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Outage in PMD compensated optical fiber line due to uncertainty of PMD estimation

Abstract. *The effect of uncertain control of a first order PMD compensator on outage probability in fiber optic communication line is analyzed. For a PMD compensator – PMD monitor set a characteristics is proposed that is capable of presenting limits of compensation. Relation between outage probability and uncertainty of estimated PMD parameters, used to control the compensator, is quantified by numerical calculations. Consequences of the uncertainty for maximum transmission reach are evaluated.*

Streszczenie. *Przeanalizowano wpływ niepewności sterowania kompensatorem PMD pierwszego rzędu na prawdopodobieństwo niedostępności w światłowodowej linii telekomunikacyjnej. Dla układu kompensator PMD – monitor PMD zaproponowano charakterystykę umożliwiającą prezentację granic kompensacji. Numerycznie przeprowadzono ocenę ilościową relacji pomiędzy prawdopodobieństwem niedostępności a niepewnością estymacji parametrów PMD, zastosowanych do sterowania kompensatorem. Oceniono skutki niepewności dla maksymalnego zasięgu transmisji.* ***(Niedostępność w światłowodowej linii optycznej z kompensacją PMD w wyniku niepewności estymacji PMD)***

Keywords: optical fiber, polarization mode dispersion, monitoring differential group delay, dispersion compensation.
Słowa kluczowe: światłowód, dyspersja polaryzacyjna, monitorowanie różnicowego opóźnienia grupowego, kompensacja dyspersji.

Introduction

In optical fiber communication the on-off keyed (OOK) signalling with symbol rates rates exceeding 10 giga-symbols per second (Gs/s) requires mitigation of Polarization Mode Dispersion (PMD) effects. If no measures are applied that counteract the PMD the received signal can be occasionally distorted to such a degree that transmitted data symbols cannot be detected with acceptably low probability of error. During this events the communication line cannot be in service, i.e. is in an outage. Episodes of outage happen randomly because PMD in an optical fiber is a random phenomenon. It is of the utmost interest to engineer the optical communication line so the outage probability is sufficiently low. PMD compensation is one of the successful mitigation methods. The compensation can cancel or substantially reduce the PMD resultant distortions, hence decrease outage probability. PMD compensators, both optical and electronic, were studied extensively. In particular, individual compensator structures were optimized and characterized with respect to achievable range of tolerance for Differential Group Delay (DGD) excursions and gain in outage probability [1].

However, one issue has not drawn explicit attention, yet. An application of a compensator can add a chance for an outage if it is controlled by uncertain data. In practice, parameters of the actual PMD, required to control the compensator, shall be estimated from PMD measurements, hence unavoidably suffer from some randomness. The problem in particular arises when the compensator is controlled in an open loop by data output from a PMD monitoring system, e.g. like a one that estimates PMD from transmitted waveforms [2,3].

The paper attempts to complete the knowledge on how uncertainty of PMD estimates, like those obtained from transmitted waveforms, used to control a PMD compensator in an open loop influences compensation. Particularly, the aim is to evaluate suitable limits for uncertainty could have acceptably low effect on outage probability. In order to achieve the goals of the study the conditional outage probability is introduced that quantifies relevant properties of a compensator. With this tool uncertainty resultant outage probability in a fiber optic line is investigated for a given first order PMD compensation scheme. Consequences of the uncertainty for maximum transmission reach are evaluated. Outcomes of the study are applicable to optical system designs in which the first order PMD compensation schemes are used.

Organization of the paper is as follows. First, concepts of the first order PMD compensation are presented and the one selected for the analysis is characterized in detail. Then, the theoretical background of the outage problem in an effect of uncertain control is given and relevant formulas are derived. On this basis outage probability is evaluated and its' behavior assessed, in particular to show relation between outage probability and uncertainty of controlling estimates. Conclusions summarize the key findings.

First order PMD compensation

PMD is a linear effect on optical power, provided only the first order PMD is considered. It is commonly recognized that higher order PMD has negligible influence on performance of direct detection OOK optical fiber communication at data rates below 40 Gs/s, which range is in focus here.

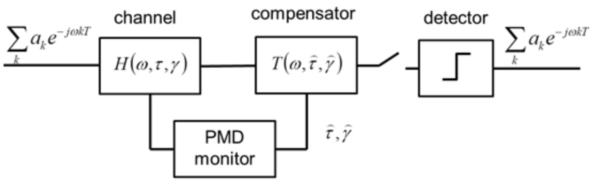

Fig.1. First order PMD compensation with the use of an electronic Feedforward Equalizer (FFE).

