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Mathematical model of asynchronous pump drive with distributed mechanical parameters

***Abstract**. The paper presents a mathematical model of a complex electromechanical system. The electromechanical system features a deep-groove asynchronous motor, which drives a vertical pump. I this case the motor and pump are connected through a long shaft with distributed parameters. The motor is powered from a power transformer. The general model of the system was developed on the basis of interdisciplinary variational approaches. The final state equations are represented in the Cauchy normal form. This methodology gives the possibility of using both explicit and implicit numerical methods.*

Streszczenie**. W pracy przedstawiono model matematyczny złożonego układu elektromechanicznego, składającego się z głębokożłobkowego silnika asynchronicznego, który przez długi wał o parametrach rozłożonych napędza pompę pionową. Silnik zasilany jest z transformatora mocy. Ogólny model układu opracowano na podstawie interdyscyplinarnych podejść wariacyjnych. Końcowe równania stanu reprezentowane są w normalnej postaci Cauchy'ego, co daje możliwość wykorzystania zarówno jawnych, jak i ukrytych metod numerycznych. **(Model matematyczny asynchronicznego napędu pompowego o mechanicznych parametrach rozłożonych).

Keywords: interdisciplinary modeling, electromechanical system, asynchronous drive, distributed parameters, numerical methods, long shaft, mathematical modeling.

Słowa kluczowe: modelowanie interdyscyplinarne, elektromechaniczny system, napęd asynchroniczny, układy o parametrach rozłożonych, metody numeryczne, długi wał, modelowanie matematyczne.

Introduction

The use of electric drives as of today is practically unlimited. The electric drive analysis should include a description of the phenomena related to: drive motor, system movement transmission, drive mechanism and the electric drive control system. Each of the system components is very important in the context of the task set up for the drive system.

The paper proposes mathematical modeling of transient dynamic processes in a drive system, consisting of a deep-groove asynchronous motor, a complex movement transmission with mechanical distributed parameters and a vertical pump. Such systems are widespread in many industrial applications, for example in the water pumping systems for cooling turbine sets in power plants. In case of these drives, it is important that the system movement transmission consists of very long shafts between the motor and the vertical pump. Multiple energy conversion process complicates the analysis electro-mechanic-hydraulic. The process is complicated itself, and there are no mathematical models of vertical pumps integrated in the electromechanical part of the drive system [4], [6], [8], [9], [10], [13], [15]. In order to solve a similar problem at a high theoretical level, it should be assumed that the movement transmission of the elastic-dissipative drive will be analyzed as a continuum with mechanical distributed parameters. Therefore, the main emphasis in the work is put on the analysis of transient processes in the pumping system including long elastic elements.

To solve this problem, the interdisciplinary method of variation was used. The method is based on the modification of the Hamilton-Ostrogradski principle [1], [2], [3], [5], [10].

The aim of the work is mathematical modeling of transient processes in complex pumping systems with long elements of flexible movement transmission based on interdisciplinary approaches.

Mathematical model of system

When analyzing the structure of the system a conclusion is drawn that it is necessary to include three areas of science in the general model: electrical engineering, applied mechanics and hydrodynamics. The mentioned fact forces creation of a team of specialists to solve the task. Another approach is to create an interdisciplinary method, using one replacement circuit of the drive system, and analyze it using the extended Lagrange function.

The solution of the task is complicated by the fact that the parameters of the transmission system are distracted, and this leads to determining densities of linear functions of the Lagrangian components in addition to the energetic components of the extended Lagrangian. The modified Hamilton- Ostogradski rule is defined by the following equations:

$$S=\int_{t_1}^{t_2}\left(L^*+\int_l L_l dl\right)dt,\;\; I=\int_l L_l dl\,, \tag{1}$$

where: S – actions according to Hamilton-Ostogradski, L^* – modified Lagrange function, L_l – density of the modified Lagrange function, I – internal functional of energy.

The extended function and its linear density are determined by equations:

$$L^*=\tilde{T}^*-P^*+\Phi^*-D^*\,,\;\; L_l=T_l-P_l+\Phi_l-D_l\,, \tag{2}$$

where: $\tilde{T}^*$ – kinetic coenergy, P^* – potential energy, Φ^* – dissipative forces energy, D^* – external and internal non-potential forces of energy, T_l, P_l, Φ_l, D_l – linear densities of energy functions.

