Szymon CYGAN[1], Mirosław ŁUKOWICZ[1]

Wrocław University of Sience and Technology (1)

doi:10.15199/48.2018.11.20

Analysis of the synchrophasor estimation problem

Abstract. *This paper discusses the analytical analysis of synchrophasor estimation employed in electrical systems. Short-time Fourier transform with a phase-locked loop and Taylor-Fourier series are analysed for signals relating to different states which may occur in real power systems. As a consequence of transients in power system signal waveforms changes may occur. This leads to inconvenient errors in any estimation algorithm. This paper presents the character of these errors and their consequences individually for any signal deviation.*

Streszczenie. *W artykule przedstawiona została analiza problemu wyznaczania fazorów w systemie elektroenergetycznym. Algorytymy bazujące na szybkiej transformacie Fouriera z pętlą synchronizacji fazy oraz z rozwinięciem w szereg Taylora-Fouriera zostały przetesotwane na sygnałach odpowiadających rzeczywistym przebiegom w systemie elektroenergetycznym. Zmiany sygnałów napięciowych i prądowych skutkują błędami estymacji parametrów. W artykule przedstawiono charakter błedów estymacji fazorów oraz ich wpływ dla różnych typów analizowanych sygnałów.* ***(Analiza problemu estymacji synchrofazorów)***

Keywords: dynamic phasor, Taylor - Fourier series, phase-locked-loop, dynamic filtering
Słowa kluczowe: fazory dynamiczne, szeregi Taylora - Fouriera, pętla synchronizacji fazy, filtracja dynamiczna

Introduction

An accurate estimation of electrical signal parameters is a fundamental issue as far as the appropriate operation and control of a power system is concerned. The imbalance between power supply and the load leads to dynamic changes in the system state, so that the voltage and current values, waveshapes and system frequency may differ from nominal. Efficient control of an electrical network requires more and more accurate information on signal parameters. This includes accurate measurement of the signal phase, amplitude, frequency, higher harmonics, rate of change of frequency and the amplitude change rate. These parameters can be represented concisely as a complex valued function referred to as a phasor. As a result, in standard C137.118.1-2011 [1], estimation algorithms have been organized and divided into P (protection) and M (measurement) classes depending on their purpose. M class algorithms are designed for measurement devices with high accuracy requirements and slow response times. P class algorithms are designed for protection purposes with a strict response time and overshoot limits, but higher estimation errors are acceptable.

The phasor estimation issue has been approached in various ways and methods. The most popular concept of a phasor estimation is based on Fourier transform algorithms, in particular short time Fourier transform (STFT). This includes the process of the optimal filter design to ensure appropriate noise damping, good dynamical properties and more effective higher harmonic damping [2], [3]. It has been observed that even a proper filter design can be insufficient for processing of abnormal signals. More advanced methods of filter design employ dynamic filtering to enhance phasor estimation for off nominal frequencies [4], [5]. The most accurate estimation method involves polynomial approximation [6], [7], [8]. Least square method (LSM) is used to evaluate polynomial coefficients of the corresponding phasor. Applying LSM ensures correct estimation for signals with sophisticated amplitude waveforms. The convergence rate of LSM algorithms is dominated by the convergence rate for an exponential function which may be insufficient for some fast varying phenomena. Phase-locked loop estimation algorithms [9], [10] employ rough frequency estimation for frequency adjustment in LSM method, extending the frequency range for off nominal frequencies.

Phasor definition

The dynamic phasor was introduced in [1], to simplify the representation of a real-valued signal. Expressing fast varying real valued signal as a slow varying complex valued signal facilities analysis and allows to obtain the core signal parameters directly from the complex form. The dynamic phasor is defined for the special set of input signals $x(t)$ as:

$$x(t) = X_m(t)\cos\left(2\pi\int f(t)dt + \varphi\right) \tag{1}$$

where: $X_m(t)$ - time dependent signal amplitude, $f(t)$ - time dependent signal frequency, φ - constant phase shift.

