0245 & -19.4905 & -0.1127 \\ -19.4905 & 250.7915 & 0.2733 \\ -0.1127 & 0.2733 & 0.0142 \end{bmatrix}.$$

The T_d –iteration including the LMI computation with the feasibility problem was implemented in MATLAB, hereby only its symbolic algorithm is shown, see below! (Note that some commands are from LMI-Toolbox)

Algorithm for finding the common minimum dwell time by means of T_d –iteration

while $i < i_{max}$ **and** $(b - a) > eps$

$T_d = a + (b - a)/2$

$$\begin{cases} Z_q > 0 \\ Z_j > 0 \\ H_q Z_q + Z_q H_q^T + Q_q < 0 \\ e^{H_q T_d} Z_j e^{H_q^T T_d} - Z_q + Y_q < 0 \\ \forall q \neq j \in \theta \end{cases}$$

$[t_{min}, x_{feas}] = feasp(lmis)$

$Z_{qs} = dec2mat(l_{mis}, x_{feas}, Z_q)$

if $t_{min} \geq 0$

$a = T_d$

else

$b = T_d$

$T_{dm} = T_d$

end if

end if

end while

$T_{dmin} = T_{dm}$

The obtained values feasible solutions for Z_q are denoted by Z_{s1}, Z_{s2}, Z_{s3} and shown below. The corresponding eigenvalues $eig(Z_q)$ are denoted by $eig(Z_{s1})$, $eig(Z_{s2})$, $eig(Z_{s3})$ and shown in the row 10 of the Table 1. The positive eigenvalues prove the positive definiteness of Z_q and the feasibility as well.

$$Zs_1 = 10^4 * \begin{bmatrix} 0.4073 & 0.0542 & 0.5203 \\ 0.0542 & 0.0518 & 0.0806 \\ 0.5203 & 0.0806 & 1.8642 \end{bmatrix},$$

$$Zs_2 = 10^4 * \begin{bmatrix} 0.5122 & 0.0832 & 0.6322 \\ 0.0832 & 0.0573 & 0.1137 \\ 0.6322 & 0.1137 & 1.8605 \end{bmatrix},$$

$$Zs_3 = 10^4 * \begin{bmatrix} 0.6249 & 0.1250 & 0.7722 \\ 0.1250 & 0.0691 & 0.1679 \\ 0.7722 & 0.1679 & 1.8750 \end{bmatrix}.$$

Table1.

i	T_d	t_{min}	eig.(Zs_1)	eig.(Zs_2)	eig.(Zs_3)
1	1	−0.0	25.9263 83.4787 246.1332	25.2094 65.1757 251.7411	24.3039 54.9130 252.8941
2	0.5000	−0.0	25.9233 83.4782 246.1332	25.2067 65.1749 251.7411	24.3024 54.9119 252.8940
3	0.2500	−0.0	25.7108 83.4469 246.1329	25.0205 65.1218 251.7407	24.1738 54.8301 252.8934
4	0.1250	−0.4	63.3241 100.7714 246.4629	52.9620 95.4908 252.1020	44.0826 99.2161 253.4098
5	0.0625	−0.5	68.0075 111.0561 246.6869	54.9368 107.9080 252.3338	45.0798 111.5681 253.6986
6	0.0313	0.01	$10^4 *$ 0.0000 0.0219 3.5277	$10^4 *$ 0.0000 0.0115 3.0118	$10^4 *$ 0.0000 0.0073 2.1199
7	0.0469	−0.7	65.5712 108.0716 246.8438	53.7206 104.2321 252.4862	44.1185 107.6819 253.8765
8	0.0391	0.01	$10^4 *$ 0.0000 0.0177 3.8775	$10^4 *$ 0.0000 0.0098 3.0578	$10^4 *$ 0.0000 0.0114 3.2479
9	0.0430	−0.6	63.1824 105.1323 246.8925	52.4740 100.2198 252.5290	43.2152 103.1864 253.9147
1	0.0410	−9.7	$10^4 *$ 0.0437 0.2442 2.0353	$10^4 *$ 0.0426 0.2679 2.1196	$10^4 *$ 0.0423 0.2640 2.2627
1	0.0400	0.01	$10^4 *$ 0.0000 0.0243 6.2884	$10^4 *$ 0.0000 0.0128 5.7631	$10^4 *$ 0.0000 0.0040 1.0035

