Abdelghani BOUKRERIS, Abdelkader MEKRI, Ben-yssaad KRALOUA, Ali HENNAD

Université des Sciences et de la Technologie d'Oran-Mohamed Boudiaf, USTO-MB, FGE-LMSE, BP 1505 El M'Naouer, 31000 Oran, Algeria

doi:10.15199/48.2018.06.05

Two-dimensional modeling positive and negative streamer discharge at high pressure

Abstract. *The propagation of negative and positive streamer in electric discharge can be described by solving of a two-fluid model for charged particles. It based of continuity equations for the positive ions and electrons (also called drift-diffusion equations) including the effects of ionization, electron diffusion, and photoionization coupled to the Poisson's equation. The validity of this model is demonstrated and presented by performing of ADBQUICKEST method in two-dimension form. This new method is employed for the solution of transport equations of charged particles by using the time splitting method. The results so obtained by numerical simulations for streamer discharge are analyzed and compared with previously published experimental data.*

Streszczenie. *Propagacja ujemnego i dodatniego przepływu w wyładowaniu elektrycznym może być opisana za pomocą modelu dwupłynowego. Opiera się ona na ciągłych równaniach dodatnich jonów i elektronów (zwanych także równaniami dyfuzji dryfu), w tym efektów jonizacji, dyfuzji elektronów i fotojonizacji sprzężonej z równaniem Poissona. Ważność tego modelu przedstawiono w metodzie ADBQUICKEST w postaci dwuwymiarowej. Metodę tę stosuje się do rozwiązania transportu cząstek za pomocą metody podziału czasu. Wyniki uzyskuje się za pomocą symulacji numerycznych dla wyładowania streamera.* ***Dwuwymiarowy model ujemnego i dodatniego przepływu w wyładowaniu elektrycznym***

Keywords: streamer discharge, continuity equation, ADBQUICKEST method, time splitting method..
Slowa kluczowe: wyładowanie Słowa kluczowe: rozładowanie strumieniowe, równanie ciągłości, metoda ADBQUICKEST,

1. Introduction

The simulation of a streamer in electric discharge is not a new problem. Indeed, for many years authors have proposed first the simulations in one dimensional considering only the phenomena on the axis. These one-dimensional models remain limited and do not fully account for the physics of discharge. This is why many authors have focused on two-dimensional modeling. The basics of the streamer theory were developed by Raether [1], Loeb and Meek [2]. In their model, we explain the movement of the discharge by that of an ionization front that propagates within space between two electrodes. Once the discharge is initialized, we notice that its propagation is assured without the help of any outside agent.

Since the propagation of a streamer depends only on its own space charge field, it can propagate towards the cathode or towards the anode. This possibility makes it possible to define two types of streamers: negative streamer (also called Anode-Directed Streamers) and positive streamer (also called Cathode-Directed Streamers) [3].

We describe the results of numerical calculations of negative and positive streamer propagation based on a fully two-dimensional algorithm, which apply of flux-corrected scheme names ADBQUICKEST to correct and follow the strong density gradients [4]. The development of this algorithm has allowed us to investigate problems in streamer propagation of considerable interest [5][19]. This work presents the results of the algorithm application to questions including the streamer propagation on ionization ahead of the streamer, on applied photoionization and ionization term, on applied field, on initial and boundary conditions for both case of streamer.

2. Model formulation

2.1 Studied configuration

The computational domain is a cylinder of radius R = 0.5 cm (Figure 1) [6]. This domain is limited by two metallic electrodes parallel, planes and circular separated by a distance d equal to 0.5 cm. The applied voltage was 26 kV giving a geometric field equal 52 kVcm^{-1}. The streamer is supposed to be cylindrically and symmetrical with the axis z (direction of the propagation) and the axis r (radial propagation).

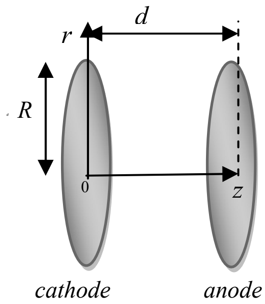


Fig.1: Schematic representation of the computational domain

The gas nitrogen is at atmospheric pressure (p=760 Torr). To initiate a streamer, a small plasma spot of Gaussian profile was placed at the cathode or anode tip for negative or positive streamer respectively. The densities of electrons and ions positives in the radial and axial directions take the same equation form:

$$n(z,r,t=0)=n_0\exp(-(\frac{z}{\sigma_z})^2-(\frac{r}{\sigma_r})^2)+10^8 \qquad (1)$$

n_0 =10^{14} cm^{-3}, σ_z =0.027 cm, σ_r =0.027 cm.

The choice of a high value of the Gaussian height n_0 equal to 10^{14} cm^{-3} in the expression provides immediate formation of the streamer in the gap space (with this value, we by pass the avalanche level).