For each signal of the form (1) the dynamic time dependent phasor $X(t)$ is defined as follows

$$X(t) = X_m(t)e^{2\pi i\left(\int(f(t)-f_0)dt+\varphi\right)} \tag{2}$$

where: f_0 - base frequency, $\omega_0 = 2\pi f_0$ - base angular speed. The expression in the exponent represents a time dependent phase of the dynamic phasor

$$\psi(t) = 2\pi i\left(\int(f(t)-f_0)dt+\varphi\right) \tag{3}$$

Therefore, the dynamic phasor is formulated as

$$X(t) = X_m(t)e^{i\psi(t)} \tag{4}$$

Moreover, the following equality holds

$$x(t) = \mathrm{Re}\left(X(t)e^{i\omega_0 t}\right) \tag{5}$$

Defining the dynamical phasor in this way is burdened by a crucial problem. Formula (2) is not uniquely defined for any real valued signal. Using trigonometric identities, any signal $x(t)$ can be expressed with many significantly different forms. Consider a sample sinusoidal signal

$$x(t) = sin(\omega t) \tag{6}$$

Applying (1) results in phasor form (7). Transforming signal through trigonometric identities leads to forms (8) and (9).

$$\begin{gathered} x(t) = \cos(\omega_0 t - \tfrac{\pi}{2}) \\ X_m(t) = 1, \quad e^{i\psi(t) = e^{-i\frac{\pi}{2}}} \end{gathered} \tag{7}$$

$$\begin{gathered} x(t) = 2\sin(\tfrac{\omega_0 t}{2})\cos(\tfrac{\omega_0 t}{2}) \\ X_m(t) = 2\sin(\tfrac{\omega_0 t}{2}), \quad e^{i\psi(t) = e^{-i\frac{\omega_0 t}{2}}} \end{gathered} \tag{8}$$

$$\begin{gathered} x(t) = 2\sin(\tfrac{\omega_0 t}{2} - \tfrac{\pi}{2})\cos(\tfrac{\omega_0 t}{2} - \tfrac{\pi}{2}) \\ X_m(t) = 2\sin(\tfrac{\omega_0 t}{2} - \tfrac{\pi}{2}), \quad e^{i\psi(t) = e^{-i\left(\frac{\omega_0 t}{2} + \frac{\pi}{2}\right)}} \end{gathered} \tag{9}$$

In fact, infinitely many miscellaneous forms can express the sinusoidal signal just by using mathematical formulas. This follows that fundamental notions i.e. amplitude and phase od a signal are difficult to be generalized for any type of a signal. This inaccuracy is a core problem for complete and precise phasor definition for any input function. The most important issue in phasor estimation is to define the phasor in such a way to obtain a unique representation. It can be claimed that not all of the previous representations are acceptable in a physical sense. Signal amplitude in (8), (9), are nonpositive functions, implying that at some instant the amplitude can become negative. This issue can be easily fixed by increasing phase shift by $180°$, when such phenomena occur.

In C37.118.2011 phasor uniqueness is achieved by restricting possible forms of both amplitude and phase shift, in terms of time dependent functions. The set of restricted functions is herein denoted as Test Functions Set (TFS). Input signals were divided into steady state and dynamic state subclasses. Signals from the steady state class are periodic functions with higher harmonic distortions

$$x(t) = X_m \cos(\omega t + \varphi) + \sum_k X_m^k \cos(k\omega t + \varphi_k) \tag{10}$$

where: ω - constant angular speed, X_m^k - k-th harmonic amplitude, φ_k - k-th harmonic phase shift. Model phasor is defined in regards to the first harmonic in the series. Higher harmonics are omitted

$$X(t) = X_m e^{i(\omega-\omega_0)t+\varphi} \tag{11}$$

Phasor model for functions from the dynamic class is defined as follows. The phasor amplitude can vary linearly, sinusoidally, or as a step function.

$$\begin{aligned} X_m(t) &= X_m(1 + at) \\ X_m(t) &= X_m(1 + a\chi_{(0,\infty]}(t)) \\ X_m(t) &= X_m(1 + a\cos(\omega_1 t)) \end{aligned} \tag{12}$$

