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# Undersampling applied for measurements of nonlinear object characteristics in presence of disturbances

***Streszczenie**. Pomiar charakterystyk statycznych elementu nieliniowego można przeprowadzić wykorzystując podpróbkowanie Σ. W przypadku, gdy na wejście elementu nieliniowego jest podany sygnał sinusoidalny, jego sygnał wyjściowy jest odkształcony. Przedstawiono algorytm cyfrowego przetwarzania sygnałów, umożliwiający redukcję błędów pomiarowych. Przeanalizowano zależność błędów pomiaru charakterystyki elementu nieliniowego od krotności podpróbkowania, liczby próbek oraz błędów częstotliwości na przykładzie 2 typowych układów nieliniowych.* ***Podpróbkowanie w zastosowaniu do badania charakterystyk obiektów nieliniowych w obecności zakłóceń***

***Abstract**. A measurement of static characteristics of nonlinear devices can be performed using a sigma undersampling. When the input signal of nonlinear element is sinusoidal, its output signal is deformed and contains an infinite series of harmonics. An algorithm of digital processing of the output signal enabling a reduction of errors is described. A dependence of the errors of the measurement on the parameters of the undersampling was analyzed for 2 typical nonlinear elements.*

**Keywords:** nonlinear element, sigma undersampling, frequency fluctuations, Gaussian noise;
**Słowa kluczowe**: podpróbkowanie sigma, szumy Gaussa

## Introduction

A continuing progress in the electronic technology comprises a progress in applications of nonlinear elements [1-2]. Contemporary electronic devices work, using lower levels of the signals and higher frequencies. To perform a correct design of the electronic device, the parameters of its components must be measured in proper conditions. It means proper levels of the signals and working frequencies. Classical methods of measurements of characteristics of the nonlinear elements could give inaccurate results for very high frequencies and low level signals. Therefore it forces an improvement of the method of measurement of the nonlinear element parameters.

A method applying a sigma undersampling is proposed in this paper. It enables a reduction of errors, caused by fluctuations of frequency and an inaccuracy of measuring sensors.

## Description of the method

An idea of the proposed method assumes a reduction of errors arising during the measurement. The output signal of the examined nonlinear element must be sampled in order to apply a signal processing of the results of the measurements. Two kinds of errors are taken into considerations. The first one arises from an inaccuracy of the measuring system. The second one is the error coming from the incoherence between the sampling generator and the generator of the input signal. To minimize the first kind of errors an integration of the output signal of the device during the sampling period is applied. According to Central Limit Theorem a variance of the noise decreases proportionally to the time of the integration. Therefore the sampling period should be as long as possible. On the other side, the extension of the sampling time causes the difference of the phases between the input and the sampling signals is getting greater. In this case the second kind of errors becomes significant. Because of this a special algorithm reducing phase errors must be applied.

A scheme of measuring system is presented in Fig. 1. When a sinusoidal signal $u(t)$ of the frequency $f_0$ is applied as the input signal of nonlinear system, its output signal $x_0(t)$ is periodic and deformed, Therefore it can be expressed in form of Fourier's series

$$(1)\qquad x_0(t) = X_0 + \sum_{m=1}^{\infty} X_m \cdot \exp\left[j\left(2\pi m f_0 t + \varphi_m\right)\right].$$

$X_0$ denotes the mean value of the signal, whereas $X_m$ and $\varphi_m$ - the amplitude and the phase of its $m^{th}$ harmonic, respectively.

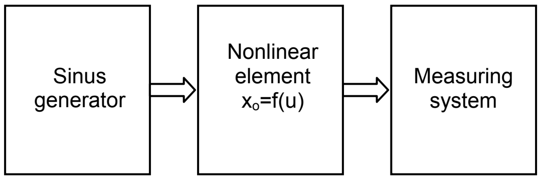


Fig.1. Scheme of system for measurements of static characteristics of nonlinear devices

When the static characteristic of the examined device is symmetrical with respect to a vertical axis, a spectrum of its output signal contains only odd harmonics. In the opposite case even harmonics appear in the spectrum of the output signal. Exemplary static characteristics of 2 nonlinear elements are presented in Fig. 2. The spectra of their output signals are shown in Figs. 3 and 4.

