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Adaptive Technique based on Fast Fourier Transform for Selecting the Modelled Harmonics' orders in Kalman filter

***Abstract**. Kalman filter is used widely in harmonics detection in power system, where, the quality of the Kalman filter depends on having accurate predicting values based on a mathematical model of the harmonics in power system. It required an exact knowledge of the harmonics' orders, and this is difficult especially that in some of power system apparatus the order of harmonics may change during their operation. For that reason an adaptive Kalman filter combined with Fast Fourier Transform FFT is proposed to determine the orders of harmonics that should be modelled in Kalman filter, in order to reduce the error in the estimated signal.*

Streszczenie**. Filtr Kalmana może być wykorzystywany do określania harmonicznych w systemie energetycznym. Dokładność określania harmonicznych zależy od dokładności predykcji. W celu poprawy dokładności adaptacyjny filtr Kalmana jest wspomagany szybką transformatą Fouriera. **Technika adaptacyjna wykorzystująca algorytm FFT wykorzystana do wyboru modelu harmonicznych w filtrze Kalmana

Keywords: Kalman filter, FFT, Harmonics, Power System.
Słowa kluczowe: filtr Kalmana, harmoniczne w systemie energetycznym, FFT

Introduction

Harmonics are existed in power system due to the power electronics devices, nonlinear loads and Photo Voltaic PV plants.. etc[1] , where the Total Harmonics Distortion THD is not fixed, and it may increase rapidly under certain operating conditions, such as low radiation in PV system [2]. The increment of THD may be due to the increment of the harmonics amplitude or the increment of the numbers of the harmonics' components in the signals[2].

Many of passive and active filters have been used to estimate the fundamental component in distorted input signal, where exact value of the fundamental signal is necessary for the protection devices to work probably.

Kalman filter is one of the most of filters that have been used in power system and power system apparatus for different purposes [3]-[6] especially for harmonic signal estimation in power system[7]-[10] and it showed a better performance compared to other filters [11], but it is required to model the system exactly which means that all the harmonics in the signal must be modelled. It has been discussed in details the effect of harmonic modelling in the Kalman filter in [12], and it was concluded that the best performance of Kalman filter achieved, when all the harmonics in the input signal are modelled, and it was also concluded that, estimating extra harmonics, which are not existed in the measured signal, doesn't affect the Kalman filter output. The error signal (difference between the input signal and the estimated output by the Kalman filter), becomes zero only when all the harmonics of the input signal are included in the Kalman model.

The problem here, when the order of harmonics are changed, while the modelled harmonics are unchanged this will cause the error signal to exceed an acceptable limit, and the order of the estimated harmonics must be modified to reduce that error, for that reason an adaptive Kalman filter is proposed in this paper to model the harmonics exist in the input signal in Kalman filter model, in the proposed filter, the Kalman filter is combined with FFT, where the FFT is used only when the error exceeds certain limits in order to determine the harmonics that must be modelled in the Kalman filter. Two approach canl be used here, the first one is that the Kalman filter matrices' sizes can't be changed, which means that, the number of estimated harmonics by kalamn filter is fixed, and the second approach that the size of Kalman filter can be modified to model all the input signal harmonics. the proposed adaptive filter in this paper is designed based on the first approach.

Kalman filter

Kalman filter is a recursive linear optimal filter, for estimating the state variables and the output Y_k for a system can be modelled as follows [13]:

$$\begin{aligned} X_{k/k} &= A_k X_{k/k-1} + B_k U_k + W_k \\ Y_k &= C_k X_{k/k} + D_k U_k + V_k \end{aligned} \tag{1}$$

where: A_k : transition matrix; B_k : input control vector; W_k : process noise; Y_k: observation state vector; C_k : observation matrix; V_k : observation noise.