The variation of the functional (1) should be determined and compared to zero to develop a mathematical object model [1]:

$$\delta S=\delta\int_0^{t_1}\left(L^*+\int_l Ldl\right)dt=\int_0^{t_1}\delta L^* dt+\int_0^{t_1}\int_l \delta Ldldt \tag{3}$$

The Euler-Lagrange equations (Euler-Poisson) obtained on this basis are the basis for creating a mathematical model of the considered object - the pumping system.

In this case, the AM drive motor is powered from the TM power transformer. The transformer is powered from an EES (electromagnetic energy source) with infinite power. To unify the structure of the system, a long shaft of movement transmission from both sides was connected to the shafts of the motor and the vertical pump with additional elastic couplings SP1 and SP2, which significantly reduces impact torques on the motor and pump, as shown in Figure 1.

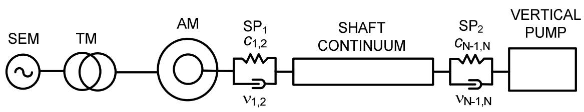

Fig.1. Basic scheme of the pumping system

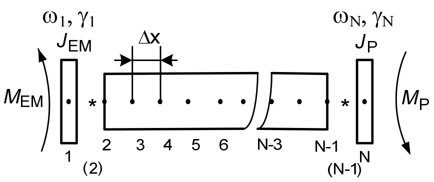

Fig.2. Calculation scheme of the shaft continuum

The first step to develop the model in the proposed method is to determine the generalized coordinates and the corresponding linear densities of the object (2).

The charges of the primary and secondary windings for the power transformer are:

$$q_{(1-6)} = Q_{1A},\, Q_{1B},\, Q_{1C}, Q_{2A},\, Q_{2B},\, Q_{2C}$$

The charges for the stator and rotor of an asynchronous motor are:

$$q_{(7-12)} = Q_{SA},\, Q_{SB},\, Q_{SC}, Q_{RA},\, Q_{RB},\, Q_{RC}$$

The rotation angle of the motor rotor for the movement transmission is:

$$q_{13} = \gamma_1$$

The rotation angle of the pump rotor is:

$$q_{14} = \gamma_{\mathrm{N}}$$

The fluid volume for a vertical pump with a pipeline is:

$$q_{15} = V$$

Linear density function of generalized coordinates, which is characterized by rotation angles of the shaft continuum, is determined by the following formula:

$$q(x,t) = \gamma(x,t)$$

Linear density function of generalized speeds is given by:

$$\frac{\partial q(x,t)}{\partial t} \equiv q_t(x,t) = \omega(x,t)$$

Generalized speeds will be as follows:

$\dot{q}_{(1-12)} = i_j$ –currents in transformer and motor windings,

$\dot{q}_{13}, \dot{q}_{14} = \omega_1, \omega_{\mathrm{N}}$ – motor and pump angular speed,

$\dot{q}_{15} = Q$ – pump flow.

All electrical parameters of transformer and motor have been reduced to quantity of the transformer secondary winding and the motor stator [3], [12].

On this basis, elements of non-conservative Lagrangian were designated [1]:

(4) $$\tilde{T}^* \equiv \left[\tilde{T}^*_{TM}\right] + \left[\tilde{T}^*_{AM}\right] + \left[\tilde{T}^*_{W}\right] + \left[\tilde{T}^*_{P}\right] =$$

$$= \sum_{j=1}^{3}\left[\int_0^{i_{1j}} \Psi_{1j} di_{1j} + \int_0^{i_{2j}} \Psi_{2j} di_{2j} + \int_0^{i_{Sj}} \Psi_{Sj} di_{Sj} + \int_0^{i_{Rj}} \Psi_{Rj} di_{Rj}\right] +$$

$$+\frac{J_1\omega_1^2}{2} + \frac{J_N\omega_N^2}{2} + \frac{L_\Sigma Q^2}{2}, \quad J_1 \equiv J_{EM},\ J_N \equiv J_P\ ;$$

(5) $$P^* = \frac{c_{1,2}(\gamma_2-\gamma_1)}{2} + \frac{c_{N-1,\mathrm{N}}(\gamma_N-\gamma_{N-1})}{2}, \qquad j = A,B,C\ ;$$

