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Modelling of multi-phase BLDC motor

Abstract*. In the paper a multi-phase BLDC motor unit, including stator winding and electronic commutation circuit, is studied on the example of five-phase BLDC motor. Mathematical model of the considered structure is formulated as well as the results of computer simulation are included and compared with the results dealing with three-phase motor.*

Streszczenie. *W niniejszej pracy przedstawiono analizę zespołu wielofazowego silnika BLDC, zawierający uzwojenie stojana i komutator elektroniczny, na przykładzie silnika pięciofazowego. Sformułowano model matematyczny oraz zaprezentowano wyniki symulacji komputerowej, które porównano z wynikami symulacji komputerowej trójfazowego silnika BLDC. (**Modelowanie wielofazowego silnika BLDC**).*

Keywords: BLDC motor, converter and inverter, mathematical modelling, control strategies.
Słowa kluczowe: silnik BLDC, przekształtnik, modelowanie matematyczne, strategie sterowania.

Introduction

Most brushless direct current (BLDC) motors include three-phase stator winding connected in star fashion. Each phase winding is constructed with numerous interconnected coils placed in the slots of stator. There are two types of motors: trapezoidal and sinusoidal [1,3,4]. Back electromotive forces (EMFs) and phase currents of trapezoidal motor are deformed, thus the output torque is rippled, which results in additional vibration and noise. The magnitude of torque ripple may be reduced by using a sinusoidal motor or a multi-phase trapezoidal motor.

The groups of stator phase windings of the considered multi-phase BLDC motor should be energized in a proper sequence in order to rotate the BLDC motor. The rotor position angle is required to determine which groups of stator phase windings have to be energized according to the abovementioned sequence. The Hall effect sensors (HES), embedded into the stator on the non-driving end of the motor, are widely used to determine the rotor position angle of BLDC motor (HES control) [1-7]. Multi-phase motor requires N_m HES, where N_m is number of motor phases. In addition to the HES control, the pulse width modulation (PWM) is widely used in order to limit the starting current as well as to control speed and torque of BLDC motor.

In the paper a multi-phase BLDC motor unit, including stator winding and electronic commutation circuit, is studied on the example of five-phase BLDC motor. Mathematical model of the considered structure is formulated as well as the results of computer simulation are included and compared with the results concerning three-phase motor.

Mathematical model of BLDC motor unit, including stator winding and electronic commutation circuit

The power electronic switches, consisting of transistors and diodes, are used in order to commutate the current in phase windings of BLDC motor. These switches are connected in a five-phase bridge for a five-phase BLDC motor shown in Fig. 1.

Two examples of five-phase stator winding for selected number of pole pairs N_p are presented in Fig. 2.

The equivalent circuit of BLDC motor unit, including stator winding and electronic commutation circuit corresponding to the real circuit (Fig. 1), is depicted in Fig. 3, where the power electronic switches are replaced by the electric switches shunted with the resistance R_{off} of turn-off transistor (compare [1,2]).

The PWM-based control strategy for the five-phase stator winding in the first sequence period:
(a) S1 = S6 = on, S2 = S3 = S4 = S5 = S7 = S8 = S9 = S10 = off
may be described by the dependencies (1).

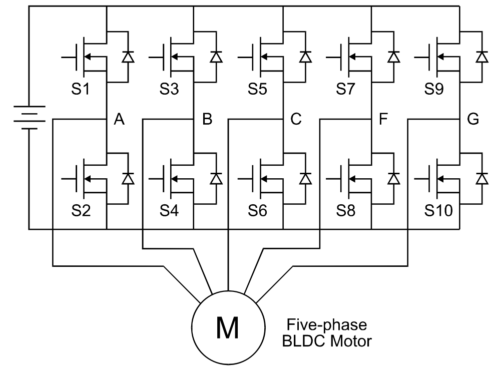


Fig. 1. Five-phase BLDC motor energized by five-phase inverter bridge

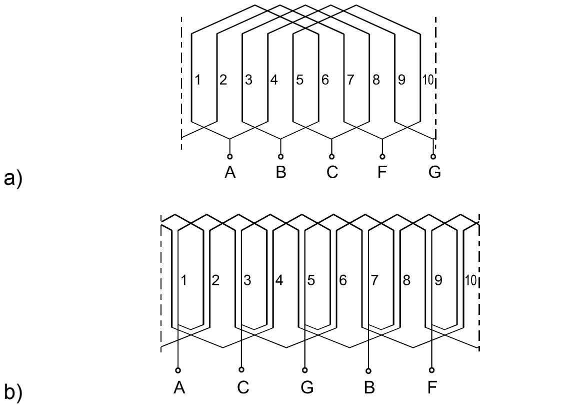


Fig. 2. Examples of five-phase windings: a) N_p = 1, b) N_p = 2

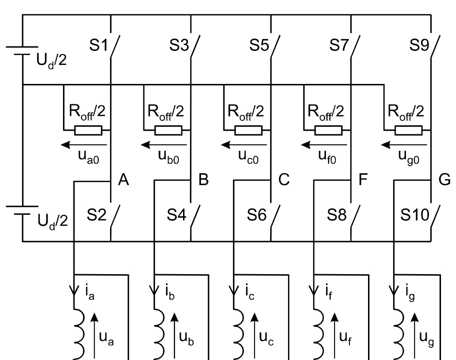


Fig. 3. Equivalent circuit of BLDC motor unit including five-phase stator winding and electronic commutation circuit

