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Influence of signal sampling period in the control system on optimal settings of PI and PID controllers

***Abstract**. The work covers the issues connected with selection of optimal settings of PID controllers and the influence of sampling frequency of the control signal on their values. The methods used to select the sampling frequency of the control signal were reviewed. In this paper, we determined the ITAE optimum settings of continuous and discrete PID regulators. The influence of the frequency of control signal sampling on the optimum settings of the regulator was analyzed.*

***Streszczenie.** Praca porusza zagadnienia związane z doborem optymalnych nastaw regulatorów PID i wpływu częstotliwości próbkowania sygnału sterującego na ich wartości. Dokonano przeglądu stosowanych metod doboru częstotliwości próbkowania sygnału sterującego. W pracy określono optymalne względem kryterium ITAE nastawy ciągłych i dyskretnych regulatorów PID. Dokonano analizy wpływu częstotliwości próbkowania sygnału sterującego na wartości optymalnych nastaw regulatora. (**Wpływ okresu próbkowania sygnałów w układzie sterowania na optymalne nastawy regulatorów PI oraz PID**).*

Keywords: PID controllers, control systems, optimization, signal sampling.
Słowa kluczowe: regulatory PID, układy sterowania, optymalizacja, próbkowanie sygnałów.

Introduction

General principles of sampling period selection are determined on the basis of the identified parameters of the controlled system model. These are most often: T_{max} - dominating time constant, L - a fixed time transport delay, T - a fixed time constant [1]. The methods by which the sampling period is determined on the basis of time based control quality indicators such as: t_s – settling time and t_r – the rise time are also known [2][3]. There are also rules that allow to estimate the sampling period based on identified parameters of the controller, such as T_i – integral time and T_d – derivative time [2][4]. These rules do not specify exactly what the value of the Δt sampling period should be. They only allow for an indicative estimation of the maximum value of this parameter, exceeding which may cause a deterioration in the quality of control.

It is assumed that the operation parameters of the continuous control system shown in Fig. 1 are the reference point. In this system, the controller continuously monitors the value of the controlled signal $y(t)$ and setpoint $x(t)$. Based on them it generates the control signal $u(t)$.

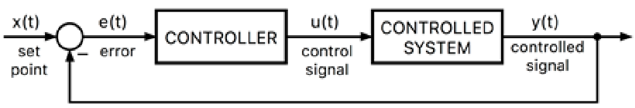

Fig. 1. Continuous control system

The discrete control system shown in Fig. 2 works differently. The controller reacts to changes in setpoint and controlled values only within specified time intervals, every Δt. This results in the loss of information about the potential change of the controlled signal value and setpoint between sampling times. This may lead to a deterioration in the quality of control.

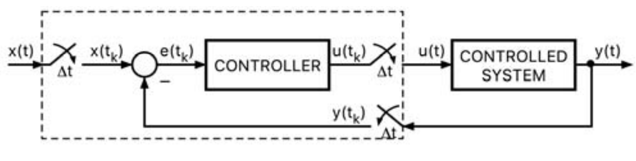

Fig. 2. Discrete control system

Comparing the optimal settings for a continuous controller with the optimal settings for a discrete controller for a set of sampling period values, makes it possible to determine its influence on the quality of system control.

Equations of the control system model

A number of simulation experiments have been carried out to determine the optimum controller settings: continuous and discrete. The following assumptions have been made. The controller is described with the equation of positional algorithm [5]

$$u_k = K_c\left[e_k + \frac{\Delta t S_k}{T_i} + \frac{T_d\left(e_k - e_{k-1}\right)}{\Delta t}\right] \tag{1}$$

where: e - error signal, $S_k = S_{k-1} + e_k$, k - current iteration, k-1 - previous iteration, K_c - proportional gain coefficient, T_i - integral time, T_d - derivative time.

The controlled system was approximated by an inertial model of the first order with transport delay

$$G_{(s)} = \frac{K\exp(-sL)}{1+sT} \tag{2}$$

where: K - static gain coefficient, T - time inertia constant, L - time constant of transport delay.

