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Analytical model of single-phase AC circuit with inductance and bridge rectifier

***Abstract**. The paper deals with a single – phase AC circuit consist of series inductance, resistance and bridge rectifier loaded by resistance and capacitance. The continuous operation mode of the rectifier is analyzed. The analytical characteristics of such circuit are presented. The output magnitudes of the rectifier, i.e. the mean value of the voltage and current have been determined. The magnitude of the first harmonic and the sum of the higher harmonics of the input current are given. The limit condition of continuous work has been described. Presented formulas are written in simplified form.*

Streszczenie.** W pracy analizowany jest jednofazowy obwód AC zawierający szeregową indukcyjność, rezystancję oraz prostownik mostkowy z obciążeniem RC. Praca ciągła prostownika jest analizowana. Charakterystyki analityczne takiego obwodu zostały zaprezentowane. Wielkości wyjściowe prostownika tj. wartość średnia napięcia i prądu zostały określone. Wartość skuteczna pierwszej harmonicznej i sumy wyższych harmonicznych prądu wejściowego zostały podane. Warunek graniczny pracy ciągłej został wyznaczony. Prezentowane zależności zapisano w postaci uproszczonej. **Model analityczny jednofazowego obwodu AC z indukcyjnością i prostownikiem mostkowym

Keywords: modeling, bridge rectifier, continuous work, nonlinear load.
Słowa kluczowe: modelowanie, prostownik mostkowy, praca ciągła, obciążenie nieliniowe.

Introduction

Rectifiers are ones of the most used loads of the power system. They are used to supply electric and electronic DC devices from AC network. In many applications full-wave bridge rectifier with output capacitor is used as basic type of rectifying of mains voltage. In general case output load of rectifier can be consider as resistance, as shown in the figure 1. In practical solution, between capacitor C and load R_L, usually DC/DC converter and linear voltage regulator are used. In the circuit under consideration, the bridge rectifier is supplied from a sinusoidal AC voltage source via series connected inductance L_s and resistance R_s, which also included equivalent impedance of the mains supply. For this circuit continuous and discontinuous working modes are possible [1],[2]. In continuous conduction mode a current I_s has countable number of points of the zero crossing. This condition is not fulfilled for discontinuous operation mode. In continuous mode higher harmonics propagation to suppling mains is significantly lower than in discontinuous operation mode.

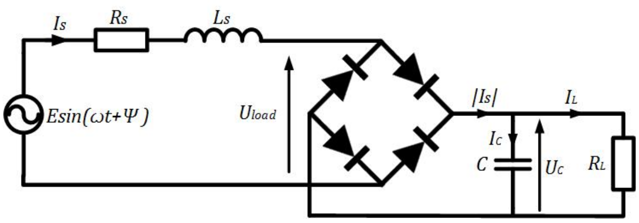

Fig. 1. AC circuit with bridge rectifier with capacitor C and resistance load R_L supplied through inductive impedance

In [3],[4] single phase circuit with bridge rectifier loaded by RC parallel connection and ideal supply voltage source was analysed without taking into account series inductance and resistance. Series inductance was taking into account in [5],[6], but only for discontinuous operation mode and only computer simulation analysis were carried out. The discontinuous conduction mode is especially undesirable in view of higher harmonics generation to the mains, as opposed to continuous operation. Therefore, determining the limit terms of continuous operation and characteristics of such load will be useful. The continuous operation is occasionally analysed, probably because of inductance. Nowadays, wide range of inductance is available and analysis of the circuit may be required.

Modeling AC circuit with nonlinear load

Analytical model of AC circuit with nonlinear load was presented in [7]. This circuit contains series inductance, resistance and nonlinear element as shown in figure 2.