Phasor phase shift can vary parabolically, sinusoidally, or as step function

$$\begin{aligned} \psi(t) &= \psi_0(1 + a_1 t + a_2 t^2) \\ \psi(t) &= \psi_0(1 + a\chi_{(0,\infty]}(t)) \\ \psi(t) &= \psi_0(1 + a\cos(\omega_1 t)) \end{aligned} \tag{13}$$

where: a, a_1, a_2 - variability coefficients, ω_1 - an alternate angular speed. The aforementioned definition under appropriate assumptions for coefficients a, a_1, a_2, ω_1 s provides a unique representation. This means that for any signal from the restricted class a unique pair of functions exists i.e. time dependent amplitude $X_m(t)$ and time dependent phase shift $\psi(t)$, which fulfils phasor identity (5). The primary problem present for this solution is that TFS does not cover all periodic functions. Reasonable signals exist which do not belong to TFS, thus corresponding phasors are not defined. The core aim for the estimation problem is to determine the method of representation for uncovered signals. The second problem is a constricted reversibility. Let us consider two model signals

$$\begin{aligned} x_1(t) &= X_{m_1}(t)\,\mathrm{Re}\left(e^{i\omega t}e^{i\varphi_1(t)}\right) \\ x_2(t) &= X_{m_2}(t)\,\mathrm{Re}\left(e^{i\omega t}e^{i\varphi_2(t)}\right) \end{aligned} \tag{14}$$

It does not follow that if signals are close to each other, in any sense, for example root mean square of signal difference $RMS(x_1 - x_2)$ is small, then signal phasors $X_1(t)$, and $X_2(t)$ are also close. The signals amplitudes and phase shifts can be significantly different. Therefore, restricted function values lead to numerically ineffective reversibility.

Problem definition

Phasor estimation is a process of finding the corresponding phasor of an input signal, based on a finite number of probes. For any input signal $x(t)$ from TFS it is necessary to find the amplitude $X_m(t)$ and the phase shift $\varphi(t)$ which are close to predefined values of the model. For any other signals estimation quality is not examined. Every phasor estimation can be verified using three primary value indicators, namely: the Total Vector Error (TVE), the Frequency Error (FE), and the Rate of Change of Frequency (RFE). TVE is defined as the relative difference between an estimated phasor $\widehat{X}(t)$ and the model phasor $X(t)$. TVE states for absolute difference between estimated and real phase shift, as well as between estimated and real amplitude.

$$TVE = \frac{|\widehat{X}(t) - X(t)|}{|X(t)|} \tag{15}$$

FE is defined as a relative difference between the model frequency and its estimated value

$$FE = \frac{|\widehat{f} - f_{real}|}{|f_{real}|} \tag{16}$$

RFE is defined analogously as

$$RFE = \frac{\left|\widehat{df} - \frac{df_{real}}{dt}\right|}{\left|\frac{df_{real}}{dt}\right|} \tag{17}$$

Indices TVE, FE, RFE can be roughly interpreted as relative differences between the zero, first, and second derivative of the complex valued function.

Model Signal

All simulations presented in this paper were focused on emphasizing behaviour of estimation algorithms for steady signals. Proper understanding of estimation process properties is crucial for solving the entire estimation problem. All tests were conducted for boundary signals that go beyond signal values typically recorded in power systems. This measure exposes critical features and simplifies analysis. The model signal is organized as composition of 32 test signals with the following properties:

- 1, sinusoidal with constant amplitude and f = 50 Hz
- 2 - 7, sinusoidal f = 50 Hz, amplitude variations

Nr.	2	3	4	5	6	7
X_m	$\nearrow$	2	$\searrow$	$\searrow$	0,5	$\nearrow$

- 8 - 13, sinusoidal with constant amplitude and frequency variation

Nr.	8	9	10	11	12	13
f [Hz]	$\nearrow$	52	$\searrow$	$\searrow$	48	$\nearrow$

- 14 - 19 sinusoidal with higher harmonics

Nr.	14	15	16	17	18	19
Harm. num.	2	3	5	10	25	50

- 20, constant amplitude, frequency oscillation
- 21, constant frequency, amplitude oscillation
- 22 - 25, sinusoidal with constant amplitude