In the frequency domain PMD results in selective fading at frequencies related to the actual DGD. Fading depths depend on an actual value of the power split factor between polarization modes. In the time domain PMD manifests as symbol spread which causes intersymbol interference (ISI). In a consequence of locally reduced signal to noise ratio (SNR) probability of error in symbol detection (commonly expressed by the corresponding Bit Error Rate - BER) can increase. Linearity of PMD allows to compensate its effects completely by the means of a linear filter that has an inverse transfer characteristics to the one resulting from the PMD. In an optical fiber both DGD and power split ratio fluctuate randomly. Hence, any PMD compensation method shall allow adaptation of the compensating filter to the actual PMD either in an open or in the closed loop. In the latter variant parameters of the PMD are sensed by a monitoring system and readouts from the PMD monitor control the compensating filter (ref. Fig. 1).

Compensation can be done optically or in the electrical domain. In case of the considered PMD monitor the polarization information is lacking which excludes a use of this monitor in conjunction with an optical compensator. For electronic mitigation of PMD effects there are two options: the Feedforward Equalizer (FFE) or the Decision Feedback Equalizer (DFE) [1]. The FFE (ref. Fig. 1) is a filter located in front of the symbol detector of a digital communication receiver. From the two possible FFE designs, the Zero Forcing (ZF) and the Minimum Mean Square Error (MMSE), the latter better performs in noisy channels [4]. In MMSE FFE total power of combined ISI and noise is minimized. Hence, this filter in an attempt to compensate PMD does not generally cancel the PMD resulting ISI. The DFE in fact is the MMSE FFE followed by a symbol detector placed in a feedback loop equipped with a feedback filter [4]. The DFE is superior to the FFE in terms of the compensation effectiveness and the scope of DGD within which the application of an equalizer reduces probability of erroneous detection. However, communication systems designers may opt for the FFE in favor of simpler practical realization when the widest DGD range is not targeted. It shall be underlined that the MMSE FFE filter affects the signal being equalized in the same way irrespectively whether it is a part of a DFE or a stand-alone FFE. Therefore, when considering effects of uncertain control of an electronic PMD compensator, it is worth focusing on the stand-alone MMSE FFE.

In practice an adaptive FFE can be realized only as a finite impulse response filter with certain number of tunable taps [4]. The number of taps and tap spacing influence the filter characteristics. In order to eliminate variants in the analysis the limiting case is examined in which the infinite impulse response is allowed and the filter structure is unrestricted. In this case the MMSE FFE transfer characteristics shall be given by [4]:

$$K(\omega)=\frac{H^{*}(\omega,\tau,\gamma)}{N(\omega)}\frac{\Xi[A(\omega)]}{\Xi[SN(\omega,\tau,\gamma)]\Xi[A(\omega)]+1}\,. \tag{1}$$

In (1) the asterisk denotes complex conjugate, ω is angular frequency, $H(\omega,\tau,\gamma)$ is the hybrid channel transfer function:

$$H(\omega,\tau,\gamma)=S(\omega)C(\omega,\tau,\gamma)\,. \tag{2}$$

where: $S(\omega)$ is spectrum of the symbol signal, $C(\omega,\tau,\gamma)$ is the transfer characteristics of the PMD affected intensity modulated (IM) optical channel, τ and γ are the actual values of DGD and power split ratio that characterize the state of the first order PMD in the fiber. The other quantities in (1): $N(\omega)$ and $A(\omega)$ are two sided power spectral densities of noise and transmitted data respectively, $SN(\omega,\tau,\gamma)$ is two sided power spectral density of signal to noise ratio. The $\Xi(X)$ is the folding function which outputs spectrum being equivalent to the one effected by signal sampling. For $X(\omega)$, being a spectrum of a signal with infinite duration, the spectrum folding operation is given by [4]:

$$\Xi[X(\omega)]=\frac{1}{T_S}\sum_{n=-\infty}^{\infty}X\left(\omega+n2\pi T_S^{-1}\right), \tag{3}$$

where: T_s is sampling time, here equal to the symbol signaling time interval. The $SN(\omega,\tau,\gamma)$ in (1) is related to quantities that characterize the optical channel through the following formula:

$$SN(\omega,\tau,\gamma)=\frac{|S(\omega)C(\omega,\tau,\gamma)|^{2}}{N(\omega)}\,. \tag{4}$$