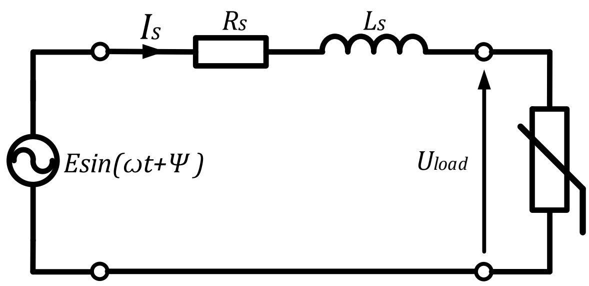

Fig. 2. AC circuit with nonlinear element

Voltage on nonlinear load U_{load} is written as a signum function of current I_s and magnitude U_a:

$$(1)\quad U_{load(t)} = U(I_{s(t)}) = U_a \cdot sign\left(I_{s(t)}\right) = \begin{cases} U_a & I_{s(t)} > 0 \\ 0 & I_{s(t)} = 0 \\ -U_a & I_{s(t)} < 0 \end{cases}$$

The circuit from figure 2 has been described equation:

$$(2)\qquad L_s \frac{dI_{s(t)}}{dt} + R_s \cdot I_{s(t)} + U_{load(t)} = E\sin(\omega t + \psi)$$

where: Ψ – denote phase shiftment angle between supply voltage and first harmonic component of nonlinear load voltage u_{h1}.

Analysis of circuit was conducted with using dimensionless variables. Hence, reference variables E, ωL_s and time scaling $\tau = \omega t$ were introduced. After transformations, equation (2) takes form:

$$(3)\qquad \frac{di_{s(\tau)}}{d\tau} + r_s \cdot i_{s(\tau)} + u_a \cdot sign(i_{s(\tau)}) = \sin(\tau + \psi)$$

where:

(4) $$i_{s(\tau)} = \frac{I_{s(t)}}{E/\omega L_s} \quad u_{a(\tau)} = \frac{U_{a(t)}}{E} \quad r_s = \frac{R_s}{\omega L_s}$$

Instantaneous waveforms voltages and current with period 2π are shown in figure 3. These waveforms are typical for continuous operation mode. The parameters of (3) are: $u_a = 0{,}45$ and $r_s = 0{,}1$.

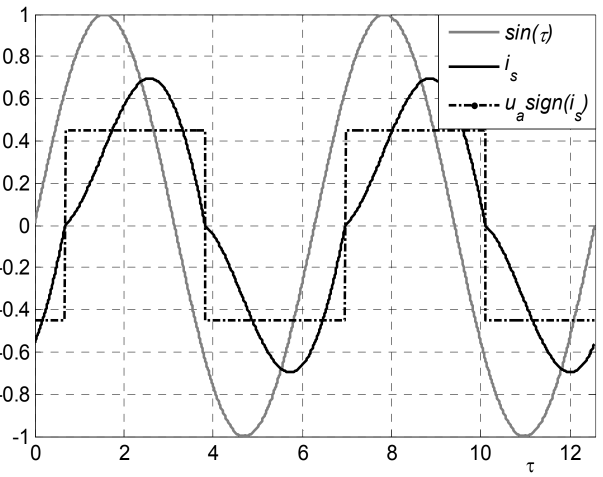

Fig. 3. Instantaneous waveforms voltages and current of circuit shown in figure 2

Amplitude of fundamental component of dimensionless current i_{sh1} has following form [7]:

(5) $$i_{sh1} = \sqrt{\left[\frac{1}{1+r_s^2}\left(\sqrt{1+r_s^2-u_{1h1}^2\left(1+W\left(r_s^2+1\right)\right)^2}-u_{1h1}r_s\right)\right]^2+\left(u_{1h1}W\right)^2}$$

where:

(6) $$W = \left(\frac{\pi^2}{8}-1\right)-0{,}0147r_s^2+0.00145r_s^4+\ldots$$

(7) $$u_{1h1} = \frac{4u_a}{\pi}$$

and u_{1h1} denote fundamental component of voltage u_{load}. The RMS value of the sum of higher harmonics of current i_{shh} may be written as:

(8) $$i_{shh} = \frac{u_{1h1}}{\sqrt{2}r_s}\sqrt{\frac{\pi^2}{8}-1-W}$$

Full wave rectified current i_s has average value in following form:

(9) $$i_{av} = \frac{2}{\pi}\left[\frac{1}{1+r_s^2}\left(\sqrt{1+r_s^2-u_{1h1}^2\left(1+W\left(1+r_s^2\right)\right)^2}-u_{1h1}r_s\right)-\frac{u_{1h1}}{r_s}\left(\frac{\pi^2}{8}-1-W\right)\right]$$

The phase shiftment angle between i_{sh1} and first harmonic of the load voltage u_{1h1} is expressed by:

(10) $$\varphi_{1h1} = -\arcsin\left(\frac{u_{1h1}W}{i_{sh1}}\right)$$

Above dependencies be in force, when countability condition set of the zero crossing points of the dimensionless current $i_{s(\tau)}$ is fulfilled. It happens, when:

(11) $$u_{1h1} < \frac{\pi}{4\cdot\left(1+W(1+r_s^2)\right)}$$

Modeling AC circuit with bridge rectifier

The analytical formulas of nonlinear load given in previous chapter describe the typical parameters of a single-phase AC circuit with nonlinear element. These parameters were determined as function of dimensionless series resistance r_s and amplitude of the first harmonic voltage on the load u_{1h1}. The analysis shows that the given equations can be useful also in the analysis of the circuit from figure 1. But for such case, the nonlinear element is a bridge rectifier with a capacitive filter C and load resistance R_L. The circuit from figure 1 was analyzed in authors' work [8]. Differential equations of such circuit were described using dimensionless variables (small letters):

(12) $$\frac{di_{s(\tau)}}{d\tau} = \sin(\tau+\psi)-i_{s(\tau)}r_s-u_c sign(i_{s(\tau)})$$

(13) $$\frac{du_{c(\tau)}}{d\tau} = \frac{1}{c\cdot r_L}\left(r_L\cdot\left|i_{s(\tau)}\right|-u_{c(\tau)}\right)$$

where dimensionless variables are defined as follows:

(14) $$u_{c(\tau)} = \frac{U_{c(t)}}{E} \quad r_s = \frac{R_s}{\omega L_s} \quad r_L = \frac{R_L}{\omega L_s} \quad c = \omega^2 L_s C$$
$$i_{s(\tau)} = \frac{I_{s(t)}}{E/\omega L_s} \quad i_{av} = \frac{I_{av}}{E/\omega L_s} \quad u_{av} = \frac{U_{av}}{E/\omega L_s}$$

For a preliminary analysis of the circuit in figure 1, equation (12) and (13) may be solved in Simulink. The voltage - current characteristic of $u_{load}=f(i_s)$ such load is shown in figure 4.

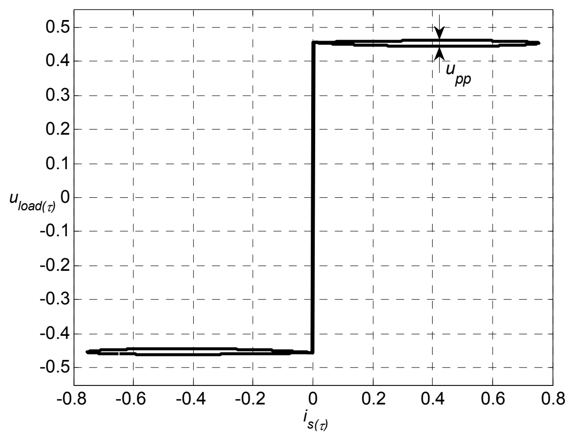

Fig. 4. Voltage-current characteristic $u_{load}=f(i_s)$, when $T_c=10$

The shape this characteristic depends on time constant $T_c=c\cdot r_L$. If $T_c=10$ it can be assumed that the output voltage has a constant value with negligible small fluctuations u_{pp}. Assuming that the ripple voltage u_{pp} have a small value relative to the average voltage value u_{av}, the characteristic of figure 4 may be approximated using the voltage-current characteristic $u_{load}=f(i_s)$ of the nonlinear load analyzed in [7]. This approximation can be used to determine the analytical characteristics of the single-phase AC circuit shown in figure 1, for operation in continuous operation mode.

Due to, that the voltage power source is sinusoidal and load is nonlinear, the dimensionless current i_s can be considered as the first harmonic i_{sh1} and the sum of the higher harmonics current i_{shh}. These quantities have been determined on the basis of formulas (5) and (8) using additional transformations and simplification operation. The final formula for the amplitude of the first harmonic of the current i_{sh1} is:

$$i_{sh1} = \frac{1}{1+r_s^2}\left(\sqrt{1+r_s^2 - u_{1h1}^2(1+2W_o)} - u_{1h1}r_s\right) \tag{15}$$

where W_o is a simplified form of the expression (6) obtained as a result of elimination of components dependent on r_s, and written in the following form:

$$W_o = \frac{\pi^2}{8} - 1 \tag{16}$$

voltage u_{1h1} is determined by the dependence:

$$u_{1h1} = \frac{4u_{av}}{\pi} \tag{17}$$

The RMS value of the sum of the higher harmonics of the current i_{shh} can be determined by substituting to (8) expression (6):

$$i_{shh} = \frac{u_{1h1}}{\sqrt{2}} \cdot \sqrt{0.0147 - 0.00145 r_s^2} \tag{18}$$