2.2. Basic Equations of Discharge Model

The most effective and useful model to study the dynamics of streamers is based on the following continuity equations for charges particles (drift-diffusion equations) coupled with Poisson's equation for the electric potential which considerate the fundamental equations for charged species and that solved are the following:

$$\frac{\partial n_e}{\partial t}+\nabla\varphi_e=\nu n_e+S \qquad (2)$$

$$\frac{\partial n_p}{\partial t}+\nabla\varphi_p=\nu n_e+S \qquad (3)$$

With:

(4) $\varphi_e = -n_e w_e - D_e \nabla n_e$

(5) $\varphi_p = n_p w_p - D_p \nabla n_p$

Poisson's equation is defined by:

(6) $$\nabla^2 V = -\frac{e}{\varepsilon_0}(n_p - n_e)$$

where n_e, w_e, D_e, φ_e, n_p, w_p, D_p, φ_p are the particle density, velocity, diffusivity and flow for the electrons and positive ions, respectively, ν is the ionization frequency, S is the rate of electron-ion pair production due to photoionization, V is the electric potential, e is the (unsigned) electronic charge, and ε_0 is the permittivity of free space. The continuity equations (2) and (3) correspond to a fluid description of streamer discharge based on a first moment of the Boltzmann equation.

We use the flux-corrected transport algorithm in two-dimensional form which allow us to solve numerically our model under strongly space charge dominated conditions such as occur at the propagating of streamer head, to follow the radial development of the streamer, and to include effects of electrons distribution resulting by photoionization or photoemission from the cathode.

All reaction rates and transport parameters in this test studies are taken from Dhali and Williams [7].

2.3. Numerical model

The studied discharge imposes that the method chosen to solve the transport equations is efficient and has the capacity to follow the strong gradients during a reasonable calculation time. In our model, the solution is obtained by using numerical technique in the finite difference flux-corrected transport called ADBQUICKEST method. This method was presented in 2009 by Ferreira and Kurokawa [8] as a new version of the TVD (Total Variation Diminishing) Quickest scheme. They have used the finite difference method in the third order to discretize the continuity equation. We notice that these two authors are the first to have applied this new technique for a two-dimensional modeling of electric discharge.

The main objective of the ADBQUICKEST technique is to control the values of the density calculated by the QUICKEST scheme, in order to make the scheme strictly positive and in such a way that no maximum or minimum densities appears on the time interval[19].

Coupling the ADBQUICKEST method with QUICKEST scheme have proven a higher stability, effective and given the best results for modeling of drift-diffusion equations in two-dimensional models [4] [10].

The system of equations (2) and (3) was converted to finite difference form with the method of control volume in cylindrical coordinates as following:

(7) $$\frac{n_{i,j}^{k+1} - n_{i,j}^{k}}{\Delta t} + \frac{\varphi_{i+1/2,j}^{k} - \varphi_{i-1/2,j}^{k}}{\Delta z} + \frac{(r_{j+1/2}\varphi_{i,j+1/2}^{k} - r_{j-1/2}\varphi_{i,j-1/2}^{k})}{r_j \Delta r} = \nu_{i,j}\; n_{e_{i,j}}^{k} + S_{i,j}^{k}$$

The electric potential must satisfy the Poisson's equation:

(8) $$V_{i,j-1}^{k}\left(\frac{1}{\Delta r^2} - \frac{1}{2 r_j \Delta r}\right) - 2V_{i,j}^{k}\left(\frac{1}{\Delta r^2} - \frac{1}{\Delta z^2}\right) + V_{i,j+1}^{k}\left(\frac{1}{\Delta r^2} + \frac{1}{2 r_j \Delta r}\right) = \rho_{i,j}^{k} - \left(\frac{V_{i-1,j}^{k} + V_{i+1,j}^{k}}{\Delta z^2}\right)$$

With

(9) $$\rho_{i,j}^{k} = -\frac{e}{\varepsilon_0}(n_{p_{i,j}}^{k} - n_{e_{i,j}}^{k})$$

The poisson's equation (8) was solved by the Symmetrical Successive Over Relaxation method (SSOR).

We know that the two-dimensional numerical solution of the continuity equation (7) is a very expensive test in computing time. In this paper, we used the Time Splitting method [11] [20]. The advantage of this method lies in the stability and fast convergence towards the desired solution.

This method consists in replacing the two dimensional equations by a succession of one-dimensional equations in each of the directions of space.

The transition from a two-dimensional equation to a system of one-dimensional equations means that the transport of the particles are done in a synchronous way linked in the space and time will be carried and solving separately.

The transport equation in the axial direction is:

(10) $$\frac{n_{i,j}^{k+1} - n_{i,j}^{k}}{\Delta t} + \frac{\phi_{i+1/2,j}^{k} - \phi_{i-1/2,j}^{k}}{\Delta z} = 0$$

The writing of the transport equation in the radial direction is:

(11) $$\frac{n_{i,j}^{k+1} - n_{i,j}^{k}}{\Delta t} + \frac{(r_{j+1/2}\varphi_{i,j+1/2}^{k} - r_{j-1/2}\varphi_{i,j-1/2}^{k})}{r_j \Delta r} = 0$$

That's mean the solution of the first equation is considered as an initial condition for solving the second equation. This procedure is valid for calculating the solution of the last system equation:

(12) $$\frac{n_{i,j}^{k+1} - n_{i,j}^{k}}{\Delta t} = \nu_{i,j} n_{i,j}^{k} + S_{i,j}^{k}$$

The equation (10) and (11) are called drift-diffusion and it was calculate by ADBQUICKEST method, and (12) calculated by EULER method.

