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# Digital filters to separate the first and second harmonics of signals in microprocessor-bases protection of electrical installations equipped with transformers

***Abstract.*** *The shortcomings of the sine and cosine digital filters used in microprocessor-based protection of electrical installations are discussed in the article; a technique to overcome the shortcomings is proposed. The mathematical description of non-recursive filters is presented. One of the models of the proposed method of formation of the orthogonal components is presented as well. The study of frequency and transient responses of digital filters at different types of input impact was performed. The reaction of various filters to inrush of magnetization currents of power transformers is investigated.*

***Streszczenie.*** *W artykule omówiono wady cyfrowych filtrów sinusoidalnych i cosinusoidalnych stosowanych w mikroprocesorowej ochronie instalacji elektrycznych, zaproponowano technikę poprawy tych wad. Przedstawiono matematyczny opis filtrów nierekurencyjnych. Omówiono również jeden z modeli proponowanej metody tworzenia składowych ortogonalnych. Przeprowadzono badania częstotliwości i odpowiedzi filtrów cyfrowych przy różnych typach sygnału wejściowego. Zbadano reakcję różnych filtrów na oddziaływanie prądów magnesujących transformatorów mocy.**(Filtry cyfrowe do separacji pierwszej i drugiej harmonicznej sygnałów w mikroprocesorowych układach zabezpieczających instalacje elektryczne z transformatorami mocy.)***

**Keywords:** a combined filter, a cosine filter, a sine filter, an orthogonal components former, inrush of a magnetization current, frequency response, transient response.

## Introduction

In electrical installations equipped with power transformers when they are turned on with no load under voltage, an inrush of the magnetizing current (IMC) occurs in some cases of voltage recovery after disconnection of external short circuits (SC) and in some other cases. Under the influence of mentioned inrush overcurrent protection of electrical installations might falsely trigger. Prevention is possible either by means of an increase of the current of protection operation (that is not always acceptable), or by the implementation of the protection blocking at the IMC occurrence.

The classic way of locking protection when IMC takes place is based on tuning-out by harmonics content. The content of low harmonics in the current can be the criterion for protection actuation, and the presence of the current harmonics of a higher order can indicate the appearance of the IMC and their use to block the action of protection.

Various options of the lock, based on the above provision, are presented in [1]. Herewith, one of these implementations involves the use of the amplitude of the second harmonic current of negative sequence and of the amplitude of the current of the first harmonic of direct sequence, according to which a lock parameter is determined [2].

In order to obtain the necessary information, from the corresponding currents with the aid of frequency filters the harmonics with frequencies of 100 and 50 Hz are separated, according to which their sine and cosine orthogonal components (OC) are formed. In accordance with the specified OC or with the readouts of instantaneous values of one of them [3] amplitudes of currents of the first and second harmonics for the subsequent determining of the lock parameter are calculated.

The article describes the principle of implementation of the respective filters that makes it possible to obtain eventually current amplitudes of the first and the second harmonics currents and examines frequency and dynamic properties of the filters.

## The main part

In order to receive the information parameters of the analog signals of currents and voltages in microprocessor-based protection, preliminary analog filtering by the filters of the lower frequencies followed by the subsequent filtering by non-recursive digital filters (DF) and separation of their OC is implemented.

The mathematical description of such DF is as follows:

$$u(n)=\sum_{i=0}^{N-1} y(n-i)A(i), \tag{1}$$

where $n$ is the number of the current sampling; $y(n)$ is the $n^{\text{th}}$ value of the input signal of DF; $u(n)$ is the $n^{\text{th}}$ value of the output signal of DF; $A(i)$ is the value of the $i^{\text{th}}$ coefficient; $N$ is the quantity of the DF coefficients.

In order to separate the OC the orthogonal components formers (OCF) based on sine and cosine DF are used. In general, the $A_C$ coefficients of the cosine DF are calculated as [4]:

$$A_c(i)=\frac{2}{N}\cos\left(\frac{2\pi ki}{N}\right), \tag{2}$$

where $k$ is the number of harmonic component being separated; $i$ is the coefficient number, $i$ = 0…$N$.