Nr.	22	23	24	25
f [Hz]	52	55	45	48

- 26 - 29, sinusoidal, f = 50 Hz

Nr.	26	27	28	29
X_m	1	2	1	0.5

- 31- 34, constant amplitude, f = 50 Hz

Nr.	30	31	32
φ	$\frac{\pi}{2}$	0	$-\frac{\pi}{2}$

Fourier Analysis

Let us consider a signal from TFS. According to Fourier series theory, each periodic sufficiently smooth function $x(t)$ with angular speed ω can be represented as a Fourier series formulated as follows

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{i\omega kt} \tag{18}$$

Fourier coefficients are obtained from

$$c_k = \int_D x(t) e^{i\omega kt} \tag{19}$$

where: D - window length. For functions from the steady state class, normalized phasor is equal to first Fourier coefficient c_1. The core idea for short time Fourier transform algorithms (STFT) is to extend phasor estimation as first Fourier coefficient for any signal from TFS. Unfortunately, applying (19) to functions with varying amplitude or varying phase shift yields high errors in phasor estimation. Functions with time dependent amplitude and phase may be no longer periodic in a mathematical sense. It follows that the integral in (19) is no longer independent of filter domain location D. It implies that higher harmonic distortion varies as time dependent functions.

Fourier series algorithms can be significantly improved by modifying the window function used in phasor evaluation. Window function corresponds to a compound of all discrete and continuous filters applied in the estimation algorithm. Proper filter design allows the reduction of the influence of higher harmonics by filtering all but the ω component. However additional filters may affect the dynamical properties of the estimation process. In general, Fourier coefficient c_1 is evaluated using the following formula

$$c_1(t) = \int_{t+D} x(\tau) e^{i\omega\tau} g(\tau - t) d\tau \tag{20}$$

where g - filter function defined on domain D. Applied filter g often causes damping and shifting of the first ω component, which is an inexpedient issue. This problem can be reduced by correcting the output phasor with expected filter properties at estimated frequency $\widehat{f}$. The process of a frequency estimation is a complex problem that has been thoroughly investigated and is not analysed in this paper. It follows that the estimated phasor which is undergone a correction is given by following formula

$$\widehat{X}(t) = \frac{c_1(t)}{h(\widehat{f}(t))} \tag{21}$$

where h - characteristics of filter g .

Iterative algorithms

The iterative approach to the phasor estimation allows the extension of the scope of accurately estimated functions. One core problem occurs when signal frequency is constant, but differs from the nominal value. Iterative approach employs previously estimated frequencies to adjust coefficients in Fourier series. In particular, window length D and a filter angular speed are set to match the estimated signal waveform. For further analysis let us consider the sample triangle filter function with a two cycle window i.e. the domain of the filter is $D_T = [-T, T]$. This function corresponds to exemplary filters for P class usage.

$$g_T(t) = \begin{cases} \left|1 - \frac{t}{T}\right|, & |t| \le |T| \\ 0, & |t| > |T| \end{cases} \tag{22}$$

where: T - cycle time. Triangle filter was set to ensure flawless phasor estimation of signals with linearly changing amplitude and a constant phase shift. The first level phasor is