The numerical resolution of the transport equations in the two-dimensional fluid model requires the introduction of boundary conditions and initial conditions. According to the literature, the numerical resolution of partial differential equations depends essentially on the nature of the conditions and the levels of integration. In this work, we used the Neumann conditions on the symmetrical axis for the computation electronic and ionic densities, and electric field.

3. Results and discussion

3.1 Test case with negative streamer

To validate the numerical code for the case of negative streamer, we will carry out a comparison the results obtained in our 2D code for the resolution of the two-dimensional model with those resulting from the study carried out in the nitrogen by Dhali and Williams [7, 12].

To facilitate the comparison of our results with those of Dhali and Williams, we presented the profiles resulting in 1D according to the propagation axis. The comparison of these results shown in Figures 2 and 3 makes it possible to observe the evolution of the propagation of the electron density and the axial electric field on the axis of propagation for the same scale and at times 1.0, 2.0 and 2.5 ns.

We can notice that the results of our 2D model are almost identical and close to those obtained by Dhali and Williams [7, 12] concerning the variations of the electronic densities and the electric field.

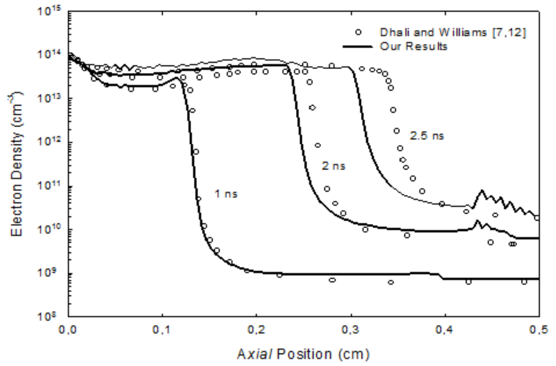


Fig.2: Electron density propagation at different times (1, 2 and 2.5 ns) for negative streamer.

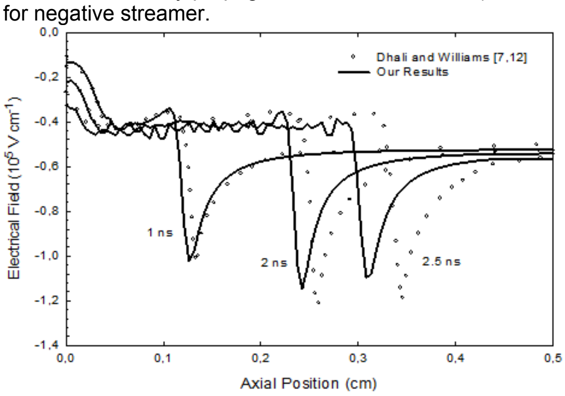


Fig.3: Axial electric field profile at different times (1, 2 and 2.5 ns) for negative streamer.

The figures 4,5 and 6 respectively represent the 2D profiles of the electron density and the axial and radial electric field at times 1, 2, 2.5 ns.

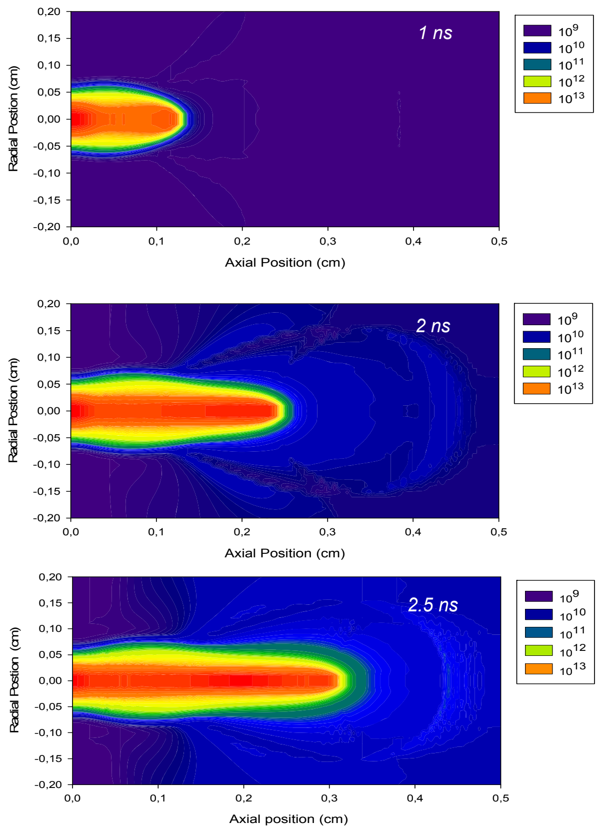


Fig.4: Electron density propagation (log10 (ne)cm^{-3}) at different times for negative streamer.