the Kalman filter calculation can be divided into two stages; predicted stage and updating stage. in predicted stage, predicted values of the state variables are calculated based on the system model as follows[13]:

$$\begin{aligned} X_{k/k-1} &= A_k X_{k-1/k-1} + B_{k-1} U_{k-1} \\ P_{k/k-1} &= A_k P_{k-1/k-1} + A_k^T + Q_k \end{aligned} \tag{2}$$

then, in the updating values will be updated based on the measurement signal as follows:

$$\begin{aligned} Y_k &= Z_k - C_k X_{k/k-1} \\ K_k &= P_{k/k-1} C_k^T [C_k P_{k/k-1} C_k^T + R_k]^{-1} \\ X_{k/k} &= X_{k/k-1} + K_k Y_k \\ P_{k/k} &= [1 - K_k C_k] P_{k/k-1} \end{aligned} \tag{3}$$

where: Q_k: process noise covariance matrix; R_k: observation noise covariance matrix.

Harmonics Modeling in Kalman Filter

The kalman filter can be used in several applications, harmonic estimation is one of the main uses of kalman filter in Power system, where the Kalman filter equations (equ.3) don't change based on the application, while the system model (A,B,C and D) is changed. The main feature that should the kalman model have is to predict the next output based on the current input, so for harmonics modelling, it is required to have a model to predict the next harmonics based on the current input signal measurement, and this is not difficult since it is not required to model the power system, the model can be as follows:

if the input signal $y(nkT) = A\cos(nwkT + \theta)$

where, K is the current sampling index, T is the sampling time, and n is the harmonic order.
then the next state of the input signal is

$$y(nkT) = A\cos(nw(k+1)T)$$

Now, let the two state variable; $x_1(nkT)$ and $x_2(nkT)$ as follows:

$$x_1(nkT) = A\cos(nwkT + \theta)$$
$$x_2(nkT) = A\sin(nwkT + \theta)$$

the next state of these states variables will be as follows:

$$x_1(n(k+1)T) = A\cos(nw(k+1)T + \theta)$$
$$= A\cos(nwkT + \theta)\cos(nwT) - A\sin(nwkT + \theta)\sin(nwT)$$
$$= x_1(nkT)\cos(nwT) - x_2(nkT)\sin(nwT)$$
$$x_2(n(k+1)T) = A\sin(nw(k+1)T + \theta)$$
$$= A\cos(nwkT + \theta)\sin(nwT) + A\sin(nwkT + \theta)\cos(nwT)$$
$$= x_1(nkT)\sin(nwT) + x_2(nkT)\cos(nwT)$$

rewrite the above equations in matrix form as follows:

$$\begin{bmatrix} x_1(nw(k+1)T \\ x_2(nw(k+1)T \end{bmatrix} = \begin{bmatrix} \cos(nwT) & -\sin(nwT) \\ \sin(nwT) & \cos(nwT) \end{bmatrix} \begin{bmatrix} x_1(nwkT) \\ x_2(nwkT) \end{bmatrix}$$
$$= A_n \begin{bmatrix} x_1(nwkT) \\ x_2(nwkT) \end{bmatrix}$$

so based on this model, the next state of any harmonic oredr can be predicted based on the current state, where, it is required to have at least two state variables.
the input signal $y((k+1)wT)$ will be as follows:

$$y(nw(k+1)T) = x_1(nw(k+1)T$$
$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(nw(k+1)T) \\ x_2(nw(k+1)T) \end{bmatrix}.$$
$$= C_n \begin{bmatrix} x_1(nw(k+1)T) \\ x_2(nw(k+1)T) \end{bmatrix}$$

the harmonic model doesn't contain input signal, then:

$$B_n = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, D_n = [0].$$

when the input signal has several harmonics' components, then each harmonic component needs two state variables to be modelled and the state variables for each harmonic component are decoupled from each others, the model will be as follows:

$$A_k = \begin{bmatrix} A_{1k} & 0 & 0 & \dots & 0 \\ 0 & A_{2k} & 0 & \dots & 0 \\ 0 & 0 & A_{3k} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & A_{nk} \end{bmatrix}, B_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$
$$C_k = \begin{bmatrix} C_{1k} & C_{2k} & C_{3k} & \dots & C_{nk} \end{bmatrix}, D = [0]$$
$$A_{nk} = \begin{bmatrix} \cos(n\omega kT) & -\sin(n\omega kT) \\ \sin(n\omega kT & \cos(n\omega kT \end{bmatrix}, \ C_{nk} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

As it is mentioned earlier that modelling one harmonic component required two state variables, one of them will appear in the output signal based on *C* matrix.

