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# Mathematical modelling of transient processes in power systems considering effect of high-voltage circuit breakers

**Abstract**. A mathematical model of a power system supplied with high-voltage electricity is developed on the basis of variational approaches. A high-voltage circuit breaker, with its mechanical part treated as a complex disaxial cranking mechanism, is the key element of the model. The model serves the purposes of electric and mechanical analysis of transient processes in the integrated system. Results of computer simulation are presented as graphics.

Streszczenie. W pracy niniejszej na podstawie podejść wariacyjnych opracowano model matematyczny układu elektrotechnicznego zasilania energią elektryczną wysokiego napięcia. Kluczowym elementem wymienionego modelu jest wyłącznik wysokiego napięcia, którego część mechaniczną rozpatruje się, jako skomplikowany mimoosiowy korbowodowy mechanizm. Za pomocą stworzonego modelu dokonuje się analizy elektromechanicznych procesów nieustalonych w scalonym układzie. Przedstawiono wyniki symulacji komputerowej w postaci rysunków. (Modelowanie matematyczne procesów nieustalonych w układach energetycznych z uwzględnieniem wpływu wyłączników wysokiego napięcia)

Keywords: oscillatory processes, high-voltage circuit breaker, mathematical simulation, the Hamilton-Ostrogradski principle, the Euler-Lagrange equation, electrical grid, oscillatory processes.

Słowa kluczowe: procesy oscylacyjne, wyłącznik wysokiego napięcia, modelowanie matematyczne, zasada Hamiltona-Ostrogradskiego, równania Eulera-Lagrange'a, sieć elektryczna, procesy oscylacyjne.

Gas circuit-breakers are used in high-voltage electrical grids [1]. Air-break circuit-breakers are part of some solutions. However, operation of these breakers involves a range of shortcomings, including: the need to operate compressors, considerable noise at the time of commutation, and large size [1].

These defects are absent from circuit-breakers of another type, where operating gas is replaced with sulphur hexafluoride - SF<sub>6</sub>. Its physical and chemical properties are better than of air since it does not react with materials of the breaker fittings, is not toxic or cause fire hazards. It is far smaller than equivalent air-break circuit-breakers. SF<sub>6</sub> circuit-breakers are used both in new power facilities and in modernised power switching stations on a mass scale. ABB circuit-breakers using SF<sub>6</sub> are the most common in high-voltage electrical grids.



Fig.1. Overview of LTB 362-800 (T) E4 circuit-breaker by ABB

LTB 362-800 (T) E4 high-voltage circuit-breaker by ABB is analysed in this paper, quite common in the European countries. Each phase of the breaker consists of two modules connected in series. Each module comprises two pairs of contacts in parallel, to which capacitors are connected in order to distribute voltage more evenly.

It is known [2] the time of arc burning in a circuit-breaker is affected by mechanical processes, in particular, the distance between contacts, dependent on the rate of their disconnection. Note gas pressure in the compression boxes, required to extinguish the arc, is generated only by mechanical means, without extra compression equipment [3]. Such circuit-breakers should have expanded drives to move contacts of the mechanism in order to overcome pressure in the circuit-breaker box, on the one hand, and to ensure necessary rate of contact movement in normal operation of a circuit-breaker. The fact pressure generated operates in the direction opposite to contacts' movement, which gives rise to parasitic oscillations, is an important point in operation of a circuit-breaker. The total time of contacts disconnection is prolonged after that, which can generate an arc [4]. A prolonged time of arc burning adversely affects not only operation of the contacts but also occasionally causes total damage to a circuit-breaker's boxes [5]. Added to all that, the arc itself and processes in the arc boxes affect operation of the contacts shift mechanism. Developing a mathematical model of such complex equipment is a highly complicated task requiring application and addressing of effects described by: applied physics, analytical mechanics, thermodynamics, plasma theory, electromagnetic field theory, and applied power engineering. For instance, the contacts shift is defined as a known function of time in [4]. This approach is somewhat simplified and the issue of oscillatory processes mentioned above is not addressed. The approach proposed here requires this function to be found experimentally, which is virtually impracticable in the case of high-voltage circuitbreakers from the viewpoint of adequacy. Which is precisely what persuaded us to work on an analytical mechanical model.