Sine coefficients are determined in accordance with the expression

$$A_s(i)=\frac{2}{N}\sin\left(\frac{2\pi ki}{N}\right). \tag{3}$$

This type of filter has certain advantages and disadvantages. The advantage of the sine DF is a better suppression of higher frequencies that are not multiples of the one being separated. The disadvantage of such filters is a worse suppression of the lower frequencies and the aperiodic component.

Cosine DF are distinguished by the almost complete suppression of the aperiodic component and the better suppression of the lower frequencies that are not multiples of the one being separated. The disadvantage of these filters is a higher sensitivity to higher frequencies that are not multiples of the one being separated.

The values of the two OC can be obtained by the use of two opposite DF or, if one OC is known, the other can be determined in accordance with the latter two readouts of the first one

$$u_y(n) = \frac{u_x(n)\cos(2\pi/N) - u_x(n-1)}{\sin(2\pi/N)}, \quad (4)$$

where $u_x(n)$ is the $n^{th}$ value of the known OC; $u_y(n)$ is the $n^{th}$ value of the OC being calculated.

If the values of the OC are known the signal amplitude is determined as

$$U(n) = \sqrt{u_x(n)^2 + u_y(n)^2}. \quad (5)$$

In existing microprocessor protection of electric power facilities the Fourier OCF and cosine OCF are widely spread.

The Fourier OCF use the cosine DF to separate cosine OC, and as for sine OC separation the sine DF is used. The cosine OCF use the DF with cosine sets of coefficients to separate the first OC, while the second OC is calculated in accordance with the two last readouts of the first one.

In order to overcome disadvantages of the cosine and sine OC it is proposed to use them together in combination with each other. One of the possible implementations of the combined OCF is shown in Fig.

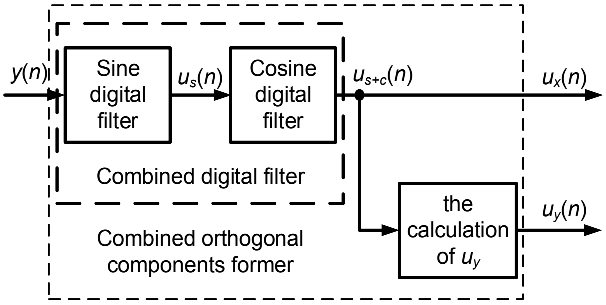


Fig. 1. Structural scheme of the combined OCF

In Fig. 1 the following indications are used: $u_s(n)$ is the signal after filtration with the sine filter; $u_{s+c}(n)$ is the same after filtering with the sine and the cosine filter.

In such OCF, the separation of the signal of the desired frequency takes place in two stages. In the first stage the input signal is filtered with the sine filter. This enables a sufficiently high degree of the higher frequencies suppression. Then, the resulting signal $u_s(n)$ is filtered by a cosine filter, resulting in the elimination of aperiodic component and additional weakening of all lower and higher frequencies.

Thereafter sine and cosine orthogonal components are separated from the obtained signal $u_{s+c}(n)$. The first OS is accepted to be equal to $u_{s+c}(n)$, and the second one is calculated according to the last two samples of the same signal.

For the complete suppression of frequencies that are multiples of the fundamental one, one of the filters must be of the full set of coefficients, the number of which is confined to the period of the fundamental frequency. Also, for complete filtering of the constant component, the sum of the coefficients of one of the DF must be zero.

## Description of the model of the study of the filters properties

The study was conducted by the method of computational experiment using the mathematical simulation environment MatLab-Simulink [5].

To obtain the frequency responses the "fvtool (A, B)" function was used [6], where an array of filter coefficients was passed as the argument "A", and the unity was passed as the argument "B".

In order to assess the transient response a model has been developed that implements a transient process in case of SC in accordance with the expression

$$i(t) = \sqrt{2} \cdot I \sin(\omega t - \varphi_F) + i_{A|0|} e^{-t/T_A}, \quad (6)$$

where $i(t)$ is the value of instantaneous current of the SC; $I$ is the effective value of current of the SC; ω is the network frequency; φ is the angle of the SC circuit; $i_{A|0|}$ is the initial value of the aperiodic component; $T_A$ is the time constant of the SC circuit.

The model of the second order filter of the lower frequencies has been chosen from the standard library of MatLab-Simulink elements as an analog filter model.