Having an accurate model for harmonics is necessary for kalman filter to estimate the harmonics accurately , and as it is mentioned before, estimating the next state of harmonic is so easy and required two state variables, the problem here is the orders of harmonics that should be modelled, especially that the harmonics' orders can't be known exactly prior the measurement process, and they may vary due to some nonlinear devices in power system.
Since the modelled harmonics in kalman filter are independent from each others, so it is easy to select which of harmonics will be modelled, where In the next section, the kalman filter will not model all the input signal harmonic, then the output of kalman filter will be analysed using FFT to understand the effect of the un modelled harmonic in kalman filter performance.

Simulation Experiment

Before illustrate the basic principle of the proposed adaptive Kalman filter, it is worth to investigate the frequencies in the error signals of Kalman filter.
two error signals are used here; error and fundamental error signals, where the error signal is defined as the difference between the input signal and the estimated output signal by Kalman filter and the fundemental error signal is defined as the difference between the input fundamental signal and the estimated fundamental signal, practically, only the error signal can be measured, so the proposed adaptive Kalman filter will be designed based on its value, while the fundamental error signal is used here to understand the effect of harmonics modelling in the estimated fundamental signal.
:
let the input signal as follows:

$$y(t) = 7\cos(100\pi t) + 5\cos(200\pi t) + 3\cos(400\pi t)$$
$$+ 2\cos(700\pi t)$$

The input signal has fundamental, 2^{nd} , 4^{th} and 7^{th} harmonics components, a Kalman model contains fundamental and all the first 14 harmonics orders will be used to estimate the fundamental input signal. The Fundamental error signal, and the Error signal are shown in Fig.1, since all the harmonic components in the input signal are modelled, both of the error and the fundamental error signal become zeros.

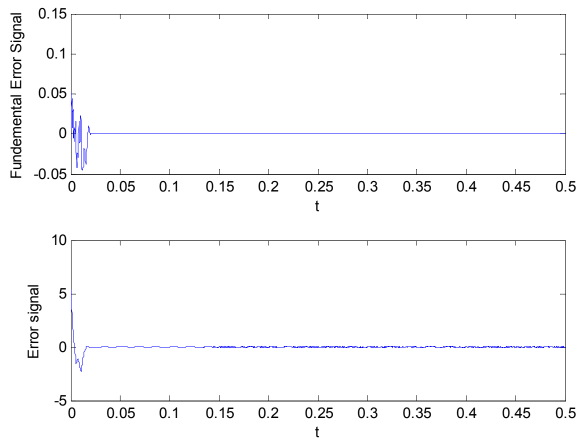

Fig.1. Fundamental error and Error signals when Kalman filter modelled all the input signal harmonic components.

Let the Kalman filter now models all the input signal harmonics except the 7^{th} harmonics. The Fundamental error signal and the Error signal are shown in Fig.2. As it is expected, since the Kalman filter doesn't model the seventh harmonic, the error signal will not equal to zero.

Fig.3 shows the FFT of the error and fundamental error signals, where both of them contain only the seventh harmonic.

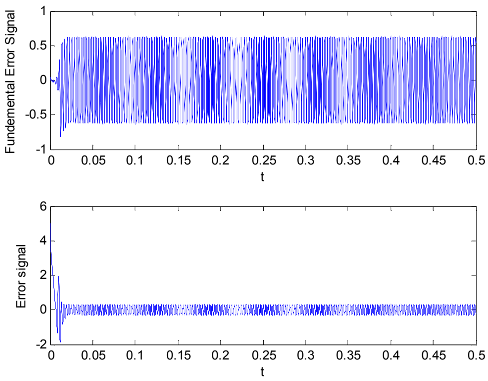

Fig.2. Fundamental error and Error signals when Kalman filter modelled all the input signal harmonic components except the 7^{th} order harmonic.