The objective of this study is to develop a mathematical model of the contacts shift mechanism in a high-voltage circuit-breaker based on Lagrange's theory of the integrated power engineering system. Effect of oscillatory processes in a circuit-breaker on transient processes in an integrated power engineering system is analysed.

### Main data

Our analysis of transient commutation processes in a part of an electrical system for transmitting energy of ultrahigh voltage will be based on the mathematical model we have developed. In order to construct mathematical models of this kind, we propose to apply an interdisciplinary modelling method based on a modification of the well-known Hamilton-Ostrogradski principle [6].

Figure 1 presents an estimated equivalent circuit of a section of 750 kV electrical network connecting two electric-power systems for parallel operation. The electric-power systems are represented by their internal active resistors, inductances, and electromotive forces. The power line is shown as a pi-equivalent circuit with lumped parameters. In order to compensate the line capacity and prevent the voltage from increasing at its terminals, shunt reactors represented by their active resistor and inductance are provided in the system under consideration. Three-phase short-circuit currents will be switched off by a 750 kV switch, whose electric equivalent circuit encompasses a form of parallel connected equivalent nonlinear active resistor and capacity (cf. Fig. 2).



Fig.2. Estimated equivalent circuit of the part of energy transmission electrical system under analysis

We intend to take into account the switching processes by means of active resistors with nonlinear characteristics based on [4], where the authors state that interrupting devices of sulphur hexafluoride switches in the electrical systems' equivalent circuit arc-fire devices can be taken into consideration by means of an equivalent nonlinear resistor  $r_D$  of the contact spacing. Parameters of this resistor are dependent on characteristics of the network, the interrupting device, and the contacts movement mechanism. It is evident the value of this equivalent non-linear resistor  $r_D$  will depend on the distance between the switch contacts, which, in turn, will depend on the disconnection rate.

Figure 3 provides a kinematic scheme of a disaxial double cranking mechanism for switch contacts shift in a LTB 362-800 (T) E4 high-voltage circuit-breaker by ABB.

What is peculiar about this mechanism is that the axis of contacts movement and the drive spring axle are not aligned with the cranking mechanism centre.



Fig.3. Kinematic scheme of the disaxial cranking mechanism for switch contacts movement

Let us note elements of the augmented Lagrange function [6]:

(1) 
$$T^* = \sum_{j=1}^{2} \left( \frac{m_j V_{x,j}^2}{2} \right) + \frac{L_1 i_1^2}{2} + \frac{L_L i_L^2}{2} + \frac{L_2 i_2^2}{2} + \frac{L_{RL} i_{RL}^2}{2} + \frac{L_{RR} i_{RR}^2}{2};$$

(2) 
$$P^* = \sum_{j=1}^{2} \left( \frac{k_j (\Delta x_j)^2}{2} + \frac{Q_{CV,j}^2}{2C_{V,j}} \right) + \frac{Q_{CL1}^2}{2C_{L1}} + \frac{Q_{CL2}^2}{2C_{L2}}$$

(3) 
$$\Phi^* = \sum_{j=1}^{2} \left( \frac{1}{2} \int_{0}^{t} \left( k_{d,j} V_{x,j}^2 \right) d\tau + \frac{1}{2} \int_{0}^{t} \left( r_{D,j} i_{D,j}^2 \right) d\tau \right) +$$

$$+\frac{1}{2}\int_{0}^{t} \left(R_{1}i_{1}^{2}+R_{L}i_{L}^{2}+R_{2}i_{2}^{2}+R_{RL}i_{RL}^{2}+R_{RR}i_{RR}^{2}\right)d\tau + +\frac{1}{2}\int_{0}^{t} \left(g_{L1}^{-1}i_{gL1}^{2}+g_{L2}^{-1}i_{gL2}^{2}\right)d\tau ;$$
(4)  $D^{*} = \sum_{j=1}^{2} \left(4F_{X,j}\Delta x_{j}\right) + \int_{0}^{t} \left(e_{1}i_{1}+e_{2}i_{2}\right)d\tau ,$ 