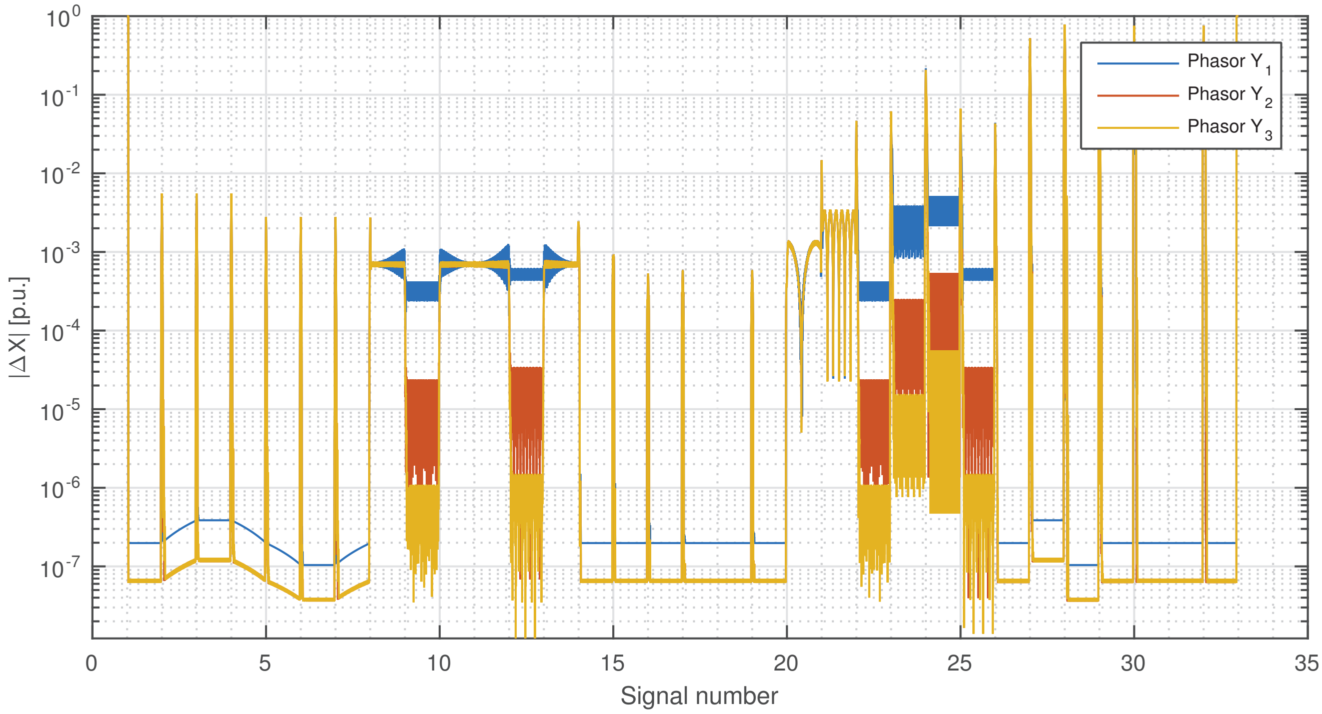

Fig. 1. Total vector error for multilevel estimators

estimated with pure Fourier parameters (20). The obtained phasor is being used to evaluate the corresponding first level frequency estimation $\widehat{f_1}$ and parameters D_1, ω_1, T_1. Next level phasors are evaluated as follows

$$\widehat{X}_{n+1}(t) = \int_{t+D_{T_n}} x(\tau)e^{i\omega_n\tau}g_{T_n}(\tau - t)d\tau \tag{23}$$

$$\omega_n = \frac{2\pi}{T_n} = 2\pi \cdot F(\widehat{X_n}) \tag{24}$$

where: $F(\widehat{X_n})$ - frequency estimator. It can be shown that for signals with a constant angular speed the phasors sequence $\widehat{X}_n(t)$ converges to the model phasor $X(t)$.

Simulation results - iterative estimation

Simulation results obtained for phase-locked-loop Fourier algorithms are shown in Fig. 1. Graph represents logarithmic difference between the estimated and model phasor. TVE is submitted for three levels of STFT. The blue line represents pure STFT without a frequency correction. Red one corresponds to the first level algorithm with a single correction and yellow one denotes a double loop. It can be observed that for signals with a frequency equal to the nominal frequency 50 Hz and constant amplitude TVEs for all estimators are low and approximately equal to 10^{-7}. According to mathematical theory it is expected to obtain error of 0 for such signals. Exact values are a result of inaccuracy in the compensation function definition for applied filters. Major improvements are achieved for signals with constant off nominal frequencies $\{9, 12, 22 - 26\}$. The first level phasor Y_1 is burdened with large errors. Applying frequency correction for the second level estimation allows evaluation of the second level phasor Y_2 with reduction of TVE by over a factor of ten. The third level phasor estimation Y_3 is evaluated with respect to the second level frequency approximation. TVE obtained for Y_3 is almost equal to TVE for phasors with nominal frequency. Since the obtained errors are not caused by window length mismatch yet frequency fluctuations, further phasor iterating is aimless. Estimation errors are preserved on acceptable levels for higher harmonic distortion $\{14 - 19\}$. Orthogonality is retained for each level of STFT. Notice, that for signals with frequency ramp $\{8, 10, 11, 13\}$, frequency fluctuation $\{20\}$ and amplitude fluctuation $\{31\}$, iterative approach does not improve the accuracy obtained. Obtained errors are not caused by frequency mismatch, so all estimation levels are performed for near nominal frequency.