In order to simplify calculations, the number of variables in the equation of controller (1) and object model (2) was reduced. This was achieved by scaling time

$$t_\circ = t/T \qquad s_\circ = sT \tag{3}$$

and the reference of all time constants in equations (1) and (2), i. e. T_i, T_d, Δt, L to the inertia time constant T of the controlled system model. The gain coefficient of the controller K_c was related to the gain coefficient K. Therefore, the controller model (1) takes the form

$$u_k = \kappa\left[e_k + \frac{\Delta t_\circ S_k}{\tau} + \frac{\gamma\left(e_k - e_{k-1}\right)}{\Delta t_\circ}\right] \tag{4}$$

where

$$\kappa = K_c K \quad \tau = T_i/T \quad \gamma = T_d/T \tag{5}$$

and the controlled system is defined by equation

(6) $$G_{(s_\circ)} = \frac{\exp(-s_\circ\theta)}{1+s_\circ}$$

where

(7) $$\theta = L/T$$

Using Tustin's transformation, the inertial part of the model (6) has been transformed into a discrete form

(8) $$G_{(z_\circ)} = \frac{\delta t_\circ z + \delta t_\circ}{(\delta t_\circ + 2)z + \delta t_\circ - 2}$$

and then into a difference equation

(9) $$y_k = \frac{\delta t_\circ(u_k + u_{k-1}) + (2-\delta t_\circ)y_{k-1}}{\delta t_\circ + 2}$$

where: $\delta t_\circ$ - sampling period of the controlled system model.

The transport delay was realized with the help of FIFO queue and its implementation in the form of cyclic buffer. The length of the FIFO queue is determined by θ value.

The ITAE index was used as the optimization criterion

(10) $$ITAE = \int_0^\infty t\left|e_{(t_\circ)}\right|dt_\circ$$

Optimization procedure

Equations of controller (4), controlled system model (9) along with appropriate operations on cyclic buffer and equation of selected quality criterion (10) were implemented in a program written in C language and performed cyclically in the loop. It is assumed that each iteration of the loop corresponds to the increment of time by a time interval equal to the period $\delta t_\circ$.

It should be noted that when the sampling period $\Delta t_\circ$ (4) is equal to $\delta t_\circ$ (9), the equations (4) and (9) should be executed successively, in the same iteration. Then, such a system behaves like a continuous control system. When $\Delta t_\circ > \delta t_\circ$ and $\Delta t_\circ$ is a multiple of $\delta t_\circ$, then the controller equation (4) should be performed every *j*th iteration of the loop, where $j = \Delta t_\circ / \delta t_\circ$. Then such a system behaves like a discreet control system.

Due to the necessity of determining optimal parameters according to criterion (10) of controller (4), the modified complex method [6] was chosen as the optimization method. It is a direct method, i. e. it does not require calculation of derivatives, simple to implement and relatively quickly convergent.

The identification process was carried out in several stages. In the first step, the optimum controller settings for the continuous system were determined. A number of simulation experiments were carried out, assuming changes of θ value in the range from 0.2 to 2.0. Set of optimal values of controller parameter settings were obtained. These values were approximated using the method of least squares. In the next step, the optimal settings of the discrete controller were determined. For this purpose, for each θ value, controller parameters were optimized within the range of changes in the sampling interval $\Delta t_\circ$ from 0.01 to 0.2. As a result, sets of optimal values of controller parameters in the function of sampling period and θ coefficient were obtained. Then, function parameters (11) were approximated, determining the relation between optimal controller settings and the sampling period for each value assumed θ coefficient.

(11) $$f^{opt}_{(\Delta t_\circ,\theta)} = C_{(\theta)} + D_{(\theta)}\Delta t_\circ \quad \forall\theta \in [0.1, 2.0]$$

The least squares were used for approximation purposes. The identified set of $C_{(\theta)}$ values corresponds to the optimal controller settings of the continuous controller for the set of θ values, which can be determined by the identified functions that have been defined in the first step of the identification procedure. The identified set of $D_{(\theta)}$ values indicates how much the optimum value of the continuous controller parameter should be modified to maintain optimum control quality when using the discrete controller for the selected $\Delta t_\circ$ sampling period. In the last stage of identification, a set of $D_{(\theta)}$ values was approximated for the accepted θ coefficient variation range. The method of least squares has been reused for this purpose.

Optimum settings

As a result of the optimization procedure, the following equations have been formulated: (12), (13), (14), (15) and (16). They represent the optimal settings of the PI and PID controllers. The identified values of the coefficients for the obtained equations are given in tables 1 and 2.