(6) $$\Phi^* = \frac{1}{2}\sum_{j=1}^{3}\left[\int_0^t r_{1j} i_{1j}^2 d\tau + \int_0^t r_{2j} i_{2j}^2 d\tau + \int_0^t r_{Sj} i_{Sj}^2 d\tau + \int_0^t r_{RLj} i_{Rj}^2 d\tau\right] +$$

$$+\frac{\nu_{1,2}(\omega_2-\omega_1)}{2} + \frac{\nu_{N-1,\mathrm{N}}(\omega_N-\omega_{N-1})}{2} + \frac{1}{2}\int_0^t R_\Sigma Q^2 d\tau\ ;$$

(7) $$D^* = \int_0^t \left(u_A i_{1A} + u_B i_{1B} + u_C i_{1C}\right) d\tau - \int_0^t \int_0^{\omega_N} M_P(\omega_N) d\omega_N d\tau\ ;$$

(8) $$T_l \equiv \frac{\partial T^*}{\partial l} = \frac{\rho J_p}{2}\left(\frac{\partial\gamma(x,t)}{\partial t}\right)^2; \quad P_l \equiv \frac{\partial P^*}{\partial l} = \frac{GJ_p}{2}\left(\frac{\partial\gamma(x,t)}{\partial x}\right)^2;$$

(9) $$\Phi_l \equiv \frac{\partial Q^*}{\partial l} = \xi\left(\frac{\partial^2\gamma(x,t)}{\partial x\partial t}\right)^2, \ D_l \equiv \frac{\partial D^*}{\partial l} = 0\ ,$$

where: TM – power transformer index, AM – asynchronous machine index, W – shaft continuum index, P – hydrosystem index;

and: $\boldsymbol{\Psi}$ – column vectors of the main transformer and motor associated fluxes; $\boldsymbol{r}$ – winding resistance matrix of the transformer, stator and end part of rotor; $\boldsymbol{i}$ – column vectors of currents, $\boldsymbol{u}$ – column vector of transformer power supply voltages; $c,\ v$ – coefficients of rigidity and dispersion of couplings, J_p – inertia moment of the pump, R_Σ, L_Σ – virtual resistance and inductance of the pump [7], G – module of transverse elasticity, ξ – internal scattering factor in the shaft; x – coordinate along the shaft; ρ – density of the shaft material.

Using the White-Woodson theory, the following assumptions were made [1], [5], [7]:

(10) $$M_{EM} = \frac{\partial\left[\tilde{T}^*_{AM}\right]}{\partial\gamma_1}, \ u_{Rj} = -\frac{\partial\left[\tilde{T}^*_{AM}\right]}{\partial Q_{Rj}}\ ;$$

(11) $$\rho_H g(H_G + S_0 \mathrm{Q}^2) = \frac{\partial\left[\tilde{T}^*_P\right]}{\partial V}\ ,$$

where: ρ_H – fluid density, g – gravitational acceleration, H_G – geometric fluid height, S_0 – pipeline hydraulic resistance.

By substituting expressions (4) - (9) in (3), functional variations - Euler equations are obtained. Taking into account the theory of transformation of Ψ-type models (in coordinates of associated streams) into the A-type model (in current coordinates) [11], and used expressions (10), (11) we obtain the equations of the object's state:

(12) $$\frac{d\mathbf{i}_1}{dt} = \mathbf{A}_{11}(\mathbf{u}_1 - \mathbf{r}_1\mathbf{i}_1) + \mathbf{A}_{12}(\mathbf{u}_S - \mathbf{r}_2\mathbf{i}_2)\ ;$$

(13) $$\frac{d\mathbf{i}_2}{dt} = \mathbf{A}_{21}(\mathbf{u}_1 - \mathbf{r}_1\mathbf{i}_1) + \mathbf{A}_{22}(\mathbf{u}_S - \mathbf{r}_2\mathbf{i}_2)\ ;$$

(14) $$\frac{d\mathbf{i}_S}{dt} = \mathbf{A}_S(\mathbf{u}_S - \mathbf{r}_S\mathbf{i}_S) + \mathbf{A}_{SR}(-\mathbf{u}_R - \mathbf{\Omega\Psi}_R - \mathbf{r}_{RL}\mathbf{i}_R);$$