We can observe, that the signal overshoot present after every step signal change (all but $\{8-13\}$) does not increase with the subsequent phasor evaluation. Resulting from such STFT characteristics, any overshoot value is almost independent of frequency for which step change occurs.

Polynomial approximation

Polynomial approximation, named often as the Taylor Fourier approximation, was introduced to improve phasor estimation accuracy. Consider a function $x(t)$ with corresponding phasor $X(t)$. By phasor definition following equality holds

$$x(t) = \mathrm{Re}\left(X(t)e^{i\omega_0 t}\right) = \frac{X(t)e^{i\omega_0 t} + \overline{X(t)e^{i\omega_0 t}}}{2} \tag{25}$$

where: $\overline{X(t)}$ - conjugation of $X(t)$. According to the Weierstrass approximation theorem, phasor $X(t)$ can be approximated with N - order complex polynomial as follows

$$X(t) \approx \sum_{k=0}^{N} a_k t^k = \sum_{k=0}^{N} b_k p_k(t) \tag{26}$$

where: a_k, b_k - complex coefficients, $p_n(t)$ - polynomial of order n. In a matrix notation the equation can be rewritten

$$X(t) \approx \boldsymbol{AT} = \boldsymbol{BP}(t) \tag{27}$$

where: $\boldsymbol{A} = \begin{pmatrix} a_0 & a_1 & \cdots & a_{N-1} & a_N \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} b_0 & b_1 & \cdots & b_{N-1} & b_N \end{pmatrix}$, - matrices of coefficients, $\boldsymbol{T} = \begin{pmatrix} 1 & t & \cdots & t^{N-1} & t^N \end{pmatrix}$, $\boldsymbol{P} = \begin{pmatrix} 1 & p_1(t) & \cdots & p_{N-1}(t) & p_N(t) \end{pmatrix}$ - matrices of orthonormal polynomials. The problem can be formulated in a