(12) $$\kappa_{PI} = \exp\left(\frac{a}{\theta^2} + b\theta^2 + \frac{c}{\theta} + d\theta + e\right) + \left(\frac{f}{\theta^2} + \frac{g}{\theta} + h\right)\Delta t_\circ$$

(13) $$\tau_{PI} = \exp\left(\frac{a}{\theta^3} + b\theta^2 + c\theta + d\right) + \exp\left(\frac{e}{\theta^4} + \frac{f}{\theta} + g\right)\Delta t_\circ$$

(14)
$$\kappa_{PID} = \exp\left(a\theta^3 + \frac{b}{\theta^2} + c\theta^2 + \frac{d}{\theta} + e\theta + f\right) + \left(\frac{g}{\theta^2} + \frac{h}{\theta} + i\right)\Delta t_\circ$$

(15) $$\tau_{PID} = \left(a\theta^3 + b\theta^2 + c\theta + d\right) + \exp\left(\frac{e}{\theta} + f\right)\Delta t_\circ$$

(16) $$\gamma_{PID} = \left(a\theta^2 + b\theta + c\right) + \left(\frac{d}{\theta^2} + \frac{e}{\theta} + f\right)\Delta t_\circ$$

Table 1. Identified coefficients *a*, *b*, *c* and *d*

	a	b	c	d
κ_{PI}	-0.026	0.094	0.43	-0.48
τ_{PI}	0.00022	-0.026	0.31	-0.088
κ_{PID}	-0.097	-0.013	0.55	0.27
τ_{PID}	0.034	-0.088	0.42	1.0
γ_{PID}	-0.029	0.32	0.013	-0.015

Table 2. Identified coefficients *e*, *f*, *g*, *h* and *i*

	e	f	g	h	i
κ_{PI}	-0.44	-0.16	0.19	0.057	
τ_{PI}	-0.00051	0.023	-0.38		
κ_{PID}	-1.27	0.6	-0.19	-0.64	0.27
τ_{PID}	0.091	-1.0			
γ_{PID}	0.13	-0.31			

In order to obtain the actual settings for the selected real controller, the values determined from the equations: (12), (13), (14), (15) and (16) should be scaled using the (5) and (7) equations.

The influence of sampling period on optimal controller settings

Fig. 3 shows changes in the optimum values of the relative gain coefficient K of the PID controller as a function of changes in the $\Delta t_{\circ}$ sampling period for several selected θ (14) values.

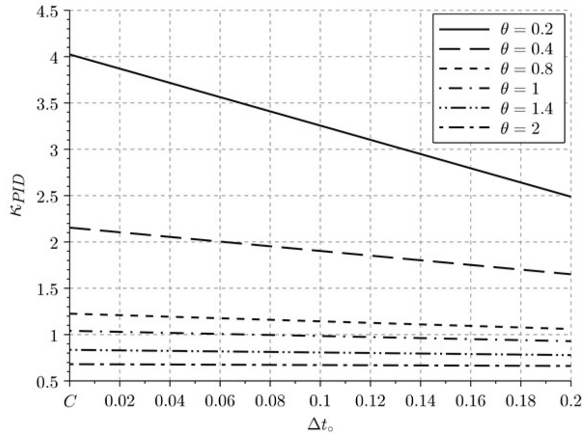

Fig. 3. Optimal settings $\kappa_{PID} = f\left(\Delta t_{\circ}, \theta\right)$ according to (14)

Values on the ordinates, for the C values on the cut-off axis, indicate the optimum settings for the continuous controller.

The greatest influence on the optimum value of the relative gain coefficient κ_{PID} can be seen for models with low θ value. They approximate controlled systems that are characterized by fast transient processes. Therefore, the loss of information about the value of signals in the system, resulting from their periodic sampling, requires correction of the examined controller parameter value. The value of the required correction increases with the increase in the sampling period.

Fig. 4 shows changes in the optimum values of the relative integral time of the PID controller in $\Delta t_{\circ}$ and θ function.