For the IM optical channel with the first order PMD the transfer characteristics is given by:

$$C(\omega,\tau,\gamma)=\gamma e^{-j0.5\omega\tau}+(1-\gamma)e^{j0.5\omega\tau}\,. \tag{5}$$

Outage in an effect of uncertainty of estimates controlling an electronic PMD compensator

The key figure that quantifies quality of digital data communication is the Bit Error Rate (BER). For certain classes of communication lines BER thresholds are agreed. If BER increases beyond a tolerable value the line is considered in an outage. In case of incomplete ISI cancellation BER is related to Signal to Noise and Interference Ratio (SNIR), measured at the receiving end. If there exist random phenomena that influence SNIR, like PMD in an optical fiber, communication system designers increase the transmitter power by some margin, a penalty δ, in order to allow some tolerance to SNIR fluctuations. Otherwise, the receiver would experience signal fades below sensitivity threshold with great probability. The penalty shall be given such a value which SNIR fluctuations can exceed only with certain tolerable outage probability, P_{out}. For a communication line could have sufficient applicability valor the probability that it can be in an outage shall be satisfactorily low (typically 10^{-8} [5]).

With MMSE FFE applied the PMD resultant ISI gets reduced and the related SNIR drop is less emphasized. Then, one may expect that a PMD compensated line shall exhibit wider tolerance to PMD fluctuations and consequently, lower or even zero P_{out}. Uncertain control of a first order PMD compensator, i.e. situation in which estimates of DGD and power split ratio, that are used to tune the compensation filter, have some random components, gives rise to another chance of an outage. Improperly tuned compensating filter both can degrade the symbol signal, so its maximum value is decreased, and can boost power level of both output noise and ISI with possibility for SNIR fall below the required minimum. Let $\boldsymbol{\mu}=[\tau,\gamma]$ and $\boldsymbol{\varepsilon}=[\Delta\tau,\Delta\gamma]$, where $\Delta\tau$, $\Delta\gamma$ are random components (errors) in the vector $[\tau+\Delta\tau,\gamma+\Delta\gamma]$ of PMD estimates that control the compensator. The SNIR drop due to random deviation of the controls can be expressed as:

$$SNIR_{drop}(\boldsymbol{\mu},\boldsymbol{\varepsilon})=SNR_0-\frac{s_{\max}^{2}(\boldsymbol{\mu},\boldsymbol{\varepsilon})}{N(\boldsymbol{\mu},\boldsymbol{\varepsilon})+I(\boldsymbol{\mu},\boldsymbol{\varepsilon})}\,, \tag{6}$$

where: $SNIR_0$ is the ultimate SNIR maximum that appears in absence of PMD and lack of control errors, s_{max} is the maximum value of the symbol signal after compensation, N and I are power of noise and ISI, respectively. All last three quantities can be computed as follows. The signal maximum reads:

$$s_{\max}(\boldsymbol{\mu},\boldsymbol{\varepsilon})=\max_{t}\mathscr{F}^{-1}\{S(\omega)C(\omega,\boldsymbol{\mu})K(\omega,\boldsymbol{\mu}+\boldsymbol{\varepsilon})\}\,. \tag{7}$$

The noise power is given by:

$$N(\boldsymbol{\mu},\boldsymbol{\varepsilon})=\frac{1}{2\pi}\int_{-\pi T_s^{-1}}^{\pi T_s^{-1}}A(\omega)|\Xi[H(\omega,\boldsymbol{\mu})K(\omega,\boldsymbol{\mu}+\boldsymbol{\varepsilon})]|^{2}\,d\omega\,. \tag{8}$$

The power of ISI reads:

$$I(\boldsymbol{\mu},\boldsymbol{\varepsilon})=\frac{1}{2\pi}\int_{-\pi T_s^{-1}}^{\pi T_s^{-1}}N(\omega)|\Xi[T(\omega,\boldsymbol{\mu},\boldsymbol{\varepsilon})]|^{2}\,d\omega\,, \tag{9}$$