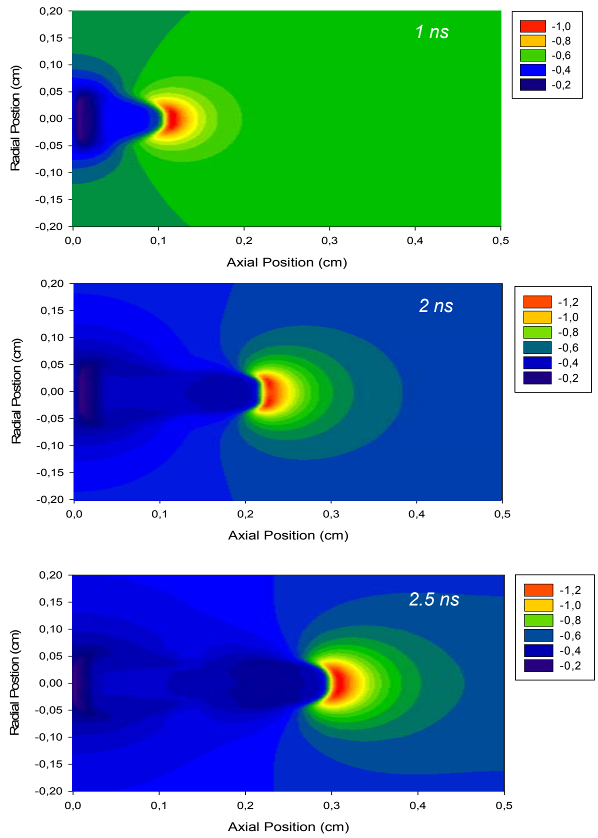


Fig.5: Axial electric field propagation (10^5 V cm^{-1}) at different times for negative streamer.

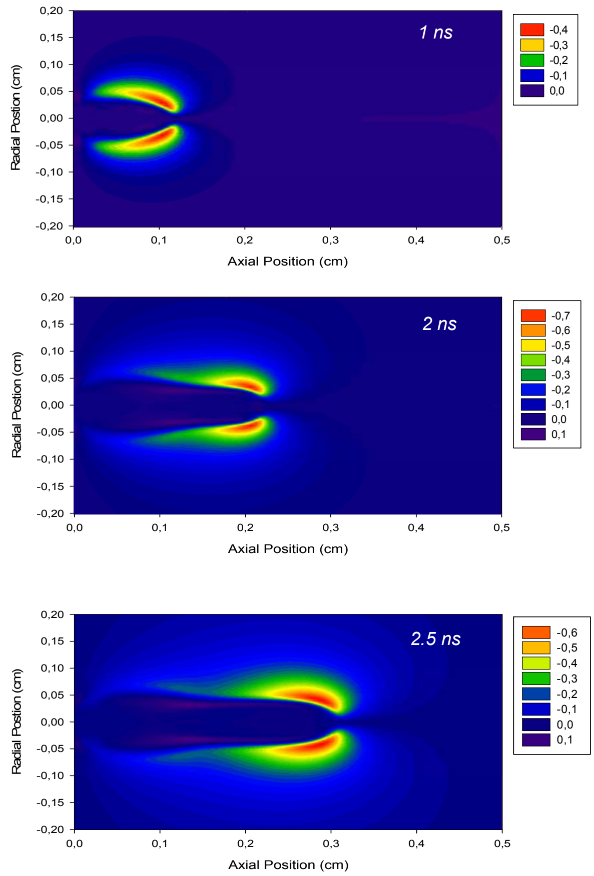


Fig.6: Radial electric field propagation (10^5 V cm^{-1}) at different times for negative streamer.

Figures 4 show the streamer discharge formation. We can see the head of streamer discharge, who plays a very important role in propagation and also favored ionization condition.

Due to the influence of the ionization and the displacement of the electrons towards the anode, the axial and radial components of the electric field shown in the figures 5 and 6 become very important at the forehead the discharge (streamer head), resulting in a permanent enlargement as a function of time of the electronic density profile. These components make the streamer confined in the inter-electrode gap and allow its propagation along the axial direction.

3.2 Test case with positive streamer

The solving of equation of the photoionization source term for simulation of positive streamer is usually calculated by classical integral methods, this description was proposed by Zheleznyak [13]. With these integral methods, the use of the photoionization term equation in two-dimensional (2D) simulation is very heavy in terms of numerical treatment and computation time, because it requires a quadrature cells on the total volume of the discharge to study a given point of this volume [14]. Then, the calculation of this equation at each time step in each cell is computationally expensive and needs a powerful CPU of computer and huge memory storage.

In order to avoid the calculation of integration over the whole field of study (total volume of the discharge) in positive streamer, two approaches models has recently been proposed to optimize the integral methods calculation of photoionization term in positive streamer. It is based on approximations of the first order photoionization term called Eddington approximation and the third order called SP3 method [15]. The second model approach has been presented by Luque and Ebert [16], which uses a set of Helmholtz equations obtained by the transformation of the integral expression of photoionization term, this approach was demonstrated and improved in Bourdon and al [17].