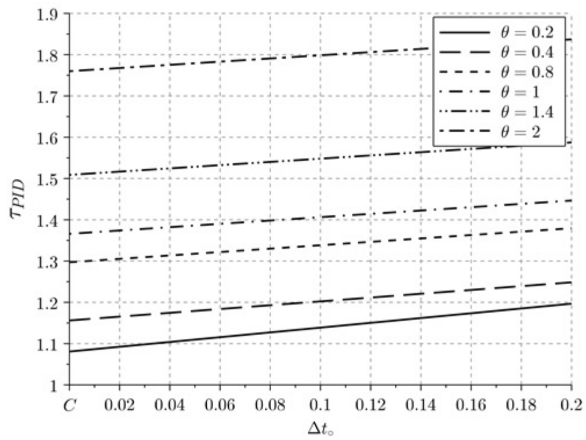

Fig. 4. Optimal settings of $\tau_{PID} = f\left(\Delta t_{\circ}, \theta\right)$ according to (15)

The slope angle of the obtained characteristics that approximate the optimum τ_{PID} values is relatively constant. This means that the influence of the sampling period on the optimum value of the analyzed controller parameter is only slightly dependent on the θ value, i. e. the speed of the transition process.

The characteristics of the equations (12) and (13) for the PI controller are shown in the Fig. 5 and Fig. 6 and are similar in shape to those shown in Fig. 3 and Fig. 4, so it can be assumed that the influence of the sampling period on the optimum PI controller parameters is analogous to that of the PID controller. However, when comparing the characteristics in graphs Fig. 3 and Fig. 5, it should be noted that the optimum gain values for the PID controller are approximately twice as high as for the PI controller.

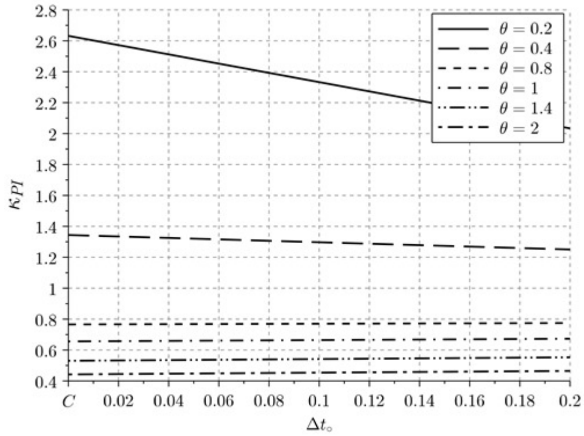

Fig. 5. Optimal settings $\kappa_{PI} = f\left(\Delta t_{\circ}, \theta\right)$ according to (12)

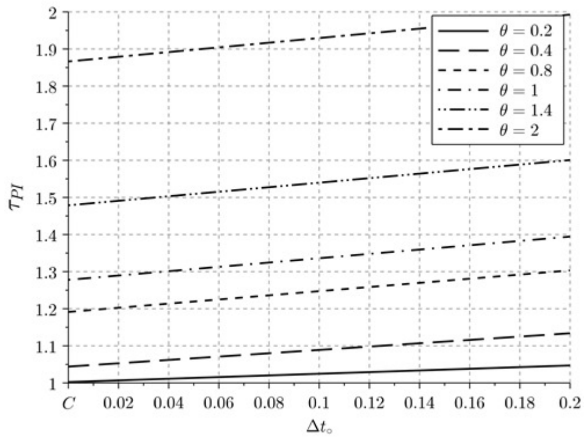

Fig. 6. Optimal settings of $\tau_{PI} = f\left(\Delta t_{\circ}, \theta\right)$ according to (13)

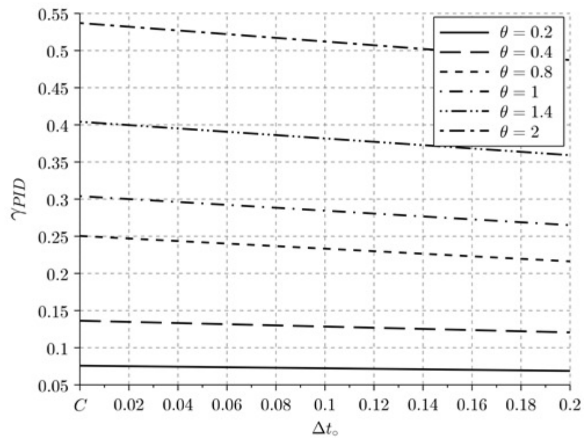

Fig. 7. Optimal settings $\gamma_{PID} = f\left(\Delta t_{\circ}, \theta\right)$ according to (16)

Fig. 7 shows the changes in the optimum values of the relative derivative time of the PID controller in the function $\Delta t_{\circ}$ and θ. The slope angle of the approximating characteristics of the optimal γ_{PID} values increases with the increase of the θ value. This means that control systems with approximating models of controlled systems, which are characterized by slower transition processes, require greater changes in the relative derivative time values as the sampling period increases.