Outage probability resulting from uncertain estimates controlling the compensator in fact is the probability that the SNIR drop exceeds the penalty δ. Outage probability, conditioned on the fixed values of momentary τ and γ in the fiber, is then given by the formula:

$$P_{out}(\boldsymbol{\mu})=\int_{\Omega(\boldsymbol{\mu})}p_{\varepsilon}(\boldsymbol{\mu},\boldsymbol{\varepsilon})d\varepsilon_1 d\varepsilon_2\,, \tag{10}$$

where: $p_\varepsilon(\mu,\varepsilon)$ is the probability density (pdf) of ε errors in the control vector, i.e. the pdf of errors of the estimates effected by the PMD monitor used. In (10) it is allowed that ε possibly depend on μ, which is the most general case. The area of integration in (10) is defined as follows:

$$\Omega(\boldsymbol{\mu})=\{(\varepsilon_1,\varepsilon_2):\; SNIR_{drop}(\boldsymbol{\mu},\boldsymbol{\varepsilon})>\delta\}, \tag{11}$$

The Ω region is the complement of the area of tolerance, determined at the penalty δ, to the compensator detuning. From $P_{out}(\mu)$ the formula for total outage probability directly flows:

(12) $$P_{tot} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_{out}(\boldsymbol{\mu}) p_{\mu}(\boldsymbol{\mu}) d\mu_1 d\mu_2 \ ,$$

where: $p_{\mu}(\mu)$ is the pdf of τ and γ in the fiber over which the communication is performed. The $p_{\mu}(\mu)$ depends on the fiber properties and environmental conditions where the fiber is laid. For fibers being concatenation of many segments, what is rather typical, the pdf of DGD usually can be described by Maxwellian distribution:

(13) $$p_{\tau}(\tau) = \frac{\pi\tau}{2a^2L}\exp\left(-\frac{\pi\tau^2}{4a^2L}\right),$$

where: a is effective PMD coefficient in the fiber and L is total fiber length. It can be assumed that τ and γ are statistically independent and that γ is evenly distributed [6].

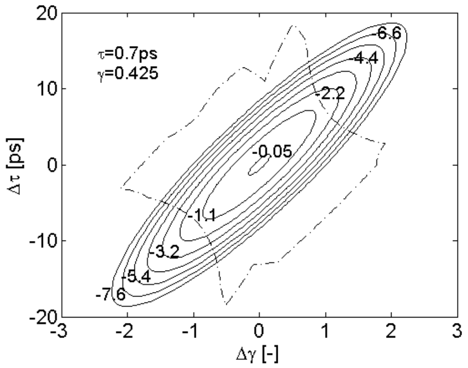

Fig.2. Contour of the tolerance area to errors of compensator control. In the background: isolines of exemplary logarithmic normalized probability density of the errors.

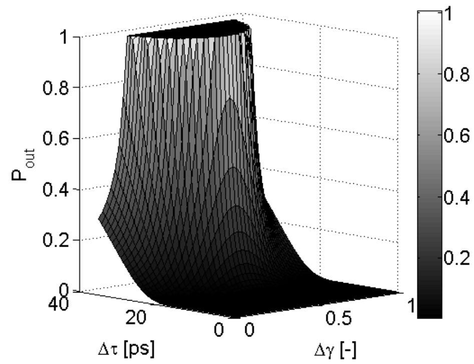

Fig.3. Conditional probability of an outage versus momentary DGD and power split ratio.

While the P_{tot} is the resultant characteristics of the fiber – compensator set the $P_{out}(\mu)$ describes properties of a PMD compensator and PMD monitor used. As can be seen from (10) $P_{out}(\mu)$ dependence on μ has two sources. The $p_{\varepsilon}(\mu,\varepsilon)$ may vary with μ what in general is influenced by the design of the PMD monitor. Also, the extent and shape of tolerance area of the compensator may be affected by μ. Having assumed some common $p_{\varepsilon}(\mu,\varepsilon)$ one can use $P_{out}(\mu)$ to characterize compensators of different types and designs in terms of effected input to the total outage probability as a function of μ. When making comparisons, of practical use can also be a unidimensional variant of $P_{out}(\mu)$ obtained by approximating it by its' maximum over γ, at any τ:

(14) $$P_{om}(\boldsymbol{\mu}) = P_{om}(\tau) = \max_{\gamma}\{P_{out}([\tau,\gamma])\}.$$

Considering that from the two components in (12) the $p_{\mu}(\mu)$ typically can be regarded constant over γ the valor of $P_{om}(\tau)$ is the capability to indicate the τ ranges where tolerance to uncertainty is reduced, hence may give rise to increased total outage probability.

Results and discussion

The computations that are explanative for the relation between *Pout* and $[\tau,\gamma]$ and δ require assumptions on other influential factors, which however are not of direct interest. Presentation of the complete effects of all these extra choices exceeds the capacity of the current communication.