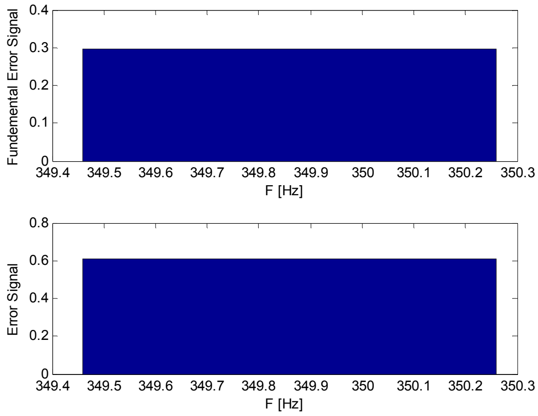

Fig.3. FFT of the Fundamental error and Error signals when Kalman filter modelled all the input signal harmonic components except the 7^{th} order harmonic.

Now if the second harmonic is also not modelled in the Kalman filter, the error signal and fundamental error signal are shown in Fig.4, the error is increased since two harmonics components that are existed in the input signal are not modelled in the Kalman filter. Fig,5 shows the FFT of the error and the fundamental error signals, again the harmonic frequencies that are not modelled in Kalman filter are appeared in the error signal and the fundamental error signal.

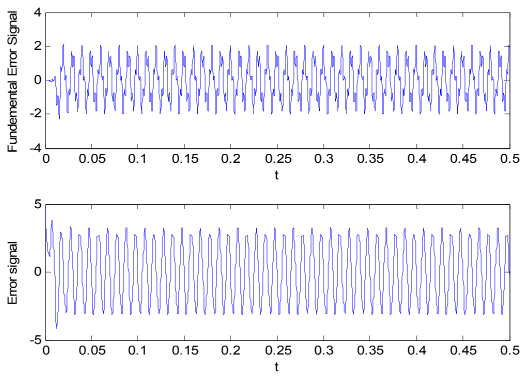

Fig.4. Fundamental error and Error signals when Kalman filter modelled all the input signal harmonic components except the 2^{nd} and 7^{th}order harmonics.

So based on these results, it can be noticed that, the un modelled harmonic frequencies of the input signal are appeared in both of the fundamental error and error signals, this may lead to two facts, the first one is, the estimated harmonics signals have the same amplitude of the harmonic amplitude in the input signal, since the error signal doesn't contain any of the modelled frequencies, the other fact that, the error signal can be used to determine the order of harmonics that must be modelled in Kalman filter.

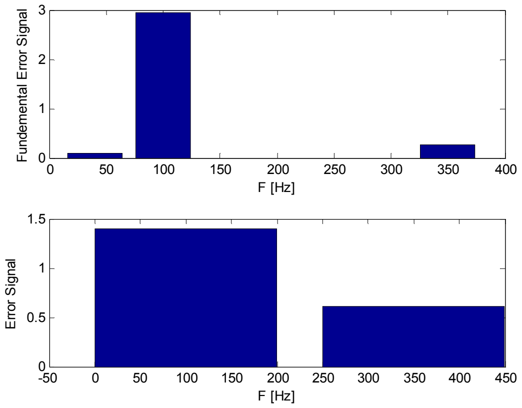

Fig.5. FFT of the Fundamental error and Error signals when Kalman filter modelled all the input signal harmonic components except the 2^{nd} and 7^{th} order harmonics.

Adaptive Kalman filter

FFT can be used to modify the Kalman filter output, this can be done in two ways, depend on the ability of increasing the size of Kalman filter matrices.