Finally, we will present in this work the simulation of a positive streamer discharge by the improved Eddington approximation model and compare it with the classic Zheleznyak integral method.

3.2.1 Eddington models for photoionization

In the paper of Segur and al [15], the approach of the Eddington approximation have been presented in detail by using direct numerical solutions for calculation of the photoionization term S. Based on [15,16 and17], we solved the following equations:

For j = 1, N_j , the photoionization term is defined by:

(13) $$S_{ph}(\vec{z}) = \sum_{j=1}^{N_j} S_{ph}^{j} = \sum_{j=1}^{N_j} A_j p_{o_2} c \xi \psi_j(\vec{z})$$

Where A_1=0.0067 cm^{-1}Torr $^{-1}$, A_2=0.0346 cm^{-1}Torr $^{-1}$, A_3= 0.3059 cm^{-1}Torr $^{-1}$ are constant coefficients.

p_{o_2} =150 Torr is an atmospheric pressure of oxygen in air, c is the speed of light.

Functions ψ are defined by:

(14) $$\nabla^2 \psi_j(\vec{z}) - 3(\lambda_j p_{o_2})^2 \psi_j(\vec{z}) = -3\lambda_j p_{o_2} \frac{n_u(\vec{z})}{c\tau_u}$$

Where λ_1=0.0447 cm^{-1}Torr $^{-1}$, λ_2=0.1121 cm^{-1}Torr $^{-1}$, λ_3=0.5994 cm^{-1}Torr $^{-1}$.

The second multiplier on the right hand side of equations:

(15) $$\frac{n_u(\vec{z})}{\tau_u} = \frac{p_q}{p+p_q}\left(\frac{\nu_u(\vec{z})}{\nu_i(\vec{z})}\right)\nu_i(\vec{z}).n_e(\vec{z})$$

p_q =30 Torr is quenching pressure,and $\xi \frac{\nu_u}{\nu_i} = 0.06$ (ξ is the photoionization efficiency).

It is interesting to note that equation (14) is an elliptic equation of the same structure as the Poisson's equation. Therefore, both Eddington approximation and poisson's equation can be solved with the same numerical method.

The boundary conditions play an important role in the solution of the Sph term using the Eddington approximation, and they are presented in Pomraning's work [18]:

(16) $$\nabla \psi_j(\vec{z}).\vec{n}_s = -\frac{3}{2}\lambda_j p_{o_2} \psi_j(\vec{z})$$

$\vec{n}_s$ is the unit outward boundary surface normal.

3.2.2 Gaussian photoionization source

To test the validity of the model described, the results are first presented for a simple test case of a Gaussian source for comparing the integral model proposed by Zheleznyak [13] with Eddington approximation model.

The photoionization source term S_i corresponds to a Gaussian distribution defined by:

(17) $$S_i(z,r) = \frac{n_u(z,r)}{\tau_u} = S_{i_0} \exp\left(-\left(\frac{z-z_0}{\sigma_z}\right)^2 - \left(\frac{r}{\sigma_r}\right)^2\right)$$

where S_{i_0} = 3.5x10^{22} cm^{-3} s^{-1}, z_0 =0,1 cm is the center position of the Gaussian along the axial coordinate, , the parameters to control the width of the source term are σ_z = σ_r =0.01 cm, a typical size of the streamer head. The calculation were carried out with a discretized grid of n_z = n_r = 251, where n_z and n_r are the number of cells along the axial and radial directions respectively.

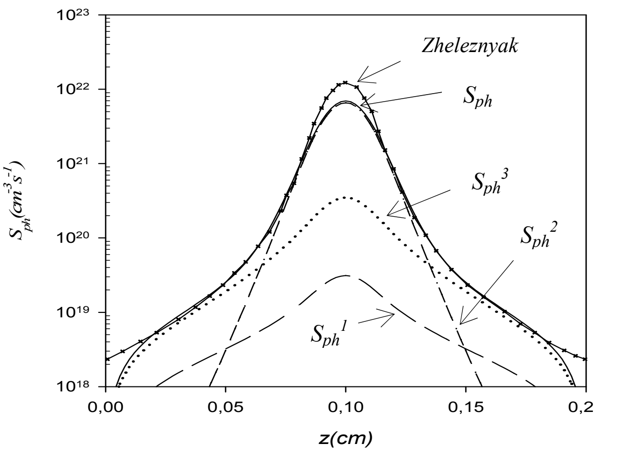


Fig.7: Profiles of the photoionizations terms S_{ph}, S_{ph}^1, S_{ph}^2,S_{ph}^3 without boundary conditions and the Zheleznyak solution.

Figure 7 shows the axial profiles of the photoionization term S_{ph} without boundary conditions, the three components S_{ph}^1, S_{ph}^2 and S_{ph}^3 are also shown. We note that the S_{ph} and Zheleznyak profiles give close results, but we note some differences appear especially near the boundaries.