For this reason, the analysis made here focuses on a possibly representative example, results of which can find practical application in optical communications. The scope of investigations is limited by the following assumptions. The transmitted signal is selected to be the 25Gb/s (40ps signaling time) return to zero (RZ), with three options of 33%, 50% and 67% form factor and, PRBS-7 pseudorandom data sequence, the last being typical for testing in data communication. No other optical or electrical filtering is considered except that related to the compensation of the PMD phenomenon. The SNR at the receiving end is assumed 20dB and the penalty δ ranges from 0.5dB to 1.5dB, which fit typical design settings. In order to have first insights into the relation between the outage probability and the uncertainty the $p_{\varepsilon}(\mu,\varepsilon)$ is assumed of Gaussian type with zero correlation between estimation errors and fixed uncertainty over the entire domain of μ. These can correspond to a typical case and if not, at least can comply with the limiting one. The uncertainties are expressed in terms of widths of confidence intervals: $w_{\Delta\tau}$ and $w_{\Delta\gamma}$ of τ (μ_1) and γ (μ_2) estimates respectively specified at 0.99 confidence level. Inspection of conditional outage probability is restricted here to 0…35ps DGD range which relates to maximum DGD excursions approaching 90% of the signaling time interval.

Any electronic PMD compensator exactly tuned to meet actual PMD shows some limit $\tau_{max}(\gamma)$ for compensation capability above which the SNIR drop can no longer be less than given penalty (ref. e.g. [5]). In case of zero uncertainty $P_{out}([\tau,\gamma])$=0 for $\tau<\tau_{max}(\gamma)$ and $P_{out}([\tau,\gamma])$=1 otherwise. In case of uncertain control $P_{out}([\tau,\gamma])$ nowhere reaches 0, although for τ=0, where it gets minimum, can be extremely small (far below 10^{-20} in the considered examples) and the transition from the $P_{out}([\tau,\gamma])$=1 area is gradual (Fig. 3 and Fig. 4). The explanation for the behavior of $P_{out}([\tau,\gamma])$ flows from the observation that uncertain control of the compensator makes the SNIR drop to fluctuate unidirectionally above that value which takes place for zero uncertainty. According to the MMSE design rule the perfectly tuned MMSE FFE compensator provides maximum SNIR at given $[\tau,\gamma]$, consequently – minimum SNIR drop (ref. eq. (6)). Detuning the compensator from perfect settings may only result in an increase of the SNIR drop. At any $[\tau,\gamma]$ the clearance between given penalty and zero uncertainty SNIR drop within an area delimited by $[\tau+\Delta\tau,\gamma+\Delta\gamma]$ excursions decides on the chance that SNIR drop fluctuations exceed given penalty. Hence this clearance determines the corresponding $P_{out}([\tau,\gamma])$ value. When $\tau>\tau_{max}(\gamma)$ the clearance is negative, because zero uncertainty SNIR drop exceeds the penalty, and none $[\Delta\tau,\Delta\gamma]$ can make the clearance increased. Consequently the tolerance area is nulled hence, $P_{out}([\tau,\gamma])$=1. When $\tau<\tau_{max}(\gamma)$ the clearance gradually increases, as τ tends to zero, resulting in progressively lower $P_{out}([\tau,\gamma])$ values. At τ=0 the clearance is maximal, however finite, therefore $P_{out}([\tau,\gamma])$ is minimal and nonzero here.

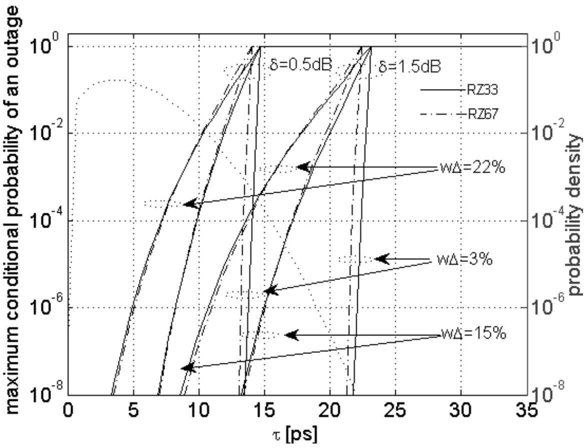

Fig.4. Conditional outage probability (solid and dash-dot lines) versus momentary DGD backgrounded by probability density of DGD in a typical 500km long single mode fiber (dotted line).