It is to note that in the rows 6, 8 and 11 we did not get a feasible solution, because the scalar t_{min} returned with a positive value, which means that the associated Z_q pencil contains eigenvalues on or very near the imaginary axis. Of course, this resulted in infeasibility. In such cases according to the algorithm interval halving, in these steps the upper - and lower bounds of an interval changed to ensure a proper distance between the t_m eigenvalues and the imaginary axix.

The iteration ran until the new interval assigned by b-a did not reach the pre-specified relative accuracy of the solution $eps = 0.001$. By performing the T_d –iteration and repeated it 10-times for the $T_{dmin} = 0.0410s$ is obtained by $t_{min} = -9.7372$. The computational cost associated to solving the q independent LMIs plus the iteration. Despite the multivariable time dependent optimization problem, by means of combination an algorithm interval halving with an LMI solver, the determining the common minimum dwell time could be applied. An evolution of measures for the feasibility t_{min} step by step during the iteration is also plotted und shown in Fig. 1.

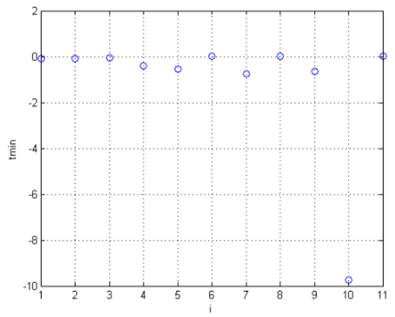

Fig.1. Changing the t_{min} step by step during the iteration

Conclusion

This paper was concerned with a numerical algorithm for determining the minimum dwell time constraint for ensuring stability of switched linear $\mathcal{H}_\infty$ fault detection filter. Despite the complexity of the multivariable time dependent optimization problem, by means of the T_d-iteration, the common minimum dwell time assuring each specified $\mathcal{H}_\infty$ level of each single filter could be calculated. The in the case study implemented algorithm resulted positive definite solutions for Z_q and also the corresponding common minimum dwell time $T_{dmin} = 0.0410s$. On the other hand the $\boldsymbol{T_d}$-iteration has to face of successive numerical computation of the quadratic matrix inequalities resulted in a proportional computation cost. We have found the solution after running the code in MATLAB after 0.5 second CPU time on a PC with Intel® Celeron® CPU B815 (1.60 GHz).

Apart from the advantage that a variety of design specifications and constraints can be expressed through LMIs, due to the combination with the interval halving algorithm, it gives more flexibility to examine the solution during the entire design process. For example, it is easy to analyse the impact of the T_d value on the number of iteration steps or the impact of changing the relative accuracy on the solution. One can easily perform experiments and get answers e.g. to the following questions: How does the iteration converge? How do the eigenvalues of the decision variable change? How close are they to the imaginary axis? Issues with such explicit conditions can be easily examined, step by step during the iterations, which can also be useful for better understanding the nature of switched systems.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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REFERENCES

[1] Paxman J., Switching Controllers: Realization, Initialization and Stability, *Ph.D Dissertation, University of Cambridge*, (2003)

[2] DeCarlo R. A., Branicky M.S., Pettersson S. and Lennartson B., Perspectives and Results On the Stability and Stabilizability of Hybrid Systems, *Proc. IEEE,* Vol. 88, (2000), pp. 1069-1082

[3] Liberzon D., Morse A. S., Basic Problems in Stability and Design of Switched Systems, *IEEE Contr. Syst.*, (1999), pp. 59-70