where j = 1, 2 are the switch numbers;  $L_1, L_2, L_L, L_{RL}, L_{RR}$  are inductances of systems 1, 2, line, and shunt reactors respectively;  $R_1$ ,  $R_2$ ,  $R_L$ ,  $R_{RL}$ ,  $R_{RR}$  are active resistors of the systems 1, 2, line, and shunt reactors, respectively; e1, e2 are electromotive forces of the systems 1 and 2;  $C_{L1}$ ,  $C_{L2}$  are the line capacities;  $C_{Vj}$  is the capacity of equivalent condenser of the  $j^{th}$  switch connected in parallel to the equivalent break of the switch contacts;  $Q_{CL1}$ ,  $Q_{CL2}$ ,  $Q_{CVj}$  are electric charges of capacities  $C_{L1}$ ,  $C_{L2}$ ,  $C_{V,j}$ , respectively;  $i_1$ ,  $i_2$ ,  $i_L$ ,  $i_{RL}$ ,  $i_{RR}$  are currents across the systems 1 and 2, current across the line, and currents of the shunt reactors, respectively;  $g_{L1}$ ,  $g_{L2}$  are active conductivities of the line;  $i_{gL1}$ ,  $i_{gL2}$  are the line leakage currents;  $i_{D,j}$  is the current of the equivalent arc of the  $j^{th}$  switch;  $r_{D1}$ ,  $\dot{r}_{D2}$  – resistance of the equivalent arcs of the breakers 1 and 2, respectively;  $\Delta x_i$  – distance of a spring switching in a *j*-breaker;  $V_{x,j}$  – speed of a spring switching in a *j*-breaker;  $k_j$  – elastic coefficient of a spring in a *j*-breaker;  $k_{d,j}$  – dissipation coefficient of a *j*breaker; m – reduced mass of the contacts;  $F_{X,i}$  is the force acting on the j<sup>th</sup> switch's contacts (normalized to the spring movement coordinates).

A variation of the action functional [6] is provided subject to the relationship:

(5) 
$$\delta S = \delta \int_{0}^{t_1} L^* dt = \int_{0}^{t_1} \delta L^* dt,$$

where *S* is a Hamilton-Ostrogradski action and  $L^*$  is the augmented Lagrange function.

A variation of the action functional according to Hamilton shall be equal to zero if and only if a dynamic system acts according to the Euler-Lagrange equations [6]:

(6) 
$$\frac{d}{dt}\frac{\partial L^*}{\partial \dot{q}_k} - \frac{\partial L^*}{\partial q_k} = 0, \qquad L^* = \tilde{T}^* - P^* + \Phi^* - D^*,$$

where  $\widetilde{T}^*$  – kinetic coenergy,  $P^*$  – potential energy,  $\Phi^*$  – energy dissipation,  $D^*$  – energy of outside nonpotential forces, q – generalised coordinate;  $\dot{q} = dq/dt$  – generalised speed; k – number of degrees of freedom (for holonomic systems).

Space does not permit us to derive the equations here. One can find out more about the methodology for obtaining similar equations in our studies, for example, [7, 8]. The final Euler-Lagrange equation results:

(7) 
$$\frac{di_1}{dt} = \frac{1}{L_1} (e_1 - R_1 i_1 - u_{V1} - u_{CL1}), \quad \frac{di_2}{dt} = \frac{1}{L_2} (u_{CL2} - u_{V2} - R_2 i_2 - e_2);$$

(8) 
$$\frac{du_{CL1}}{dt} = \frac{1}{C_{L1}} (i_1 - i_L - i_{gL1} - i_{RL}), \quad \frac{du_{CL2}}{dt} = \frac{1}{C_{L2}} (i_L - i_{gL2} - i_2 - i_{RR});$$

(9) 
$$\frac{u_L}{dt} = \frac{1}{L_L} (u_{CL1} - R_L i_L - u_{CL2});$$

(10) 
$$\frac{di_{RL}}{dt} = \frac{1}{L_{RL}} (u_{CL1} - R_{RL}i_{RL}), \quad \frac{di_{RR}}{dt} = \frac{1}{L_{RR}} (u_{CL2} - R_{RR}i_{RR});$$

(11) 
$$i_{gL1} = g_{L1}u_{CL1}$$
,  $i_{gL2} = g_{L2}u_{CL2}$ ;