Table 1. Sets of coefficients for separation sine signal with frequency of 50 and 100 Hz

| № | cos(50 Hz) | sin(50 Hz) | cos(100 Hz) | sin(100 Hz) |
|---|---|---|---|---|
| 0 | 0,0833 | 0,0000 | 0,0833 | 0,0000 |
| 1 | 0,0805 | 0,0216 | 0,0722 | 0,0417 |
| 2 | 0,0722 | 0,0417 | 0,0417 | 0,0722 |
| 3 | 0,0589 | 0,0589 | 0,0000 | 0,0833 |
| 4 | 0,0417 | 0,0722 | –0,0417 | 0,0722 |
| 5 | 0,0216 | 0,0805 | –0,0722 | 0,0417 |
| 6 | 0,0000 | 0,0833 | –0,0833 | 0,0000 |
| 7 | –0,0216 | 0,0805 | –0,0722 | –0,0417 |
| 8 | –0,0417 | 0,0722 | –0,0417 | –0,0722 |
| 9 | –0,0589 | 0,0589 | 0,0000 | –0,0833 |
| 10 | –0,0722 | 0,0417 | 0,0417 | –0,0722 |
| 11 | –0,0805 | 0,0216 | 0,0722 | –0,0417 |
| 12 | –0,0833 | 0,0000 | 0,0833 | 0,0000 |
| 13 | –0,0805 | –0,0216 | 0,0722 | 0,0417 |
| 14 | –0,0722 | –0,0417 | 0,0417 | 0,0722 |
| 15 | –0,0589 | –0,0589 | 0,0000 | 0,0833 |
| 16 | –0,0417 | –0,0722 | –0,0417 | 0,0722 |
| 17 | –0,0216 | –0,0805 | –0,0722 | 0,0417 |
| 18 | 0,0000 | –0,0833 | –0,0833 | 0,0000 |
| 19 | 0,0216 | –0,0805 | –0,0722 | –0,0417 |
| 20 | 0,0417 | –0,0722 | –0,0417 | –0,0722 |
| 21 | 0,0589 | –0,0589 | 0,0000 | –0,0833 |
| 22 | 0,0722 | –0,0417 | 0,0417 | –0,0722 |
| 23 | 0,0805 | –0,0216 | 0,0722 | –0,0417 |

Table 2. The sets of cosine coefficients for combined filters

| № | half cos(50 Hz) | cos(100 Hz) |
|---|---|---|
| 0 | 0,1667 | 0,1667 |
| 1 | 0,1610 | 0,1443 |
| 2 | 0,1443 | 0,0833 |
| 3 | 0,1179 | 0,0000 |
| 4 | 0,0833 | –0,0833 |
| 5 | 0,0431 | –0,1443 |
| 6 | 0,0000 | –0,1667 |
| 7 | –0,0431 | –0,1443 |
| 8 | –0,0833 | –0,0833 |
| 9 | –0,1179 | 0,0000 |
| 10 | –0,1443 | 0,0833 |
| 11 | –0,1610 | 0,1443 |

For digital filters the sampling rate of the input signal was chosen equal to 1200 Hz. Their models were implemented on the basis of the above-described formulas. The values of the sine and cosine coefficients for the separation of frequencies 50 and 100 Hz are presented in Table. 1.

For combined filters (Fig. 1) a DF was used as a sine filter, the number of coefficients of which is confined to the period of the fundamental frequency. The values of these coefficients are presented in Table. 1. As cosine filters to separate the first harmonic the DT with a half set of cosine coefficients was used, and to separate second harmonic the DT with a full set was used. The values of these coefficients are presented in Table. 2.

The results of the study of filters are presented by their frequency and dynamic characteristics.

**Research results**

In Fig. 2 the frequency responses of a sine, cosine, and combined filters that are configured to the separation of the signal of 50 Hz frequency are shown.

In accordance with the figure cosine filter has the weakest characteristics of suppression of the higher frequencies. The disadvantage of this filter is that at small upward deviations the fundamental frequency to the larger the signal being separated will be amplified.