[4] Hespanha J. P., Morse A.S., Stability of switched systems with average dwell-time, *Proc. 38th Conf. Decision and Contr.*, (1999), pp. 2655-2660

[5] A. Alessandri A., Coletta P., Switching observers for continuous-time and discrete-time linear systems, *In Proceedings of the American Control Conference Arlington*, VA (June, 2001)

[6] Colaneri P., Analysis and control of linear switched systems, *Lecture notes, Politecnico Di Milano,* (2009)

[7] Geromel J. C., Colaneri P., H_∞ and Dwell Time Specifications of Switched Linear Systems, *Proceedings of the 47th IEEE Conference on Decision and Control, Cancun,* (Dec. 9-11.2008)

[8] Chen W., Saif M., Observer design for linear switched control systems, *American Control Conference Proceedings of the* 2004, 0-7803-8335-4, Boston, (2004)

[9] Colaneri P., Geromel J.C. and Astolfi A., Stabilization of continuous-time switched nonlinear systems, *Systems Control Lett., (*2008), 57-1, 95-103

[10] Geromel J.C. and Colaneri P., Stability and stabilization of continuous time switched linear systems, *SIAM J. on Contr. Optim.,(*2006), 45-5, 1915-1930

[11] Karabacak O. and S¸ engo¨r N.S., A dwell time approach to the stability of switched linear systems based on the distance between eigenvector sets, *Int. J. System Science, (*2009), 40(8):845-853

[12] Edelmayer A., Fault detection in dynamic systems: From state estimation to direct input reconstruction, *Universitas-Győr Nonprofit Kft.*, Győr, (2012)

[13] Edelmayer A., Bokor J., Keviczky L., An H_∞ Filtering Approach to Robust Detection of Failures in Dynamical Systems, *in Proc. 33th Annual Decision and Control, Conf.* , Buena Vista, USA, (1994), pp. 3037-3039

[14] Edelmayer A., Bokor J., Keviczky L., An H_∞ Filter Design for Linear Systems: Comparison of two Approaches, *IFAC 13th Triennial World Congress, San Francisco, USA*, (1996)

[15] Horváth Zs., Edelmayer A., Robust Model-Based Detection of Faults in the Air Path of Diesel Engines, *Acta Universitatis Sapientiae Electrical and Mechanical Engineering*, (2015), Vol.7 pp. 5-22

[16] Horváth Zs., Edelmayer A., Solving of the Modified Filter Algebraic Riccati Equation for H-infinity fault detection filtering, *Acta Universitatis Sapientiae Electrical and Mechanical Engineering, (*2017), Vol. 9, pp. 57-77

[17] Bokor J., Gáspár P., Szabó Z., Robust Control Theory, *Typotex, Budapest*, (2013)

[18] Boyd S., Ghaoui L.E., Feron E. and Balakrishnan V., Linear Matrix Inequalities in System and Control Theory, *SIAM, Philadelphia*, (1994)

[19] Nesterow Y., Nemirovski A., Interior Point Polynomial Methods in Convex Programming: Theory and Applications, *SIAM, Philadelphia*, (1994)

[20] Gahinet P, Apkarian P., A linear matrix inequality approach to H_∞ control, *International Journal of Robust and Nonlinear Control*, (1994), Vol.4, pp.421–448

[21] Duan G., Yu R., H. H., LMIs in Control Systems: Analysis, Design and Applications, *CRC Press*, Boca Raton, (2013)

[22] See supplementary material at https://www.mathworks.com/help/robust/ref/feasp.html

[23] Gahinet P., Nemirovski A., Laub A.J. and Chilali M. , LMI Control Toolbox for Use with Matlab, *The MathWorks Inc., Natick, (1995)*

[24] See supplementary material at http://home.deib.polimi.it/prandini/file/2015_06_16%20hybrid%20systems_2.pdf

[25] See supplementary material at https://www.mathworks.com/help/robust/ref/eig.html