Figure 8 shows axial profiles of the photoionization term S_{ph} including the correct boundary conditions. The results obtained are clearly very close to The Zheleznyak integral model to those provided in figure 7. In this case, the boundary conditions taken in equation (15) have given the exact correction to Eddington approximation model with the simplicity to implement in the distribution functions.

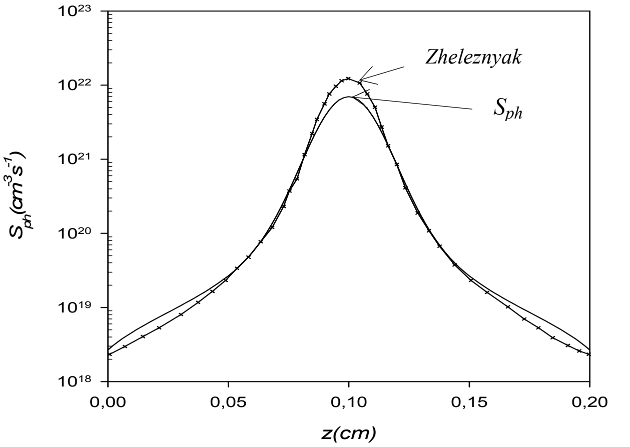


Fig. 8: Profiles of the photoionization term S_{ph} with boundary conditions and the Zheleznyak solution.

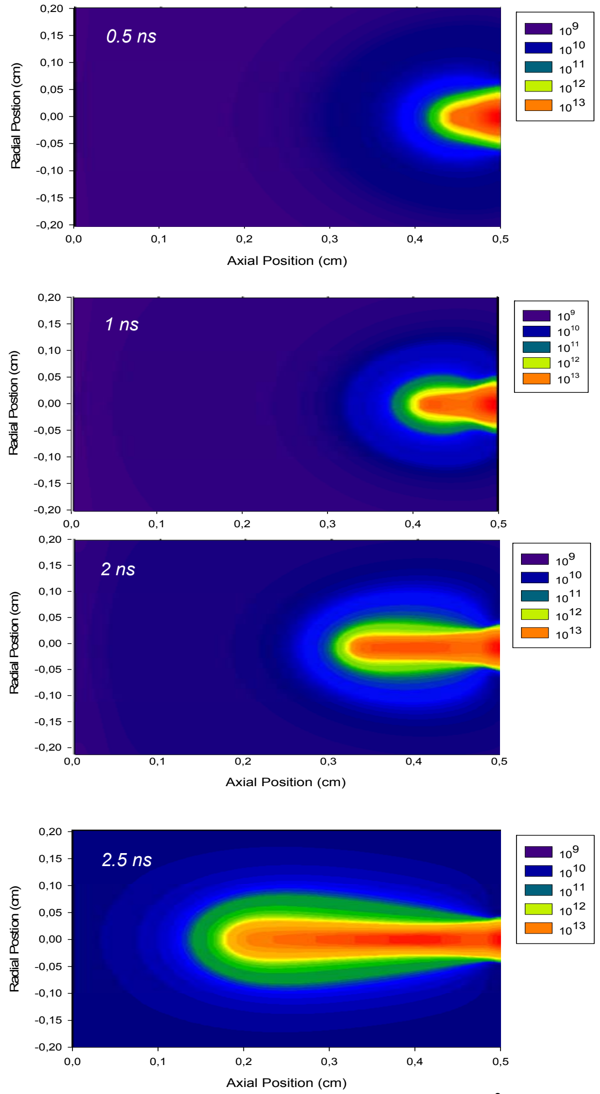


Fig. 9. Electron density propagation (log10 (ne) cm^{-3}) at different times for positive streamer.

3.3.3 Streamer simulations

For nitrogen-oxygen mixtures such as air, the figures 9,10 and 11 show the two-dimensional electron density propagation along symmetry axis and those of the axial and radial electric field at different times 0.5, 1, 2 and 2.5 ns respectively. The streamer expands as it propagates toward to cathode, we note that the photoionization source term reaches the maximum in the regions of head streamer.