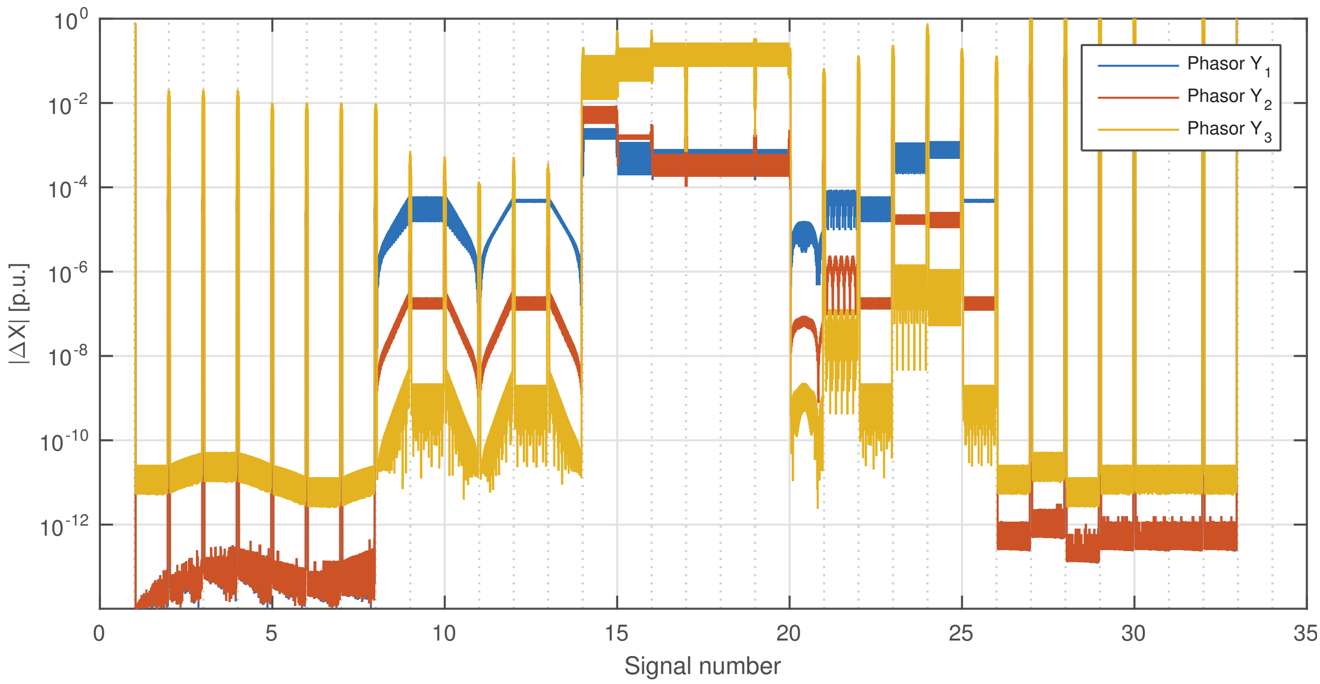

Fig. 2. Total vector error for polynomial approximation - model signal

matrix notation as

$$x(t) \approx \frac{1}{2}\begin{pmatrix} \boldsymbol{B} & \overline{\boldsymbol{B}} \end{pmatrix} \begin{pmatrix} \boldsymbol{P}e^{i\omega t} \\ \boldsymbol{P}e^{-i\omega t} \end{pmatrix} \tag{28}$$

In order to obtain the polynomial approximation of the phasor, matrix $(\boldsymbol{B}\ \overline{\boldsymbol{B}})$ has to be evaluated. Applying least square method (LSM) leads to the following equation

$$\begin{aligned} &\frac{1}{2}\begin{pmatrix} \boldsymbol{B} & \overline{\boldsymbol{B}} \end{pmatrix} \approx \boldsymbol{X_P} * \boldsymbol{G_p} \\ &\boldsymbol{X_P} = \int x(t) \begin{pmatrix} \boldsymbol{P}'e^{i\omega t} & \boldsymbol{P}'e^{i\omega t} \end{pmatrix} \\ &\boldsymbol{G_p} = \begin{pmatrix} \int \boldsymbol{P}\boldsymbol{P}'e^{i2\omega t} & I \\ I & \int \boldsymbol{P}\boldsymbol{P}'e^{-2i\omega t} \end{pmatrix}^{-1} \end{aligned} \tag{29}$$

The dynamic phasor is defined as b_0 coefficient in $\boldsymbol{B}$ matrix. Integral properties in matrix $\boldsymbol{G_p}$ are a vital issue in LSM method, because good matrix inversion properties need to be ensured.

Simulation results - polynomial approximation

Simulation results obtained for polynomial approximation are shown in Fig. 2. Analyses were performed for polynomials of order 2 (Y_1), order 4 (Y_2) and order 6 (Y_3). Integral domain in matrix $\boldsymbol{G_p}$ is set to match 2 cycles filter. It can be observed that for signals with nominal frequency and without higher harmonics distortion $\{1-7, 26-32\}$ proper accuracy is achieved regardless of the polynomial order. Minor differences in estimation accuracy are related to the efficiency of the matrix inversion. When signal frequency differs from the nominal value $\{9, 12, 22-25\}$, frequency changes linearly $\{8, 10, 11, 13\}$, frequency or amplitude fluctuations occurs $\{20, 21\}$, estimation accuracy is significantly improved with higher polynomial order. Higher polynomial order ensures a more accurate approximation for sophisticated signals as long as matrix $\boldsymbol{G_p}$ is sufficiently invertible. Increasing polynomial order compounds the numerical reversibility property. The selection of polynomial order is a trade-off between acceptable errors and numerical efficiency.

Notice that when the input signal is distorted with higher harmonics $\{16-19\}$, then estimation accuracy deteriorates. It follows both from LSM properties and phasor non uniqueness. Each signal with higher harmonics can be reformulated as signal without higher harmonics but with time varying amplitude and phase shift. The corresponding phasor is polynomially approximated instead of being damped. Orthogonality relation is no longer preserved, so estimation errors are amplified with increased accuracy. The application of additional input filters modifying the integral property of matrix $\boldsymbol{G_p}$ or extending function basis $\boldsymbol{P}$ by the harmonic component can be countermeasure for this issue.