If the Kalman filter matrices can't be increased, then FFT will be used to determine if the modelled harmonic order is exist in the input signal or not, and this can be done by determing the frequencies in each of the estimated signals, if the estimated harmonic signal has its own frequency, this means that the input signal has this frequency, if it contains only the un modelled frequency, this means that, input signal doesn't have this frequency. so based on that, the proposed adaptive kalman filter can determine which modelled harmonic should be replaced.

if the size of Kalman filter can be modified, and this is not difficult, since A matrix can be written as decoupled of sub matrices, each one of them contains two states, to model one harmonic signal, FFT will be used to determine un modelled harmonic from the error signal or from any of estimated harmonic signal, these harmonic will be added to the Kalman model, the block diagram of the adaptive Kalman filter is shown in Fig.6.

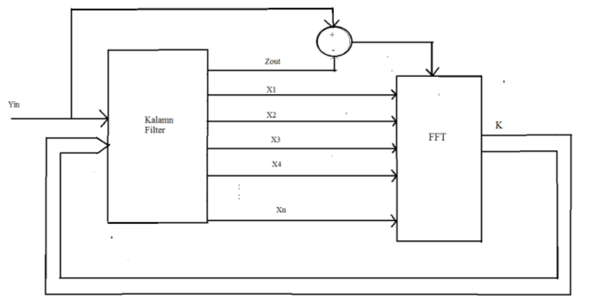

Fig.6. Block diagram of the adaptive Kalman filter.

FFT will be used only when the error exceeds a certain limit, and in this case the source of error will be modified, not the error by itself, where as it has been concluded from the previous results that as soon as the un modelled harmonics in the input signal are modelled, the error becomes zero, and if the orders of harmonic of the input signal are not changed, then FFT will be not used at all. But as soon as the orders of the harmonic are increased in the input signal and their amplitudes causes the error to exceeds certain limits, then FFT will be used to modified the orders of modelled harmonics. The output of FFT is K, where K is a vector contains the modified orders of harmonic, the length of K equal to n, where n is the size of A matrix.

In case of having order of harmonics exceeds n, the most significant harmonic will be modelled, so in this case the error will be minimum based on this size of A matrix.

Even when the FFT is not used, the values of the estimated harmonics of one cycle length is stored, the length is kept constant, and the most recent measured value of the error signal is replacing the oldest stored value. This can be done, to ensure that FFT will not cause a delay for the Kalman filter, when the error exceeds the limit, the required data for the FFT will be already stored, and the results can be obtained immediately.

A Kalman filter that can model the first 8 order of harmonic will be used to illustrate the proposed adaptive Kalman filter, for different type of input signals, the first input signals will be as follows:

$$\begin{aligned} y_{in1} &= 10\cos(100\pi t) + 9\cos(300\pi t) + 8\cos(500\pi t) \\ &+ 7\cos(700\pi t) + 6\cos(900\pi t) + 5\cos(1100\pi t) \\ &+ 4\cos(1300\pi t) + 3\cos(1500\pi t) \end{aligned}$$

The input signal has 8 harmonic components, which is equals to the size of Kalman filter matrices, so in this case the error can be minimized to be zero.

Again, the Kalman filter can model 8 harmonics (fundamental and the first seven order of harmonic beside the d.c value), the initial value of the order of harmonics are 1^{st} , 2^{nd}, 3^{rd}, 4^{th}, 5^{th}, 6^{th}, 7^{th} and 8^{th} . Fig. 7 shows the error and fundamental error signals for the adaptive Kalman filter, where both of error signals are reduced to zero. Fig.8 shows the order of harmonic in the estimated signal of Kalman filter, where the order of estimated signal harmonic were adaptively modified and they are converge to the same order of harmonics in the input signal.

The second input signal as follows:

$$\begin{aligned} y_{in2} &= 10\cos(100\pi t) + 9\cos(300\pi t) + 8\cos(500\pi t) \\ &+ 7\cos(700\pi t) + 6\cos(900\pi t) + 5\cos(1100\pi t) \\ &+ 4\cos(1300\pi t) + 3\cos(1500\pi t) + 2\cos(1700\pi t) \\ &+ \cos(1900\pi t) \end{aligned}$$

The second input signal has 10 harmonic components, which exceeds the size of Kalman filter, so in this case the error can be reduced but it can't become zero. The plot of the error signal and the fundamental error signal are shown in Fig.9, the error signals are decreased but they are never go to zero. The harmonic orders in the Kalman filter are shown in Fig.10. where the proposed adaptive kalman filter model the most significant harmonic orders of the input signal.