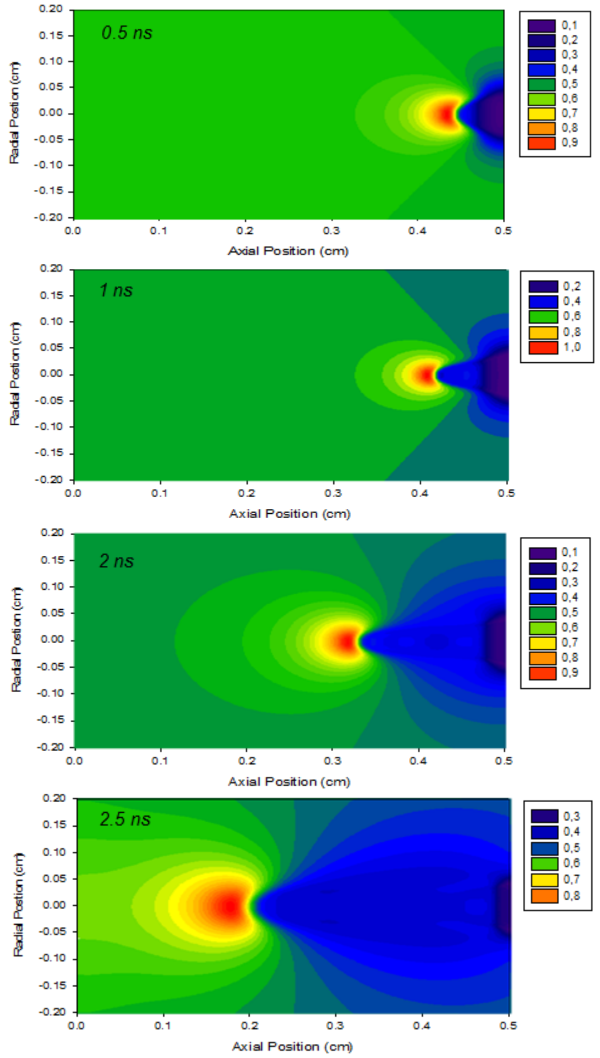
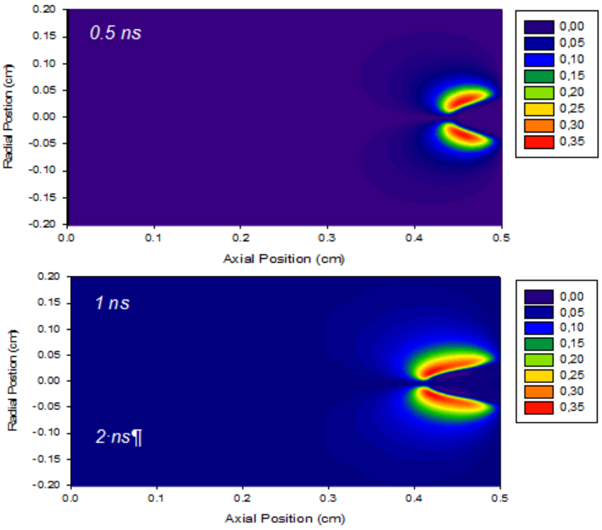


Fig.10: Axial electric field propagation (10^5 V cm^{-1}) at different times for positive streamer.



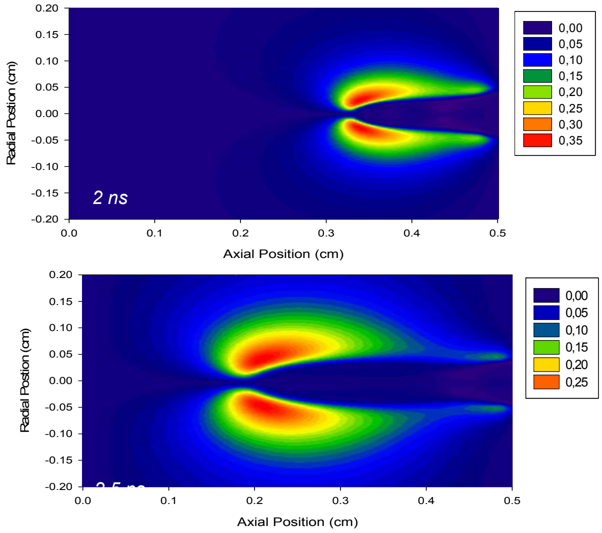


Fig.11 :Radial electric field propagation (10^5 V cm^{-1}) at different times for positive streamer.

4. Conclusion

The objective of this work is to develop a performance numerical method for solving the continuity equations and the Poisson's equation in 2D geometry, this method is ADBQUICKEST scheme. This allowed us to study the dynamics of the charged particles in the case of a negative streamer and positive for high pressure, with a view to a better understanding of the evolution and propagation of ionization waves in situations of large variations in density and electric field. The use of the ADBQUICKEST method ensured the propagation of a negative streamer under the same Dhali and Williams simulation conditions. In the case of positive streamer, we have demonstrated the performance of Eddington approximation models to calculate the photoionization term by using a simple test of gaussian source model and comparing it with the Zheleznyak integral model, we have verified the performance of a set of boundary conditions. This numerical experiment once again had the merit of verifying the performances of ADBQUICKEST scheme. Finally. The results obtained in this work will be a source of data for the validation of the electric and hydrodynamic models of the streamer discharge. Such models will be considered as complementary tools to the various experimental studies and will constitute a valuable aid in the various industrial applications of the electric discharge.