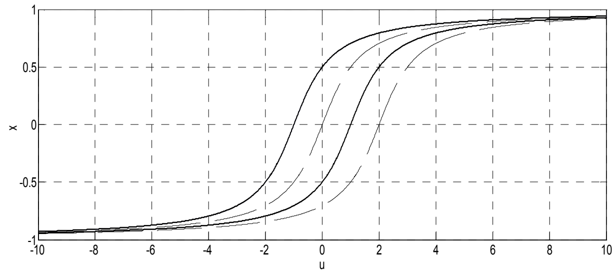


Fig. 2. Static characteristics $x=f(u)$ of 2 nonlinear elements

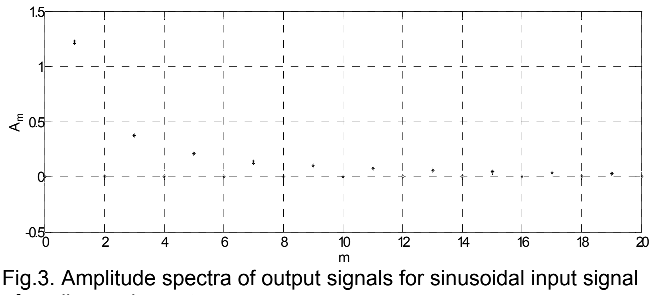


Fig.3. Amplitude spectra of output signals for sinusoidal input signal of nonlinear elements

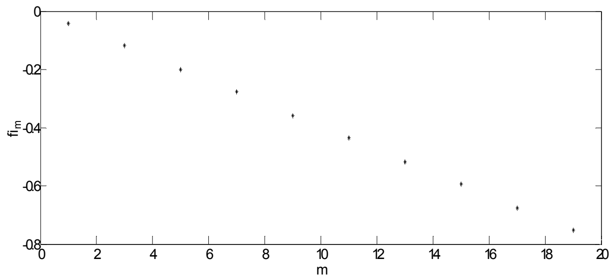


Fig. 4. Phase spectra of output signals of nonlinear elements for sinusoidal input signal

As it was mentioned above, in order to reduce the noise of the detector, the time of the integration of the output signal ought to be maximally long. Therefore an undersampling of the output signal should be applied. The time of integration $T_{d0}$ must satisfy a following condition [3-12]

(2) $$T_{d0} = \frac{M + N^{-1}}{f_0},$$

where $M$ is an undersampling factor and $N$ - a number of samples per period. The output signal is integrated during the sampling period. The output signal of the integrator $y(t)$ can be expressed as

(3) $$y(t) = \int_{t}^{t+T_{d0}} x(t)\cdot dt = \frac{X_0(MN+1)}{Nf_0} + f_0^{-1}\sum_{m=1}^{\infty} X_m \frac{\sin\left(\frac{m\pi}{N}\right)}{m\pi}\cdot \exp\left[j\left(\frac{2m\pi t}{T} + \frac{m\pi}{N} + \varphi_m\right)\right]$$

A comparison of (1) and (3) shows that the spectra of the signals are similar, whereas the phase of signal $y(t)$ disagrees with the phase of the signal $x_0(t)$. The difference between the phases of both signals is constant for the particular harmonic and it equals $m\pi/N$. Therefore to obtain the original signal a digital filter should be applied. Its transfer function $H(f)$ must satisfy a following condition [13-15]

(4) $$X_0(mf_0) = H(mf_0)\cdot Y(mf_0),$$

where $X_0(f)$ and $Y(f)$ denote the spectra of the signals $x_0(t)$ and $y(t)$, respectively. Basing on (1) and (3) $H(mf_0)$ can be expressed as:

(5) $$H(0) = \frac{Nf_0}{MN+1},$$

(6) $$H(mf_0) = \frac{m\pi f_0}{\sin\left(\frac{m\pi}{N}\right)}\cdot \exp\left(-\frac{m\pi}{N}\right).$$

Exemplary spectral characteristics of the filter are shown in Figs 5 and 6.

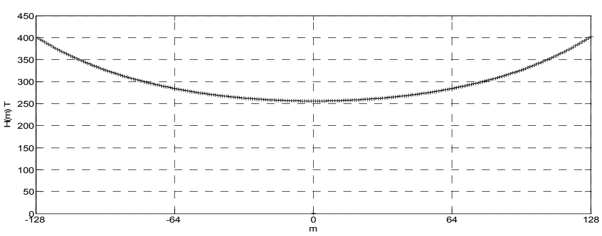


Fig. 5. Amplitude characteristic of FIR filter $H(z)$ for $N$=256, $M$=100

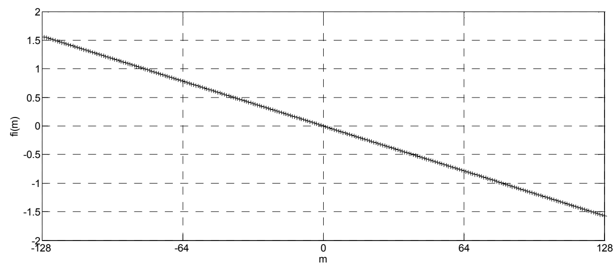


Fig. 6. Phase characteristic of FIR filter $H(z)$ for $N$=256, $M$=100

Considerations of the transfer function of the filter prove that it also works as an anti-aliasing filter, because its attenuation for the frequency $f_0 /2N$ exceeds 3 dB [13-15]. The transfer function of the filter is given by:

(7) $$H(z) = \sum_{n=0}^{N-1} h(n)\cdot z^{-n},$$

(8) $$h(n) = \frac{f_0}{MN+1} + (-1)^n\cdot\frac{\pi f_0}{2} + \frac{2\pi f_0}{N}\sum_{m=1}^{N/2-1}\frac{m\cdot\cos\left(\frac{2mn\pi}{N} - \frac{m\pi}{N}\right)}{\sin\frac{m\pi}{N}}.$$

The method generates no errors itself on the condition, that the frequencies of the generators of the input and the sampling signals correctly satisfy condition (2) and the synchronization between the generator of the sinusoidal signal and the sampling generator is perfect. In the opposite case the phase of the measured signal fluctuates. Relative fluctuations of the frequency of the input signal and the sampling period are denoted $\delta f$ and $\delta T_d$, respectively. The real frequency of the signal and the sampling period can be expressed as [13-15]:

(8) $$f = f_0\cdot(1+\delta f),$$

(9) $$T_d = T_{d0}\cdot(1+\delta T_d).$$

An application of the integrator reduces the values of errors caused by the noise on the condition, that its mean value tends to 0. To reduce a probability of accidental errors, a multiple repetition of measurements should be applied. However it requires a perfect synchronization between the output signal of the examined nonlinear element and the sampling generator. This problem is solved by the application of the proper algorithm of the signal processing [13-15]. The single period of signal consisting of $N$ samples is measured and, in the next step, FFT of the signal is calculated by means of the set of FIR digital filters. The results are memorized. This procedure is repeated $P$ times. A geometrical mean of all FFTs is calculated in the next step. Finally, the results of the measurements are obtained using IFFT, calculated by another set of FIR digital filters.

The procedure described above can lead to the deformation of the measured characteristic, when the successive series of the samples are measured starting in different phases of the output signal. This defect appears mainly in the case, when the static characteristic of the examined nonlinear element contains a hysteresis loop and the non-zero mean value (the characteristic marked with green colour on Fig. 2). An elimination of this phenomenon is possible, when a preliminary synchronization of the signal is applied. The value $X(1)$ of FFT for $m$=1 can be utilized to this purpose. Every series of the measurements must be filtered by means of the FIR filter, which transfer function is given by

(10) $$H_S(z) = N^{-1} \cdot \sum_{n=0}^{N-1} z^{-1} .$$

The result is compared to the corresponding value of the spectrum obtained in the first measurement. The minimum difference between both values is taken to further calculations. A scheme of preliminary synchronization is presented in Fig. 7.

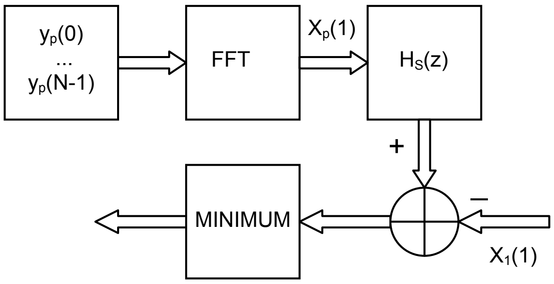


Fig. 7. Algorithm of preliminary synchronization system

The complete scheme of the digital processing system is shown in Fig. 8

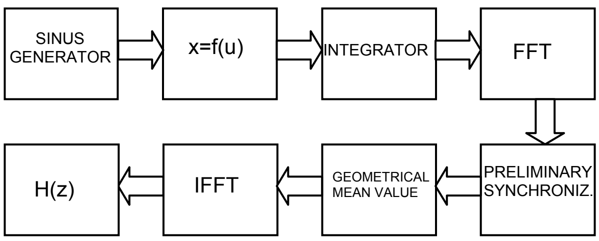


Fig. 8. Algorithm of signal processing system for measurements of static characteristics of nonlinear devices

**Simulations of the method and analysis of errors**

The method described in the previous section is applied to the simulations of the measurements of the static characteristics of two nonlinear elements shown in Fig 2.