Let the third input signal to be as follows:

$$\begin{aligned} y_{in3} &= 10\cos(100\pi t) + 9\cos(300\pi t) + 8\cos(500\pi t) \\ &+ 7\cos(700\pi t) + 6\cos(900\pi t) + 5\cos(1100\pi t) \\ &+ 4\cos(1300\pi t) + 3\cos(1500\pi t) + 2\cos(1700\pi t) \\ &+ 4\cos(1900\pi t) \end{aligned}$$

The main difference of the second and the third input signals is that the 19^{th} harmonic amplitude is greater than the amplitude of the 15^{th} and the 17^{th} harmonics, the adaptive Kalman filter will choose in this case the most significant harmonics that affected the estimated fundamental component, the priority of the un modelled frequencies to be included in the Kalman model, will be depend on the amplitude of them on the estimated fundamental signal. The error and the fundamental error signals are shown in Fig.11. Harmonic orders in the kalamn filter are shown in Fig.12, where the adaptive Kalman filter modelled the largest amplitude of the harmonics signal, again the error can't be zero since the Kalman filter modelled only harmonics' order.

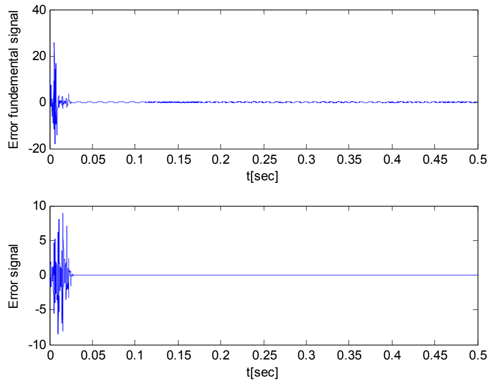

Fig.7. Fundamental error and Error signals of the adaptive Kalman filter for the first input.

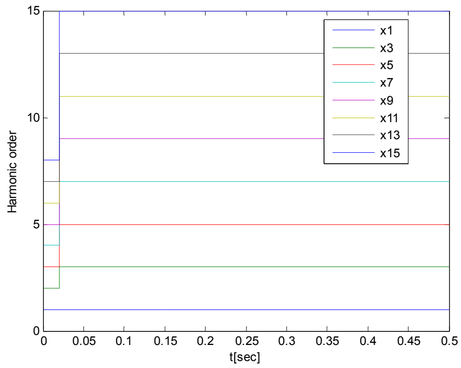

Fig.8. Estimated States order in adaptive Kalman filter for the 1^{st} input signal.

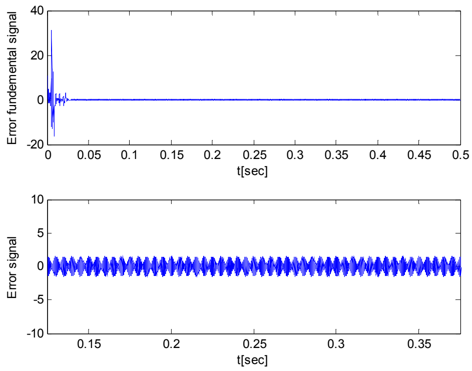

Fig.9. Fundamental error and Error signals of the adaptive Kalman filter for the 2^{nd} input signal.

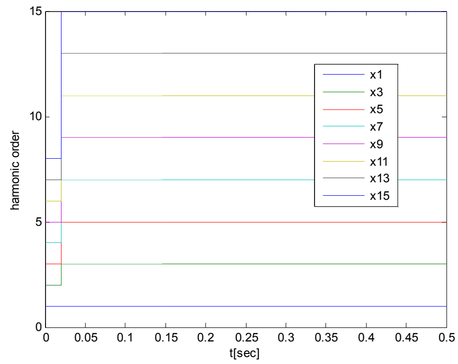

Fig.10. Estimated States order in adaptive Kalman filter for the 2^{nd} input signal.