Authors

Abdelghani Boukreris was born in Oran, Algeria, on january , 19, 1985. His scientific interests include the theoretical study and modeling of physical processes in gas discharge plasma. E-mail: ghani_boukreris@usto-univ.dz

Ali Hennad was born in Oran, Algeria, on January, 27, 1964. He received the PH.D degree with a thesis on the kinetics of ions in molecular gases to determine ion basic data in air form Monte Carlo simulation, University Paul Sabatier of Toulouse, France in 1996. He is currently a Professor at University of Sciences and Technology of Oran, Algeria. His field of expertise is more particularly the modeling of the electric and hydrodynamic behaviors of the plasmas generated in low and high pressure. He has also a good expertise on the numerical analysis of the nonlinear and strongly coupled elliptic transport equations, and the swarm parameters determination of the charged particles in no equilibrium reactive plasma. He is the author of various international publications, a lot of them in collaboration with the LAPLACE Laboratory, Toulouse, France.

E-mail: ali.hennad@gmail.com

REFERENCES

[1] Raether H., Die Entwicklung der Elektronentawine in den Funkenkanal(in german), *Z .Physik,(*1939).112:464

[2] Loeb L B, Meek J M. The Mechanism of Spark Discharge in Air at Atmospheric Pressure, *J. Appl Phys,*11(1940),438

[3] Capeillere J et al, The finite volume method solution of the radiative transfer equation for photon transport in non-thermal gas discharges: application to the calculation of photoionization in streamer discharges, *J. Appl Phys,* 41(2008), Nr 23, 234018

[4] Benaired N, Hennad A. ADBQUICKEST Numerical Scheme for Solving Multi-Dimensional Drift-Diffusion Equations,*Przegląd Elektrotechniczny*, ISSN 0033-2097, R. 90 (2014), No.8

[5] Boukreris A, Hennad A, Résolution numérique des équations de continuité appliquées pour les décharges électriques (in french), *the 4th international conference of the thermal sciences AMT'2016 (Morocco)*, 19-20th April 2016.

[6] Boukreris A, Hennad A, Modélisation bidimensionnelle de la décharge streamer par la technique ADBQUICKEST (in french). *Congrés international sur les matériaux et l'énergie cimaten2016 -Sousse, Tunisia*, 17-19 Décembre 2016

[7] Dhali,S. K. and Williams, P. F, Two-dimensionat studies of streamers fn gases, *P.F(Paul Frazer) Williams Publications* (1987),paper 22.

[8] Ferreira V.G., Kurokawa F et al. Assessment of a high-order finite difference upwind scheme for the simulation of convection-diffusion problems. *International Journal of Numerical Methods in Fluids*, ISSN 0271-2091, 60(2009), pp. 1-26.

[9] Leonard. B. P, A stable and accurate convective modelling procedure based on quadratic upstream interpolation, *Computer Methods in Applied Mechanics and Engineering*, 19(1979), p. 59-98

[10] Boukreris A, Mekri A, Hennad A, Numerical methods for solving two-dimensional continuity equation applied for the electrical discharges, *International conference on material & energy (ICOM2017)*, Tianjin, China, July 06-09,2017.

[11] Flitti A., Modélisation numérique 1.5D et 2D de la propagation d'une décharge filamentaire haute pressio. *Phd Thesis. University Of Science And Technology Mohamed Boudiaf Oran-Algeria-(in French)*, (2007)

[12] Dhali, S. K. and Williams, P. F. Numerical Simulation of Streamer Propagation in Nitrogen at Atmospheric Pressure, *Phys. Rev.A*, (1985), 31:1219.

[13] Zheleznyak M. B, Mnatsakanyan A. Kh., Sizykh S. V. Photo-ionization of nitrogen and oxygen mixtures by radiation from a gas-discharge, *Teplofizika Vysokikh Temperatur (TVD)*, 20(1982), issue. 3,423-428

[14] Kulikovsky A A. The role of photoionization in positive streamer dynamics, *Appl. Phys*, 33(2000), No.12,1514

[15] Segur P, Bourdon A, Marode E, Bessieres D, Paillol J. The use of an improved Eddington approximation to facilitate the calculation of photoionization in streamer discharges, *Plasma Sources Sci. Technol,* 15(2006), Nr 4, 648

[16] Liu, N.Y., Celestin, S., Bourdon, A., Pasko, V.P., Segur, P., Marode, E.,Application of photoionization models based on radiative transfer and the Helmholtz equations to studies of streamer inweak electric fields, *Appl. Phys. Letters*., (2007), 91, pp.211501

[17] Bourdon A, Pasco V.P, Liu N Y, Celestin S, Segur P, Marode E. Efficient models for photoionization produced by non-thermal gas discharges in air based onradiative transfer and the Helmholtz equations, *Plasma Sources Sci. Technol*, 16 (2007), Nr 3, 656

[18] Pomraning G,The Equations of Radiation Hydrodynamics, *Pergamon Press, New York,* (1973).

[19] Boukreris A., Résolution Des Equations Dérive-Diffusion Appliquées Pour La Modélisation Des Décharges Electrique. *Master's Thesis. University Of Science And Technology Mohamed Boudiaf Oran-Algeria- (in French)*,(2014)

[20] B.Kraloua, A.Hennad, Multidimentional numerical simulation of glow discharge by using the N-BEE-Time splitting method, *Plasma Science andTechnology*,14(2012), No.09.