Fig. 8 shows the transient processes of the three control systems. The first diagram shows the response of a continuous control system for which the controller parameters are optimal in relation to the ITAE criterion. The process is characterized by minimal overshoot and settling time. The second diagram shows the response of a discrete control system, for which the controller parameters correspond to the optimal parameters of a continuous controller. The sampling period was $\Delta t_{\circ} = 0.2$.

The parameters of the controlled system did not change. A significant deterioration of the control quality can be noticed. The amplitude of overshoot and the settling time increased. The oscillations of the controlled value appeared. The third graph shows the response of a discrete control system, for which the parameters were selected using dependencies (14), (15), and (16), thus taking into account the impact of sampling period $\Delta t_{\circ} = 0.2$. Parameters of the controlled system model have not changed. There is a significant improvement in control quality. Both the overshoot amplitude and settling time are comparable to the values for a continuous system.

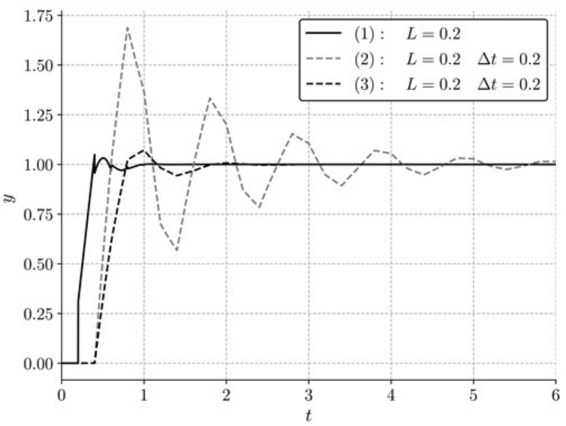

Fig. 8. Transient processes of the controlled value

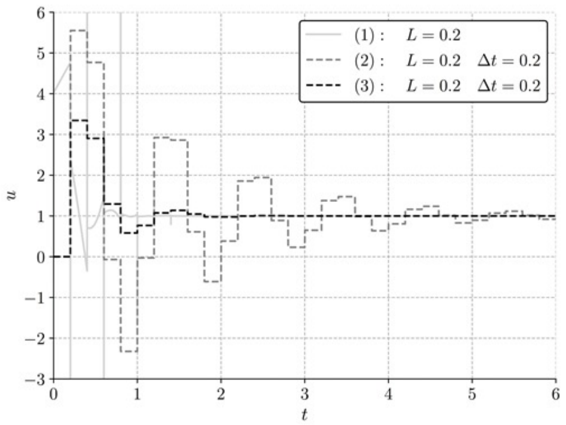

Fig. 9. Control signals $u(t)$

In Fig. 8 it can be noted that changes in the values of the controlled signals presented for the second and third characteristics are delayed. This is due to the fact that the controller performs its algorithm every Δt period, so that its reaction is delayed by Δt time. This phenomenon can be seen in Fig. 9, where the characteristics of control signals are shown. A continuous controller reacts to the values of x and y signals and generates a control signal which is updated every δt period, which is the basic operating time resolution of the entire control system. When the controller works with a resolution resulting from Δt period, which is significantly greater than the basic δt period, it will react to the signals at its inputs after the period Δt .

Conclusion

The way the discrete control system works causes loss of information about the control signal between the moments of its sampling. If the controller settings are selected without taking into account the sampling period, a significant loss of control quality is possible. The equations presented here make it possible to select optimal parameters of the controller taking into account the sampling period for a system whose controlled system can be approximated with an inertial model of the first order with transport delay. For this purpose, the PI and PID controller settings can be selected using the method described in the paper [7]. In that paper, the relationships determining the optimal controller settings are presented in a simplified form and are adequate for the real model of a continuous controller. By replacing these equations with more accurate relationships (12), (13), (14), (15) and (16), the method from paper [7] can be adapted to the system with the discrete controller described with use of model (1).

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