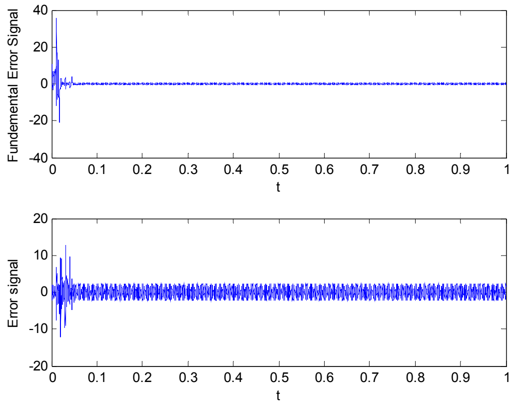

Fig.11. Fundamental error and Error signals of the adaptive Kalman filter for the 3rd input signal.

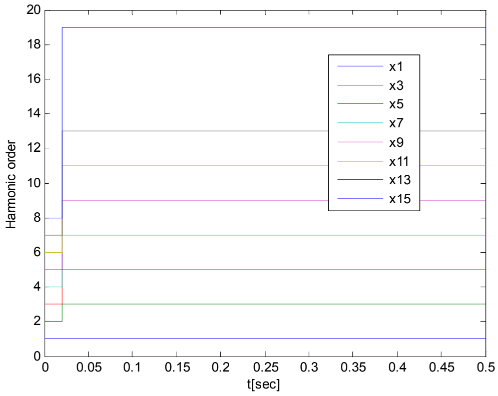

Fig.12. Estimated States order in adaptive Kalman filter for the 3rd input signal.

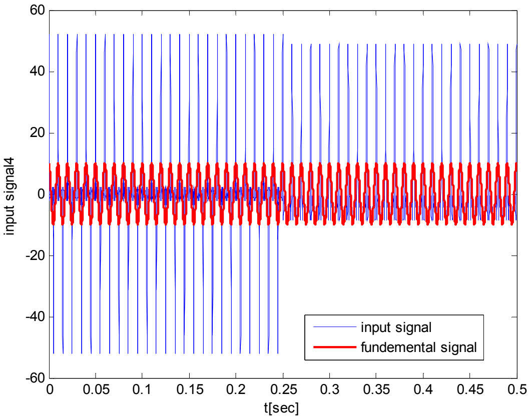

Fig.13. Input signal that has variable order harmonics.

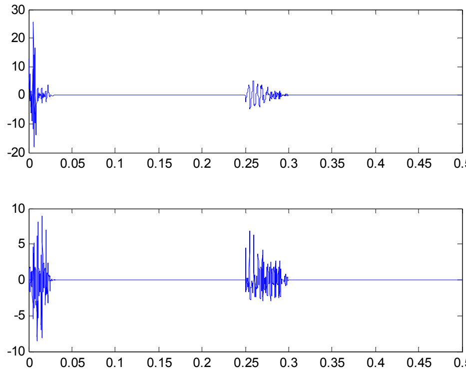

Fig.14. Fundamental error and Error signals of the adaptive Kalman filter for the 4th input signal.

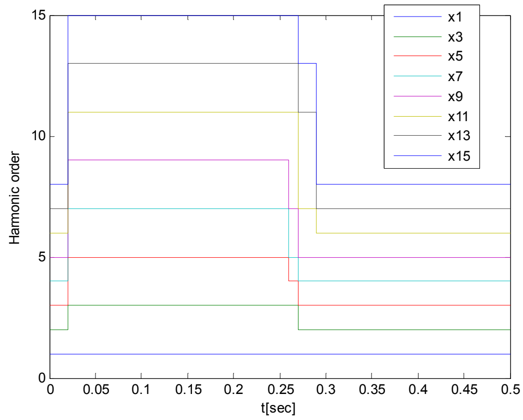

Fig.15. Estimated States' order in adaptive Kalman filter for the 4th input signal.

The fifth input signal is the same as the first input signal but with additive white Gaussian noise with Signal to Noise (SN) ration equal to 30, the input signal and the fundamental signal are shown in Fig.16.