(15) $$\frac{d\mathbf{i}_R}{dt} = \mathbf{A}_{RS}(\mathbf{u}_S - \mathbf{r}_S\mathbf{i}_S) +$$
$$+\mathbf{A}_R(-\mathbf{u}_R - \mathbf{\Omega\Psi}_R - \mathbf{r}_{RL}\mathbf{i}_R) + \mathbf{\Omega i}_R;$$

(16) $$\frac{d\omega_1}{dt} = \frac{1}{J_1}\left(c_{1,2}(\gamma_2 - \gamma_1) + \nu_{1,2}(\omega_2 - \omega_1) + M_{EM}\right);$$

(17) $$\frac{d\omega_N}{dt} = -\frac{1}{J_N}\left(M_P - c_{\mathrm{N-1,N}}(\gamma_N - \gamma_{N-1}) -\right.$$
$$\left.-\nu_{\mathrm{N-1,N}}(\omega_N - \omega_{N-1})\right);$$

(18) $$\frac{dQ}{dt} = \frac{1}{L_\Sigma}\left(\rho g(\mathrm{H}_G - S_0Q^2) - R_\Sigma Q\right);$$

(19) $$\frac{\partial^2\gamma(x,t)}{\partial t^2} = \frac{G}{\rho}\frac{\partial^2\gamma(x,t)}{\partial x^2} + \frac{\xi}{\rho J_p}\frac{\partial^2\gamma(x,t)}{\partial x^2},$$

where: $\mathbf{A}_{\mathrm{ij}}$ – matrices, elements which depend on the inverse transformer and motor inductance, Ω – rotor rotation speed matrix [1].

The boundary conditions were determined using d'Lambert's law in equation (19) [1], [14]

(20) $$GJ_p\left.\frac{\partial\gamma(x,t)}{\partial x}\right|_{x=0} + \xi\left.\frac{\partial\omega(x,t)}{\partial x}\right|_{x=0} +$$
$$+c_{1,2}(\gamma_1 - \gamma_2) + \nu_{1,2}(\omega_1 - \omega_2) = 0;$$

(21) $$GJ_p\left.\frac{\partial\gamma(x,t)}{\partial x}\right|_{x=l} + \xi\left.\frac{\partial\omega(x,t)}{\partial x}\right|_{x=l} -$$
$$-c_{\mathrm{N-1,N}}(\gamma_{N-1} - \gamma_N) - \nu_{\mathrm{N-1,N}}(\omega_{N-1} - \omega_N) = 0.$$

For equations (19) - (21) the finite difference discretization was made and it was obtained:

(22) $$\frac{d\omega_2}{dt} = \frac{2\left(c_{1,2}(\gamma_1 - \gamma_2) + \nu_{1,2}(\omega_1 - \omega_2)\right)}{\rho J_p\Delta x} -$$
$$-\frac{2\left(J_pG(\gamma_2 - \gamma_3) + \xi(\omega_2 - \omega_3)\right)}{\rho J_p(\Delta x)^2};$$

(23) $$\frac{d\omega_{N-1}}{dt} = \frac{2\left(J_pG(\gamma_{N-2} - \gamma_{N-1}) + \xi(\omega_{N-2} - \omega_{N-1})\right)}{\rho J_p(\Delta x)^2} -$$
$$-\frac{2\left(c_{N-1,N}(\gamma_{N-1} - \gamma_N) + \nu_{N-1,N}(\omega_{N-1} - \omega_N)\right)}{\rho J_p\Delta x};$$

(24) $$\frac{d\omega_k}{dt} = \frac{\xi}{\rho J_p(\Delta x)^2}\left(\omega_{k-1} - 2\omega_k + \omega_{k+1}\right) +$$
$$+\frac{G}{\rho(\Delta x)^2}\left(\gamma_{k-1} - 2\gamma_k + \gamma_{k+1}\right),\ k = 3,\ldots,N-2.$$

Equations of rotational speed functions of generalized discrete shaft units were added to equations (16), (17), (22) - (24) [1].