On the basis of (15) and (18) it is possible to calculate the RMS value of current i_{srms} using the relationship:

$$i_{srms} = \sqrt{\frac{i_{sh1}^2}{2} + i_{shh}^2} \tag{19}$$

The average value of the output current of rectifier i_{av} is also important parameter. The value of the current can be determined using formula (9) by written in simplified form:

$$i_{av} = \frac{2}{\pi}\left[\frac{1}{1+r_s^2}\left(\sqrt{1+r_s^2 - u_{1h1}^2\left(1+W_o\left(1+r_s^2\right)\right)^2} - u_{1h1}r_s\right)\right] \tag{20}$$

The mean value of the output voltage u_{av} may be determined using:

$$i_{av} = \frac{u_{av}}{r_L} \tag{21}$$

Substituting (17) and (21) to (20), gives voltage u_{av} in function dimensionless resistance r_s and load resistance r_L :

$$u_{av} = \frac{\pi}{\sqrt{\left(1+r_s^2\right)\left(\frac{\pi^4}{4r_L^2} + 16W_o^2\right) + 16\cdot\left(\frac{\pi^2 r_s}{4r_L} + 2W_o + 1\right)}} \tag{22}$$

The condition of continuous operation is met when:

$$u_{1h1} < \frac{\pi}{4\cdot\left(1+W_o(1+r_s^2)\right)} \tag{23}$$

From (22) it is possible to determine the additional formula for the resistance r_L as function of r_s and u_{av}:

$$r_L = \frac{\pi^2\left(1+r_s^2\right)}{8\cdot\left(\sqrt{\frac{\pi^2}{16u_{av}^2}\left(1+r_s^2\right) - \left(1+W_o\left(1+r_s^2\right)\right)^2} - r_s\right)} \tag{24}$$

Analysis of the analytical circuit model

The magnitude of the first harmonic of the current i_{sh1} as a function of the first harmonic of voltage of the nonlinear load u_{1h1} and series resistance r_s is presented in figure 5. For larger resistance r_s the magnitude i_{sh1} decreases.

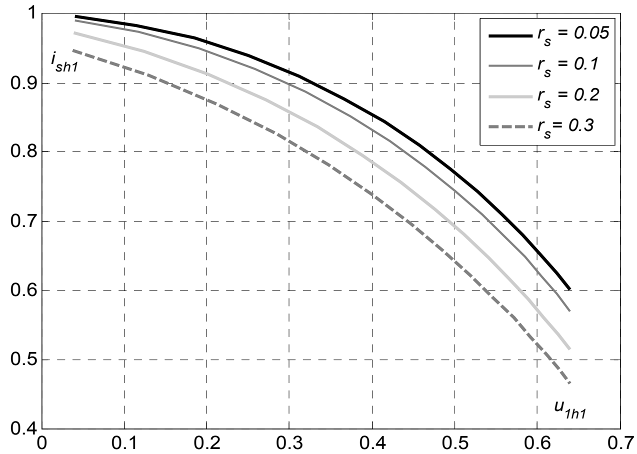

Fig. 5. Magnitude of the fundamental harmonic of the current i_{sh1} as function u_{1h1} and different series resistance r_s

Mean value of the output voltage u_{av} as function of load resistance r_L and series resistance r_s is shown in figure 6. Theoretical results may be compared with results of computer simulation.