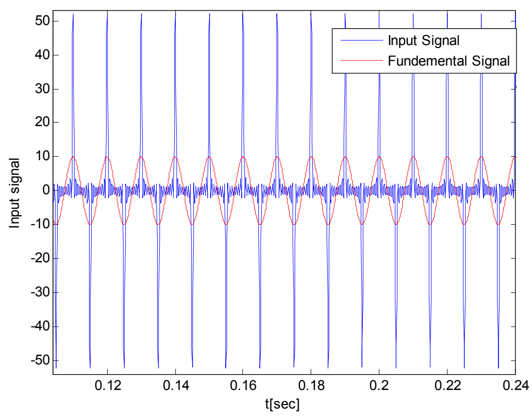

Fig.16. 5th Input signal SNR equals 30.

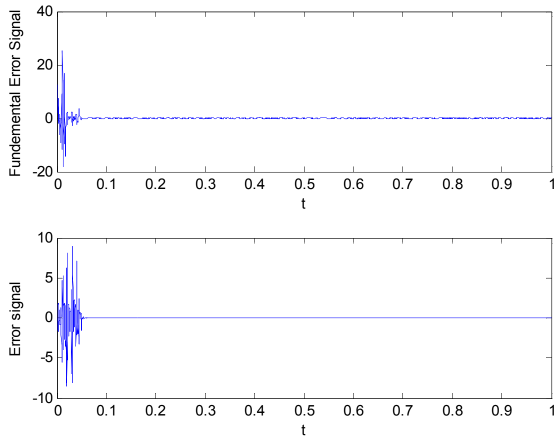

Fig.17. Fundamental error and Error signals of the adaptive Kalman filter for the 5th input signal.

The fourth input signal is shown in Fig.13, the order of harmonics are changed at t=0.25sec, from t=0 to 0.25, the input signal has 1st, 3rd, 5th, 7th, 9th, 11th, 13th and 15th harmonic components, for t=0.25 to 0.5 the input signal has 1st, 2nd, 3rd, 4th, 5th, 6th, 7th and 8th harmonic components. The fundamental error and error signals are shown in Fig.24, where both of them go to zero, The estimated harmonics' orders are shown in Fig.25, where the adaptive

Kalman filter changes the modelled harmonics' orders to minimize the error signal.

The Error and the fundamental error signals for the adaptive kalamn filter are shown in Fig.17, both of the error signal and the fundamental error signal become zero since the Kalman filter is modelling all the harmonics' orders in the input signal, even with existence of the noise, where SN ratio equals 30, and this is considered as a high value in power system. Fig.18 shows the order of harmonics of the estimated states in the Kalman filter, where the adaptive Kalman filter modelled all the input signals harmonics.

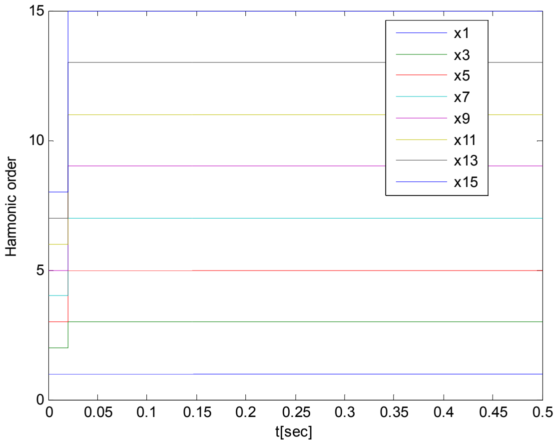

Fig.18. Estimated States' order in adaptive Kalman filter for the 5th input signal.

Conclusions

Kalman filter is combined with FFT to propose An adaptive kalamn filter for harmonic estimation in power system, the proposed filter is adaptively modify the order of harmonics in the kalman model to reduce the error in the estimated signal, in this paper they sizes of kalman matrices are not modified, the results show that the proposed adaptive kalman filter reduces the error for several input signals with different features.

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