An estimation of the accuracy of described method was performed using the simulation in MATLAB. Calculations were performed $P$=$10^6$ times. The amplitude of the input signal is chosen so that the measurements of the full characteristic of examined element were possible. Results of simulations are evaluated by means of an error $\delta$, defined as

(11) $$\delta = \frac{\sum_{p=1}^{P}\sum_{k=0}^{N-1}\left\| \left[ \bar{x}(k) - x_0\left(\frac{k}{f_0 \cdot N}\right)\right]\right\|}{x_{0\max} \cdot P \cdot N},$$

where $x_0(k)$ are the samples of the output signal in ideal conditions (fluctuations of the frequency and errors of measuring sensors are equal 0) and $x_{0max}$ is the maximum value of the signal $x_0(t)$.

Fluctuations of the frequency of the input signal $u(t)$ and the sampling period are random values described by Gaussian distribution with the variance $\delta f$ (the frequency of the input signal) or $\delta T_d$ (the sampling period) and the mean values 0. A Gaussian noise of the mean value 0 and the variance $\sigma$ was added to the output signal of the nonlinear element as the simulation of the errors of the deceiver. The undersampling factor $M$, the number of samples per period $N$ and signal to noise ratio, defined as

(12) $$SNR = \frac{2 \cdot \sigma^2}{A_1^2} ,$$

were chosen as the parameters of the calculations. The results of the simulations for the element marked with solid line are shown in Figs. 9-12 and the results for element marked with dash line - in Figs. 13-16.

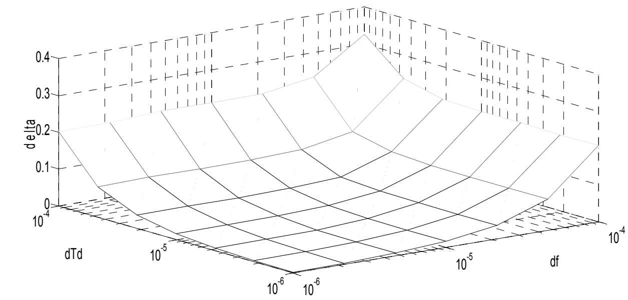


Fig. 9. Dependence of error $\delta$ on the fluctuations of frequencies for 1st nonlinear element, $SNR^{-1}$ = 0,2, $N$ = 256 and $M$=100

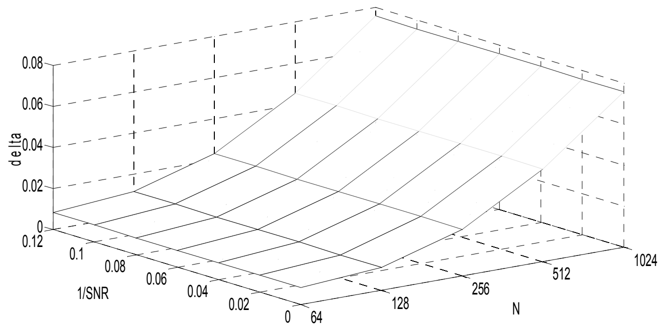


Fig. 10. Dependence of error $\delta$ on relative power of noise and $N$ for 1st nonlinear element, $M$=100, $\delta f$=$\delta T_d$=$10^{-6}$

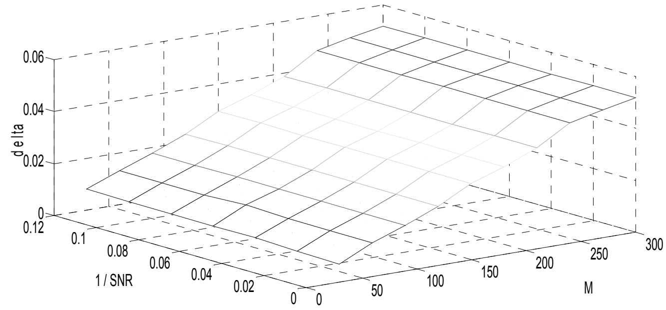


Fig. 11. Dependence of error $\delta$ on relative power of noise and $M$ for 1st nonlinear element, $N$=256, $\delta f$=$\delta T_d$=$10^{-6}$