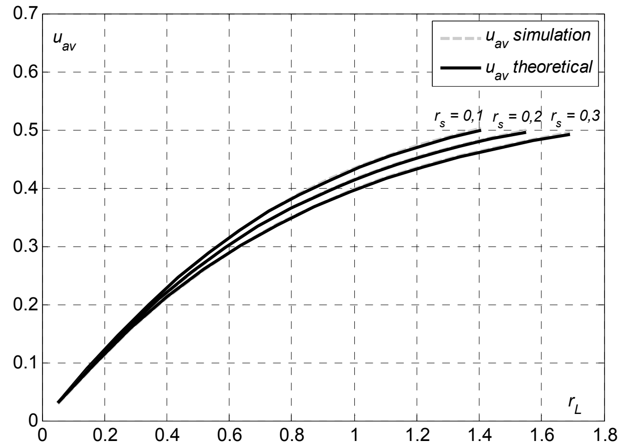

Fig. 6. The voltage u_{av} as function of r_L and r_s

The solid line represents the voltage u_{av} calculated on the basis of the theoretical formula (22) whereas the dotted line presents u_{av} obtained in the computer simulation with using Simulink program. The maximum value of dimensionless resistance r_L has been calculated on the basis of equation (24), assuming the limit value of average voltage u_{av} calculated on the basis of (23) and (17). The curves $u_{av} = f(r_L, r_s)$ show that the theoretical characteristics in the considered continuous operation mode cover with the curves of the simulation model created in Simulink on the basis of equations (12) and (13).

According to standard IEEE519 [9] the total harmonic distortion *THD* of current i_s is expressed as following:

$$THDi_s = \frac{\sqrt{i_2^2 + i_3^2 + i_4^2 + \cdots}}{\sqrt{i_1^2 + i_2^2 + i_3^2 + \cdots}} \tag{25}$$

where: i_n – is RMS values subsequent harmonic current.

The expression in the numerator refers to the sum of the RMS values of the higher harmonics of the current, whereas in the nominative its RMS value. Using dependencies (18) and (19), the formula (25) may be written as:

$$THDi_s = \frac{i_{shh}}{i_{ssk}} \tag{26}$$

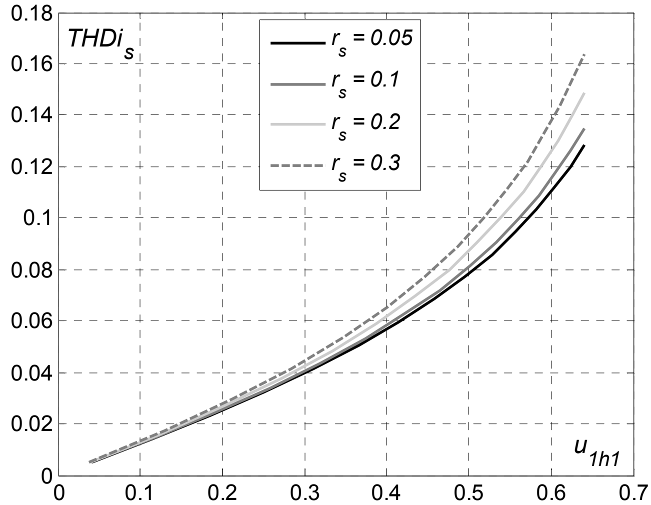

Fig. 7. The $THDi_s$ factor of current i_s in function u_{1h1} and different resistance r_s

On the basis formula (26), $THDi_s$ factor of the current i_s as function u_{1h1} and different values of resistance r_s is shown in figure 7. These curves show that the factor of higher harmonics content $THDi_s$ in current i_s increases with u_{1h1} values. Its value depends on series resistance r_s, especially for $u_{1h1} > 0,3$. It can be seen that for upper u_{1h1} values the $THDi_s$ value for $r_s = 0,3$ is about 30% greater than for $r_s = 0,05$.

Theoretical relations written in dimensionless form are functions of the amplitude of the dimensionless first harmonic voltage u_{1h1} on rectifier, series resistance r_s and load resistance r_L. Such representation allows to limit the number of parameters of the considered circuit, and consequently to simplify its analysis. Given formulas in dimensionless variables can be easily transformed to physical units, using (14) relations. It can be seen, that the effect of series resistance r_s on the circuit parameters is important, although in many papers is omitted.

Conclusion

The static characteristics of a single-phase AC circuit with series inductance, resistance and full-wave bridge rectifier have been presented. The basic quantities of currents and voltages in circuit related to continuous operation mode are given. The range for such mode has been defined. The analytical formulas are related to case when the fluctuations of the output voltage are relatively small. The accuracy of theoretical results compared to simulation ones is met with error greater than ±0,01, when the series resistance is $r_s \leq 0,3$.

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