The $\tau_{max}(\gamma)$ line demarcates the areas of possible ($P_{out}([\tau,\gamma])<1$, preferably close to zero) and impossible ($P_{out}([\tau,\gamma])=1$) compensation. Above all, the limiting values depend on the penalty applied. The lower is the penalty the smaller is the area of possible compensation. In this aspect the shape of the symbol signal also matters. Results of computations disclose that an increase of the form factor from 33% to 50% and 67% gradually and slightly pushes the demarcation downwards (Fig. 4). This however has minor effect on the total outage probability, P_{tot}. For this reason in the Fig. 5 curves for RZ33% represent P_{tot} behavior of all signaling variants.

The steepness of the $P_{out}([\tau,\gamma])$ slope, in the region where $P_{out}([\tau,\gamma])<1$, depends on the uncertainty of control, as expected. The computations provide valuations for this effect (Fig. 4) which allow worthwhile observations. The $P_{out}([\tau,\gamma])$ quite closely approaches that one for zero uncertainty when the relative widths of confidence intervals of the controlling $[\tau,\gamma]$ estimates are in the range of a few per cent. Larger uncertainties result in substantially lower steepness allowing for $P_{out}([\tau,\gamma])$ to have non-negligible values over reasonably large range of τ values. This may account for unacceptably high P_{tot} when probability of τ to fall within that range is high, which in a fiber optic communication can materialize for sufficiently long fibers.

The consequences of uncertain control of a MMSE FFE compensator on optical fiber communication are illustrated in Fig. 5. where P_{tot} behavior versus fiber length (communication line distance) is quantified. Naturally, for a given penalty higher distances suffer from higher P_{tot} even if uncertainty is zero ("plus" marked solid line in Fig. 5). Allowing for smaller penalty one has to accept higher outage probabilities. This is the result of the unavoidable $\tau_{max}(\gamma)$ limit of compensation capability. Uncertainty of control adds own component to P_{tot}. This extra outage probability is more emphasized for smaller distances Uncertainty of a compensator control has direct economic consequences. It forces systems designers to limit the transmission distance in order to keep outage probability at acceptable level. According to the results, for the 25G line with RZ33 signaling and tolerated $P_{tot}=10^{-8}$, as requested by many telecomm operators [5], the maximum distance achievable at 1.5dB penalty would be pushed from 600 km down to 360 km, if estimation errors of control values were allowed within confidence intervals corresponding to 22% of the maximum measurands. The uncertainty reduction by 32% from this figure relaxes this effect considerably, as the maximum distance is shortened to only 530 km. Again, it can be observed that P_{tot} quite closely approaches that for zero uncertainty when relative widths of confidence intervals of the controlling τ and γ estimates are in the range of a few per cent.

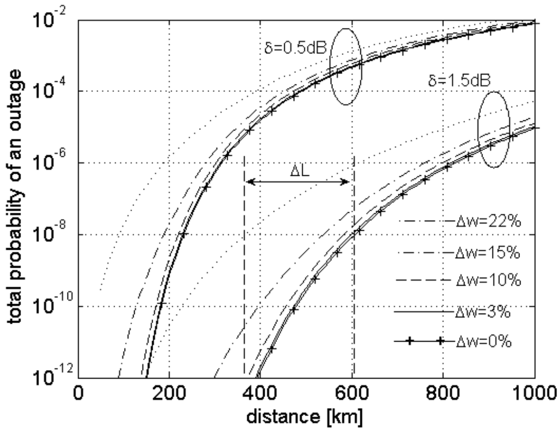

Fig.5. Total probability of an outage versus length of an optical fiber.

Conclusions

Although an application of an electronic PMD compensator can be of choice, if low outage probability is targeted in a PMD affected optical fiber communication line, unreasonable uncertainty of compensator control can have destructive effect on the targeted figure. The amount of extra outage probability due to uncertain PMD estimates controlling the compensator depends on assumed penalty, symbol signal shape, statistics of estimation errors and statistics of PMD in the compensated line. In applications where only first order PMD is of concern the relevant properties of a compensator can be described by conditional outage probability for given DGD and power split ratio and assumed estimation errors statistics. Uncertainty of control can force to reduce transmission distance achievable with a compensator applied. In order the loss of maximum distance was acceptable the uncertainty shall be kept within limits. For a 25G communication line this translates to the requirement for PMD estimation errors be kept within confidence intervals that have widths at the level of a few per cent of the maximum measurands.

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