(12) 
$$\frac{du_{V1}}{dt} = \frac{1}{C_{V1}} \left( i_1 - \frac{u_{V1}}{r_{D1}} \right), \ \frac{du_{V2}}{dt} = \frac{1}{C_{V2}} \left( i_2 - \frac{u_{V2}}{r_{D2}} \right);$$

(13) 
$$\frac{d\Delta x_j}{dt} = V_{x,j}, \quad \frac{dV_{x,j}}{dt} = \frac{k_j \Delta x_j + 4F_{X,j} + k_{d,j} V_{x,j}}{m_j}, \quad j = 1, 2,$$

where  $u_{CL1}$ ,  $u_{CL2}$  are voltages at the start and at the end of the line, respectively;  $u_{V,j}$  is the voltage between the contacts of the  $j^{th}$  switch.

We developed and verified a mathematical model of the contacts switching mechanism of LTB 362-800 (T) E4 sulphur-hexafluoride circuit-breaker by ABB in [12], therefore, we give only the final equations characterising the mechanical condition of the switching mechanism of the circuit breaker contacts here (Fig. 3).

(14) 
$$\Delta y = \Delta y_{fakt} - \Delta y_{gran},$$

where  $\Delta y_{fakt}$  – the distance which occurs after switching of the point *A* ' to *A* '' before the contacts break  $\Delta y_{fakt} = \Delta y_{gran}$ .

(15) 
$$\Delta y_{fakt} = \sqrt{(|OB'| + |B'A'|)^2 - e_1^2} - \left[ |B'A'| \sqrt{1 - \left[ \frac{|OB'| \sin((\varphi + \psi) - 90^\circ) - e_1}{-|B'A'|} \right]^2} - |OB'| \cos((\varphi + \psi) - 90^\circ) \right].$$

(16) 
$$\varphi = -250024\Delta x_{fakt}^6 + 169265\Delta x_{fakt}^5 - 45136\Delta x_{fakt}^4 +$$

$$+6044\Delta x_{fakt}^3 - 427, 4\Delta x_{fakt}^2 + 22,78\Delta x_{fakt} + 0,6194.$$

(17) 
$$\Delta x_{fakt} = \Delta x_{gran} - \Delta x \; .$$

We interpolated third order splines to approximate the given function  $r_D(\Delta y)$  for both the breakers and arrived at the formulation:

(18) 
$$r_{D_{0+0,01}} = 0,1 + 21990 \Delta y_{0+0,01};$$

(19) 
$$r_{D_{0,01+0,02}} = 220 + 21990(\Delta y_{0,01+0,02} - 0,01) + 8,01 \cdot 10^7 (\Delta y_{0,01+0,02} - 0,01)^3;$$

(20) 
$$r_{D_{0,02 \div 0,07}} = 520 + 46020 \left( \Delta y_{0,02 \div 0,07} - 0,02 \right) +$$

+ 2,403 \cdot 10<sup>6</sup> 
$$(\Delta y_{0,02 \div 0,07} - 0,02)^2$$
 +  
+ 3,6886 \cdot 10<sup>10</sup>  $(\Delta y_{0,02 \div 0,07} - 0,02)^3$ .

The formulas (18) - (20) describe resistance of the arc at different intervals of the contact breaking. (18) applies to the interval [0; 0.01]m, (19) to [0.01; 0.02]m, and the equation (20) to [0.02; 0.07]m.

The following system of differential equations is subject to joint integration: (7) - (10), (12), (13) including (11), (14) - (20).

## **Computer simulation findings**

Computer simulation was carried out for an actual section of the energy transmission electrical system of the South-Ukrainian NPP (Ukraine) – "Vinnytska" Substation (Ukraine). The 750 kV transmission line is 304 km long. Its element parameters are as follows:  $e_1 = 639 \sin(\omega t + 17.7^{\circ})$  kV,  $e_2 = 598 \sin(\omega t + 4.07^{\circ})$  kV,  $R_1 = 6.803$  Om,  $R_2 = 2.41$  Om,  $L_1 = 0.413$  H,  $L_2 = 0.141$  H,  $R_{RL} = R_{RR} = 3.415$  Om,  $L_{RL} = L_{RR} = 5.974$  H,  $R_L = 5.75$  Om,  $L_L = 0.28$  H,  $C_{L1} = C_{L2} = 2.02 \cdot 10^{-6}$  F,  $g_{L1} = g_{L2} = 4.94 \cdot 10^{-6}$  Sm,  $C_V = 400 \cdot 10^{-12}$  F,  $k_i = 650000$  N/m, m = 10 kg, BO = 0.13 m, AB = 0.11 m, e = 0.13 m, B'O = 0.6 m, A'B' = 0.2 m,  $e_1 = 0.13$  m,  $\psi = 86.5^{\circ}$ ,  $\Delta x_{gran} = 0.16$  m,  $\Delta y_{gran} = 0.06$  m,  $k_{d,i} = 0$  Ns/m.