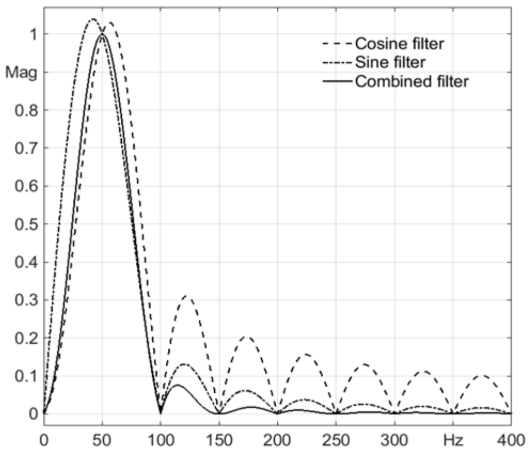


Fig. 2. The frequency responses of the filters configured for 50 Hz

The advantage of a sine filter compared to a cosine one is its better suppression of higher harmonics. In accordance with the frequency response of this filter, one of its drawbacks is the increase of the signal amplitude with a small decrease in frequency relative to the main frequency.

The combined filter has the best frequency response. From Fig. 2 it is visible that this DF the most effectively suppresses the higher harmonics, and starting with the fourth harmonic, ensures almost complete filtration.

In Fig. 3 the frequency response of a sine, cosine, and combined DF, configured to separate frequency signal of 100 Hz, are shown.

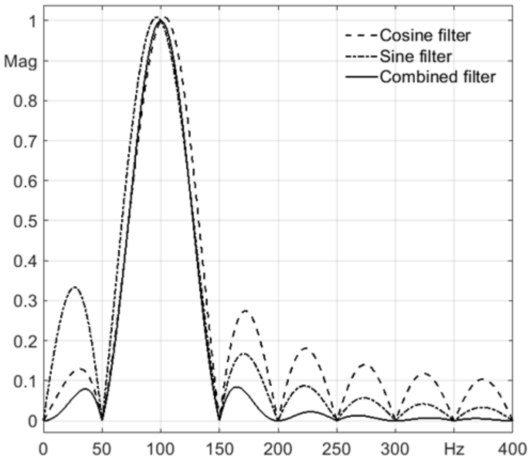


Fig. 3. The frequency response of filters configured to 100 Hz

They are characterized by the same advantages and disadvantages that are characteristic for the filters with frequency response and shown in Fig. 2. It should be noted herewith that the combined filter has the best filtering properties relative to the lowest harmonic.

In the form of the amplitude value, Fig. 4 shows the reaction of the Fourier OCF, the cosine OCF and the combined OCF to the appearance of a sinusoidal signal.

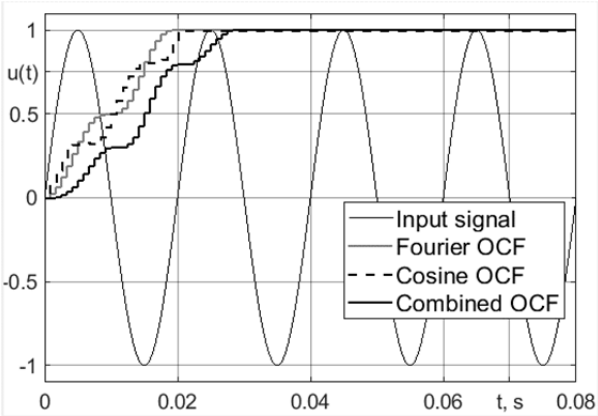


Fig. 4. The reaction of the OCF to a sinusoidal signal of the fundamental frequency

In accordance with Fig. 4 the combined OCF has the worst dynamic properties. This is due to the fact that the number of coefficients of its digital filter is more than the one of the other OCF.

In Fig. 5 in the form of the amplitude value, the reaction of the Fourier OCF, the cosine OCF, and the combined OCF to the occurrence of a sinusoidal signal with an aperiodic component being contained in it in accordance with the formula (6) is demonstrated.