(25) $$\frac{d\gamma_i}{dt} = \omega_i,\quad i = 1,2,\ldots,N.$$

On the basis of the theorem with the voltage value on the bars of the rotor cage [1], the voltage was determined:

(26) $$u_{R,j} = -l\frac{k_uk_i}{2\gamma\Delta z}(-3H_{j,1} + 4H_{j,2} - H_{j,3}),\ j = A,B,$$

where: H – units of magnetic field intensity discretization in the groove, l – groove length, k_u, k_i –motor windings coefficients, γ – specific conductivity of the groove winding.

The asynchronous motor electromagnetic torque is determined from the formula:

(27) $$M_E = \sqrt{3}p_0(i_{SB}i_{RA}^{\Pi} - i_{SA}i_{RB}^{\Pi})/\tau,$$

where:, $^{\Pi}$ – indicates a transformed system of diagonal coordinates [1], [2].

The hydraulic moment is determined by the equation [7]:

(28) $$M_P(Q) = \frac{\rho gQ(H_G + S_0Q^2)}{\omega_N}$$

or based on the similarity theory using the least squares method (for geometric height H_G=4m):

(29) $$M_P(\omega_N) = 0{,}73\omega_N^2 + 23{,}17\omega_N$$

The system of differential equations: (12) - (17), (22) - (25) is integrated taking into account equations: (26) - (29).

Computer simulation results

The computer simulation was made for the electrohydraulic system shown in Figure 1. The TOc 4000/35 power transformer primary winding was supplied from the infinite power source with a rated voltage of 35kV. The deep groove motor 12-52-8A rated data are as follows: P_N = 320 kW; U_N = 6 kV; I_N = 39 A; ω_N = 740, s^{-1}, p = 4, J_R = 49 kg·m^2. The next parameters are: r_S = 1,27 Ω, R_{RL} = 0,21 Ω, α_S = 38,9 H^{-1}, αRL= 70 H^{-1}, h = 0,038 m, l = 0,23 m, a = 0,005 m. The motor magnetization curve is approximated by the equation: $\Psi_m = 12{,}4\ \mathrm{arctg}(0{,}066 i_m)$. The long shaft parameters are as follows: G = 8,1·10^{10} N·m, ρ = 7850 Kg/m^3, d = 0,05m, l = 4,5m, ξ =0,5 $N\cdot m^2\cdot$ s, Δx = 0,05m. The load torque of the pump was approximated according to equation (29). The vertical pump works with a gear ratio k_T = 750/585.

During the computer simulation, the following assumptions were made:
1) all rotor parameters of the motor have been recalculated to the stator site;
2) the power transformer primary winding has been recalculated to the secondary winding;
3) mechanical system parameters were recalculated to the pump gear.

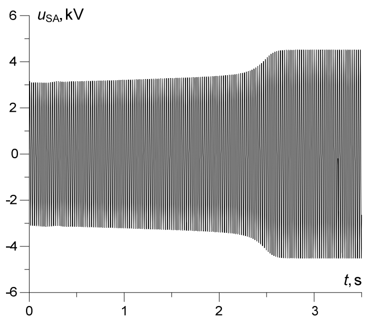

Fig.3. Instantaneous supply voltage of the asynchronous motor stator - phase A

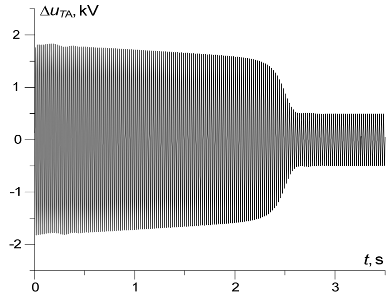

Fig.4. Instantaneous power transformer voltage drop - phase A

Figures 3, 4 show the instantaneous realation of the asynchronous motor supply voltage and the voltage drop on the power transformer. It can be seen that the presence of a transformer in the motor power supply system significantly reduces the motor power supply voltage. On the one hand, this is not good for the motor, but on the other hand it improves the working conditions of the pumping system from the point of view of the vertical pump.