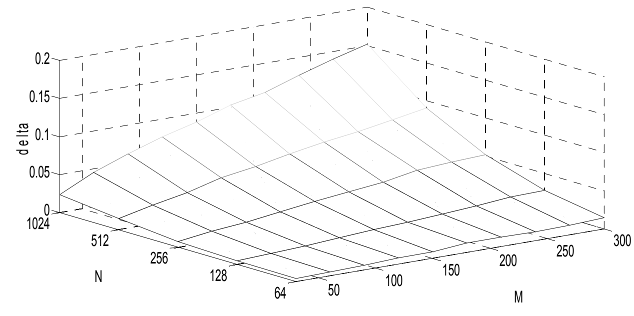


Fig. 12. Dependence of error $\delta$ on $N$ and $M$ for 1st nonlinear element, $SNR^{-1}$ = 0,2, $\delta f$=$\delta T_d$=$10^{-6}$

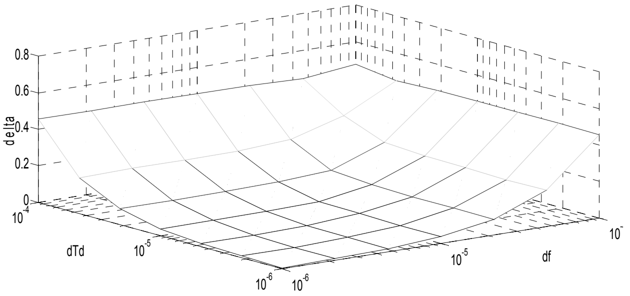


Fig. 13. Dependence of error $\delta$ on the fluctuations of frequencies for $SNR^{-1}$ = 0,2 for 2nd nonlinear element, $N$ = 256 and $M$=100

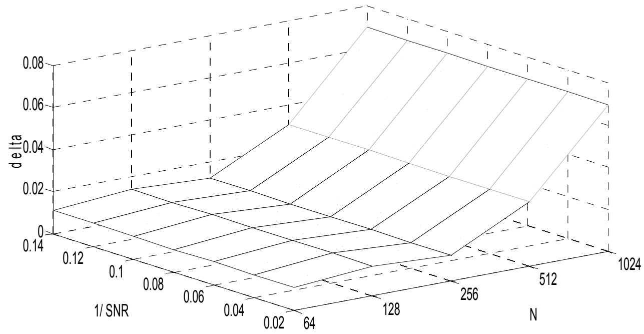


Fig. 14. Dependence of terror $\delta$ on relative power of noise and $M$ for 2nd nonlinear element, $N$=256, $\delta f = \delta T_d$=$10^{-6}$

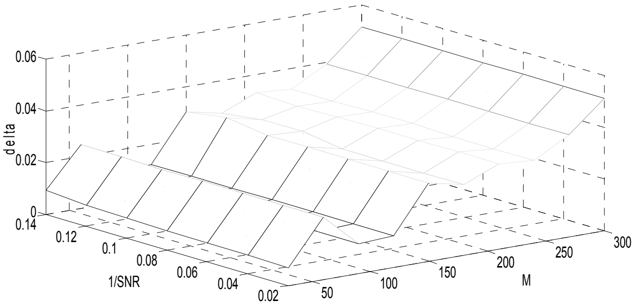


Fig. 15. Dependence of error $\delta$ on relative power of noise and $M$ for 2nd nonlinear element, $N$=256, $\delta f = \delta T_d$=$10^{-6}$

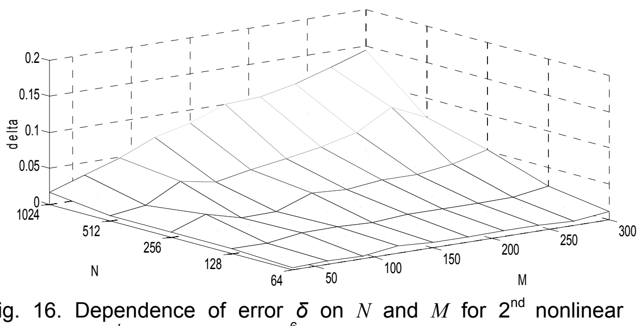


Fig. 16. Dependence of error **δ** on $N$ and $M$ for 2nd nonlinear element, $SNR^{-1}$ = 0,2, $\delta f = \delta T_d$=$10^{-6}$

## Conclusions

Presented results of calculations prove, that the sigma undersampling can be the effective method of the identification of nonlinear systems. The integration of the output signal reduces the influence of the errors of the detector, whereas the appropriate algorithm of the sampling enables the significant reduction of the errors caused by the fluctuations of the frequencies. For the numbers of samples per period in range 256-512 and the undersampling factors in the range of 100, the mean error of the measurement of static characteristic does not exceed 1 % for frequency disturbances in the range of $10^{-6}$.

Although presented method requires the complicated digital data processing system its practical realization is possible. The significant advantage of presented method is the possibility of the measurements using very high frequencies and low levels of input signals.

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