It can be observed that polynomial algorithms are much more vulnerable to overshoots than STFT algorithms, without any additional overshoot damping tools. Recorded overshoot is significantly higher for the same input signal. The overshoot increases with higher polynomial orders. It is followed by the property that a step change is considered as a valid signal phasor, which is not equal to the mean value of the pre- and post change phasors.

Conclusion

This work has demonstrated the reasons behind synchrophasor estimation errors in electrical power systems. STFT and polynomial approximation based algortihms were studied to determine their advantages and disadvantages as well as possibilities for further efficiency improvement. Investigations have revealed that algorithms based on STFT phase locked loop are efficient as long as frequency of the processed signal is constant and amplitude changes linearly. According to previous phasor analysis one signal can have many phasor representations. It follows that a signal with varying amplitude or frequency, can be considered as a different signal with harmonic distortion. In consequence, important signal parameters may be omitted on the level of higher harmonics damping.

Efficiency of polynomial approximation algorithms is strongly affected by polynomial order involved in phasor estimation. Appropriate order selection is determined by requirements for estimation accuracy, overshoot restrictions and harmonic damping capabilities. High order polynomials require good inversion properties of $\boldsymbol{G_p}$ matrix, which can be achieved by modifying appropriate integrals with suitable filter functions, or integration domain, analogously like with STFT method. Estimation accuracy can be also improved by substituting polynomial approximation with non linear approximation and by the modification of LSM, however further research needs to be conducted.

***Authors**: Szymon Cygan, M.Sc.; Mirosław Łukowicz, Ph.D., D.Sc., Wrocław University of Science and Technology, Faculty of Electrical Engineering, Wybrzeże Wyspiańskiego Str. 27, 50 - 370 Wrocław, Poland; email: szymon.cygan@pwr.edu.pl, miroslaw.lukowicz@pwr.edu.pl*

REFERENCES

[1] IEEE Standard for Synchrophasor Measurements for Power Systems, IEEE Standard C37.118.1-2011, Dec. 2011
[2] Macii D., Barchi G., Petri D.: Design Criteria of Digital Filters for Synchrophasor Estimation, Instrumentation and measurement technology conference (I2MTC), 2013 IEEE International, Minneapolis, MN, USA
[3] Romano P., Paolone M.: Enhanced Interpolated-DFT for Synchrophasor Estimation in FPGAs: Theory, Implementation, and Validation of a PMU Prototype, IEEE Transactions on instrumentation and measurement, VOL. 63, NO. 12, December 2014
[4] Roscoe A., Abdulhadi I., Burt G.: P-Class Phasor Measurement Unit Algorithms Using Adaptive Filtering to Enhance Accuracy at OffNominal Frequencies, Smart measurements for future grids (SMFG), 2011 IEEE International conference on, Bologna, Italy
[5] Roscoe A. J., Abdulhadi I. F., Burt G. M.: P and M Class Phasor Measurement Unit Algorithms Using Adaptive Cascaded Filters, IEEE Transactions on power delivery, vol. 28, No. 3, July 2013
[6] Platas-Garz M., Serna J.: Dynamic Phasor and Frequency Estimates Through Maximally Flat Differentiators, IEEE Transactions on instrumentation and measurement, vol. 59, No. 7, July 2010
[7] Mai R. K., He Z. Y., Fu L., Kirby B., Bo Z. Q.: A Dynamic Synchrophasor Estimation Algorithm for Online Application, IEEE Transactions on power delivery, vol. 25, No. 2, April 2010
[8] Belega D., Fontanelli D., Petri D.: Low-Complexity Least-Squares Dynamic Synchrophasor Estimation Based on the Discrete Fourier Transform, IEEE Transactions on instrumentation and measurement, vol. 64, No. 12, December 2015
[9] Serna J.: Synchrophasor Measurement With Polynomial Phase-Locked-Loop Taylor–Fourier Filters, IEEE transactions on instrumentation and measurement, vol. 64, No. 2, February 2015
[10] Fu L., Zhang J., Xiong S., He Z., Mai R.: A Modified Dynamic Synchrophasor Estimation Algorithm Considering Frequency Deviation, IEEE Transactions on smart grid, vol. 8, No. 2, March 2017