To cut the discussion short, we will only analyze operation of switch 1.

The computer simulation followed the sequence below. The system output was initially set to standby mode, after which a three-phase short circuit arose at t = 0.819 s at point *K* (see Fig. 2). Switch 1 turned off the short-circuit current after  $\Delta t = 42$  ms.

The force  $F_{X,i}$  is considered functional dependencies of the latter on time (see Fig. 4). Some experiments were conducted. As part of the first experiment, the switch contacts were influenced by an impulse force of 15 kN in the time interval [0.875; 0.878]s, while as part of the second and third experiments, these forces became increasingly rectilinear and exponential up to 20 kN and 30 kN, respectively, in the time interval [0.875; 0.88]s.



Fig.4. The force affecting the contacts (normalized to coordinates of the spring movement) is: 1 – impulse; 2 – increasing according to the rectilinear law; 3 – rising according to exponentially law.

Figure 5 shows that, due to the impact of the impulse force of 15 kN on the contacts, the time of the contacts' complete disconnection (7 cm) was extended by 5 ms (Fig. 5, curve 1) compared to the results obtained without considering oscillatory processes (Fig. 5, curve 4).



Fig. 5. Dependencies of contacts movement of switch 1 on different natures of the impact force: 1 – impulse; 2 – rectilinear; 3 – exponential; 4 – regardless of the impact force.

As far as the rectilinear and exponential character of the effect are concerned, that they have less effect on the mechanism operation irrespective of higher values of their forces, i.e. 20 kN and 30 kN, respectively (Fig. 5). When rectilinear, the time of total disconnection of the contacts as prolonged by 2.7 ms, when exponential, by 2 ms.



Figures 6 and 7 illustrate temporary phase voltages across the circuit-breaker towards system 1 and on the side of the line (i.e. at its start). Analysis of Figure 6 implies voltage amplitude was 612 kV, comparable to its default value, until short-circuiting. The voltage declined to 300 kV afterwards. In Figure 7, voltages across the circuit-breaker on the side of system 1 and of the line are the same until the end of the short-circuit, which is completely logical. Declining voltage oscillations follow the short-circuit.

Analyzing the transient current across a switch for the second experiment (Fig. 8) demonstrates the current chopping disappeared as the contacts disconnection time diminished by 2.3ms compared with the first experiment. The same applies to the surge voltage between the switch contacts (Fig. 9). Due to absence of the current chopping, the surge voltage between the contacts of switch 1 was 810 kV, which is 1.26  $U_{m.w}$ .



Fig.10. Functional dependences of resistance between the contacts on time: 1, 2, 3 – for the different characters of the influencing forces; 4 – without considering oscillating processes

Since the contacts disconnection time of the second and the third experiments is only 0,7 ms, demonstrating results of the third experiment is not relevant since they scarcely differ from those of the second one. However, the value of force in the third experiment is 10 kN greater than in the second one (cf. Fig. 4).

We model the resistance between the contacts by means of the functional dependences (18) - (20). Variations of the resistance as a function of time are presented in Figure 10.

## Conclusion

- Application of the modified Hamilton-Ostrogradsky principle to modelling of complex power systems allows for creation of a mathematical model of an integrated power system with equations relying on a unified energetic approach only, ignoring various decompositions of the system
- Use of a circuit-breaker as a complicated electric and mechanical unit in mathematical models of power engineering systems enables to analyse virtually all dynamic processes in an integrated system. Transient oscillatory processes can be addressed as a result.
- Analysis of the computer simulation results suggests impulse force has maximum impact on oscillatory processes in the mechanism of contacts shifting, whereas the exponential force diminishes approximately twice.

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