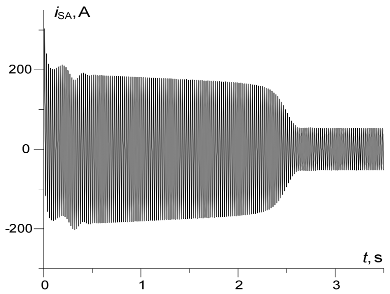

Fig.5. Asynchronous motor stator instantaneous current - phase A

Figures 6, 7 show the instantaneous motor rotational speed and the elasticity moment in the first flexible coupling SP_1. Figure 6 shows quite significant fluctuations which is related to the parameters of both couplings and the thickness of the shaft. Analyzing figure 7, very good suppression of shock moments can be noticed in a long shaft. Also there are no resonance processes in the drive system. It is influenced by correctly selected couplings parameters.

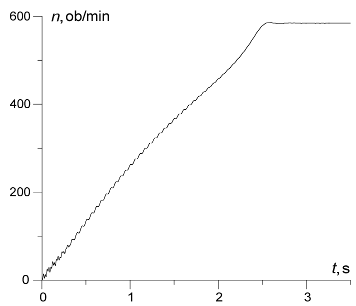

Fig.6. Instantaneous motor rotational speed

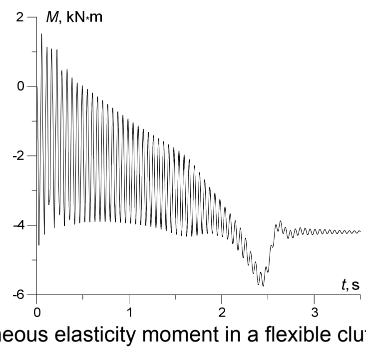

Fig.7. Instantaneous elasticity moment in a flexible clutch

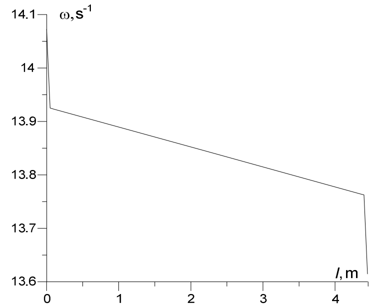

Fig.8. The angular speed spatial distribution of the integrated long shaft at the time $t = 0{,}5$ s.

Figure 8 shows the angular speed in the spatial coordinate at the time $t = 0{,}5$s. The sequence presented at figure 8 shows the long shaft elasticity. It is clearly visible that the maximum flexibility of the movement transmission system occurs in two flexible couplings.

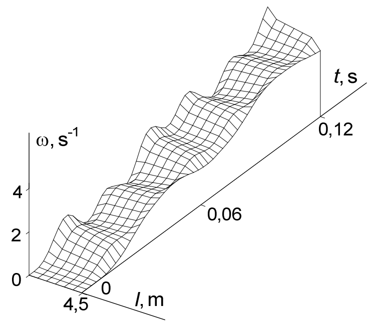

Fig.9. The angular speed spatial distribution of the integrated long shaft in the time interval [0; 0,12] s

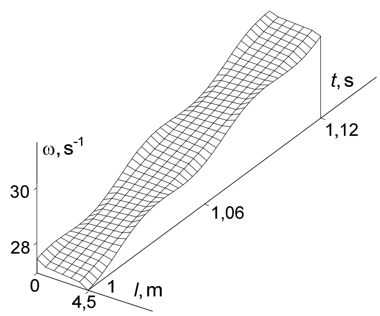

Fig.10. Time-spatial distribution of the angular speed of the integrated long shaft in the time interval [1; 1,12] s

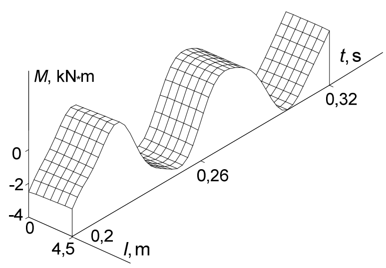

Fig.11. Time-spatial distribution of the elasticity moment of the integrated long shaft in the time interval [0,2; 0,32] s

Figures 9 - 11 present time-spatial distributions of functional movement transmission relations. Figures 9 and 10 show the angular speed of the integrated long shaft in the time interval [0; 0,12] s, [1; 1,12] s. Figure 9 shows the significant fluctuations in the rotor speed that do not occur in a vertical pump. This demonstrates the practical effect of resilient-flexible movement transmission in pumping systems. Figure 11 shows the elastic moment function in the long shaft in the time range [0.2; 0.32] s.