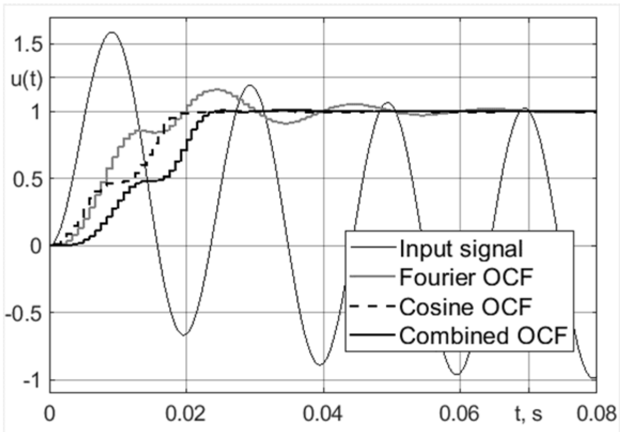


Fig. 5. The reaction of the OCF to a sinusoidal signal with an aperiodic component being contained in it

As it can be seen from Fig. 5, the cosine and combined OCF are not sensitive to the presence of aperiodic co-component and do not change their dynamic properties. The Fourier OCF under the same conditions demonstrates an increase of time of transition. This time depends on the period of attenuation of the aperiodic component and can increase several times as compared with the previous situation. Since most cases of short-circuit are accompanied by the presence of aperiodic component of current, it is a significant disadvantage of these OCF.

**The separation of the second harmonic in the inrushes of the magnetizing currents**

Modeling of the IMCs of a power transformer has been produced in accordance with [7]. Fig. 6 represents the oscillogram of the IMC of one phase.

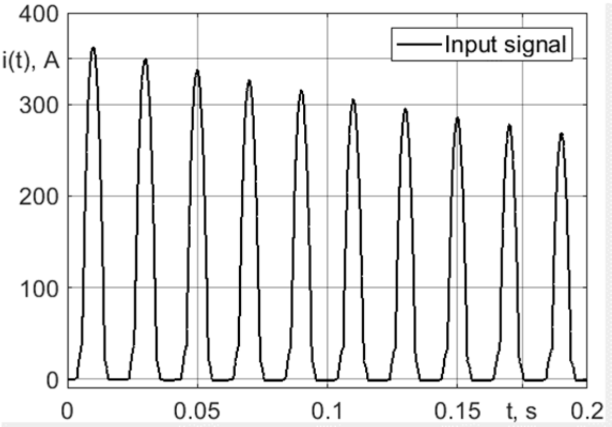

Fig. 6. The inrush of magnetizing current of one phase of a power transformer

Fig. 7 shows the reaction in the form of the amplitude value of the second harmonic of the Fourier OCF, the cosine OCF and the combined OCF to the inrush of the magnetizing current.

Unlike the cosine and combined OCF, the Fourier OCF demonstrates a burst of amplitude values of the second harmonic at the initial moment of transition. This can have a significant affect on the functioning of the microprocessor protection. Besides, the Fourier OCF features high oscillativity of the output signal; that is the fact that must be considered as undesirable.

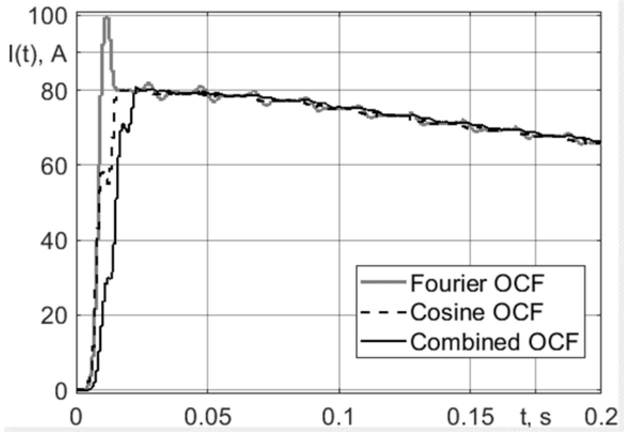

Fig. 7. The separation of the amplitude of the second harmonic in the inrush of the magnetizing current

The combined OCF provides the determining of the amplitude of the second harmonic later than other formers. However, its advantage is the lowest oscillativity of the output signal.

### Conclusion

In microprocessor protection of power installations with transformers, provided with a lock of magnetizing current inrushes that is based on the use of signals of the first and second harmonics, it is advisable to use combined formers of orthogonal components, which possess the optimal combination of frequency and dynamic properties.

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