Conclusions

1. The use of interdisciplinary approaches to modeling of complex dynamic objects significantly simplifies the development of mathematical models. This is especially true in the case of analysis of systems that combine problems of several science fields.
2. Taking into account complicated movement transmission with both focused and distributed parameters gives the possibility of analyzing physical processes in the whole drive system at a higher level. This applies to deep-groove motors and wave processes in mechanical systems in the context of a shaft continuum.
3. Based on the computer simulation results it was found:
    - including the power transformer in the drive system leads to a reduction of the motor starting torque and significantly reduces impact moments and parasitic fluctuations of functional dependencies in the system;
    - Flexible couplings in the movement transmission system significantly improve the operation of the entire pumping system, practically without the pump parameters changing;
    - in the mathematical transient processes modeling in complex pumping systems with long elastic movement transmission elements, a complicated mathematics apparatus is used in the field of the differential equations theory with ordinary and partial derivatives. Such equations are solved only with the numerical methods.

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REFERENCES

[1] Czaban A., Zasada Hamiltona-Ostrogradskiego w układach elektromechanicznych, *Wydawnictwo T. Soroki*, Lwów 2015, 464
[2] Rusek A., Czaban A., Lis M., Klatow K., Model matematyczny układu elektromechanicznego z długim sprężystym wałem napędowym, *Przegląd Elektrotechniczny,* 91 (2015), nr 12, 69-72
[3] Szafraniec A., Modelowanie matematyczne procesów oscylacyjnych w napędzie elektrohydraulicznym o podatnej transmisji ruchu, *Przegląd Elektrotechniczny,* R. 93 NR 12/2017, 167-170
[4] Domek S., Zintegrowany system monitorowania warunków pracy układu napędowego obrabiarki sterowanej numerycznie, *Przegląd Elektrotechniczny*, 86 (2010), nr 6, 113-115
[5] Ortega R., Loria A., Nicklasson P.J., Sira-Ramirez H., Passivity-Beast Control of Euler-Lagrange Systems: Mechanical, Electrical and Electromechanical Applications. *Springer Verlag*, London 1998, 543
[6] Jędral W., Karaśkiewicz K., Szymczyk J., Badanie nieustalonych stanów pracy i charakterystyk zupełnych pomp wirowych, *Instal* 11 (2013), 21-24
[7] Mandrus W., Żuk W., Hydraulika, napędy hydrauliczne i pneumatyczne maszyn wojskowych, Lwów, *ACB*, (2013), 372
[8] Szafraniec A., Głuch K., Analiza efektywności energetycznej układów pompowych z przemiennikami częstotliwości i z regulacją upustową, *Technika Transportu Szynowego* (2016), nr 12, 507-512
[9] Szafraniec A., Sterowanie optymalne wielosilnikowych układów napędowych w systemach transportowych i przemysłowych, *Zeszyty Naukowe Syberyjskiej Akademii Samochodowej* nr T08D07/2003/8, Omsk, (2003), 16-19
[10] Łukasik Z., Czaban A., Szafraniec A., Żuk V., - The mathematical model of the drive system with asynchronous motor and vertical pump *Przegląd Elektrotechniczny,* R. 94 NR 1/2018, 133-138
[11] Czaban A., Lis M., Klatow K., Nowak M., Patro M., Model matematyczny napędu synchronicznego o podatnej transmisji ruchu w fizycznych współ, rzędnych prądów (A-model), *Przegląd Elektrotechniczny*, 92 (2016), nr 12, 29-32
[12] Czaban A., Lis M., Mathematical modeling of transient states in a drive system with a long elastic element, *Przegląd Elektrotechniczny*, 88 (2012), nr 12b, 167-170
[13] Sieklucki G., Orzechowski T., Sykulski R., Model matematyczny napędu z silnikiem indukcyjnym – metoda DTC-SVM, *Elektrotechnika i Elektronika*, 29, (2010) nr 1-2, 33-39
[14] Rubinowicz W., Królikowski W., Mechanika Teoretyczna, *Państwowe Wydawnictwo Naukowe*, Warszawa, nr 9 (2012), 462
[15] Zhang D., Shi W., Chen B., Guan X., Unsteady flow analysis and experimental investigation of axial-flow pump, *Journal of Hydrodynamics*, (2010), V. 22(1), 35−43