(1)
$$\begin{aligned}&\text{if}(K=1)\,u_d=U_d;\ \text{else}\ u_d=0;\\&\text{if}(0°<\theta_e\le 36°)\{u_{b0}=\tfrac{1}{2}R_{off}(i_b-i_c);\\&u_{f0}=\tfrac{1}{2}R_{off}(i_f-i_g);\ u_{g0}=\tfrac{1}{2}R_{off}(i_g-i_a);\\&\text{if}(u_{b0}>\tfrac{1}{2}u_d)\,u_{b0}=\tfrac{1}{2}u_d;\ \text{if}(u_{b0}<-\tfrac{1}{2}u_d)\,u_{b0}=-\tfrac{1}{2}u_d;\\&\text{if}(u_{f0}>\tfrac{1}{2}u_d)\,u_{f0}=\tfrac{1}{2}u_d;\ \text{if}(u_{f0}<-\tfrac{1}{2}u_d)\,u_{f0}=-\tfrac{1}{2}u_d;\\&\text{if}(u_{g0}>\tfrac{1}{2}u_d)\,u_{g0}=\tfrac{1}{2}u_d;\ \text{if}(u_{g0}<-\tfrac{1}{2}u_d)\,u_{g0}=-\tfrac{1}{2}u_d;\\&u_a=\tfrac{1}{2}u_d+u_{g0};\ u_b=-\tfrac{1}{2}u_d-u_{b0};\ u_c=-\tfrac{1}{2}u_d+u_{b0};\\&u_f=\tfrac{1}{2}u_d-u_{f0};\ u_f=u_{f0}-u_{g0};\}\end{aligned}$$

where $\theta_e = N_p\theta_m$, $\theta_e \in [0°; 360°)$, θ_m is angle of rotor rotation, K is PWM output [1]. For the next sequence periods:
(b) S1 = S8 = on, S2 = S3 = S4 = S5 = S6 = S7 = S9 = S10 = off
(c) S3 = S8 = on, S1 = S2 = S4 = S5 = S6 = S7 = S9 = S10 = off
(d) S3 = S10 = on, S1 = S2 = S4 = S5 = S6 = S7 = S8 = S9 = off
(e) S5 = S10 = on, S1 = S2 = S3 = S4 = S6 = S7 = S8 = S9 = off
(f) S5 = S2 = on, S1 = S3 = S4 = S6 = S7 = S8 = S9 = S10 = off
(g) S7 = S2 = on, S1 = S3 = S4 = S5 = S6 = S8 = S9 = S10 = off
(h) S7 = S4 = on, S1 = S2 = S3 = S5 = S6 = S8 = S9 = S10 = off
(i) S9 = S4 = on, S1 = S2 = S3 = S5 = S6 = S7 = S8 = S10 = off
(j) S9 = S6 = on, S1 = S2 = S3 = S4 = S5 = S7 = S8 = S10 = off
the dependencies are analogical.

In the case of control strategy based on PWM and low-pass filter LC one modification is taken into account:

(2)
$$u_d = u_{ref}U_d / U_n$$

The model of BLDC motor with back EMF approximated to trapezoidal waveform was used in computer simulation. Equations of armature winding voltages:

(3)
$$\begin{bmatrix}u_a\\u_b\\u_c\\u_f\\u_g\end{bmatrix}=R\begin{bmatrix}i_a\\i_b\\i_c\\i_f\\i_g\end{bmatrix}+\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\psi_a\\\psi_b\\\psi_c\\\psi_f\\\psi_g\end{bmatrix}+\begin{bmatrix}e_a\\e_b\\e_c\\e_f\\e_g\end{bmatrix}$$

(4)
$$\begin{bmatrix}\psi_a\\\psi_b\\\psi_c\\\psi_f\\\psi_g\end{bmatrix}=\begin{bmatrix}L&M_1&M_2&M_2&M_1\\M_1&L&M_1&M_2&M_2\\M_2&M_1&L&M_1&M_2\\M_2&M_2&M_1&L&M_1\\M_1&M_2&M_2&M_1&L\end{bmatrix}\begin{bmatrix}i_a\\i_b\\i_c\\i_f\\i_g\end{bmatrix}$$

where $L = L_\sigma + L_\mu$, $M_1 = L_\mu/5$, $M_2 = -3L_\mu/5$, L_μ is inductance of main magnetic circuit (magnetization inductance), L_σ is leakage inductance. The following dependency may be used in order to derive the phase currents from the matrix dependency (4):

(5)
$$\begin{aligned}i_{a,b,c,f,g}&=\alpha_1\psi_{a,b,c,f,g}+\alpha_2(\psi_{c,f,g,a,b}+\psi_{f,g,a,b,c})+\\&+\alpha_3\sum_j\psi_j,\quad j=a,b,c,f,g\end{aligned}$$

where:
$$\alpha_1=\frac{L-M_2}{DN},\ \alpha_2=\frac{M_1-M_2}{DN},\ \alpha_3=-\frac{M_1^2-M_1^2+M_1(L-M_2)}{DN(L+2M_1+2M_2)},$$
$$DN=L(L-M_1-M_2)+M_1M_2-(M_1-M_2)^2.$$

These phase currents should be substituted in the equation (4) to reduce the number of unknowns.

Back EMFs:

(6)
$$e_j = N_p\Psi_P\omega_m f_j(\theta_e),\quad j=a,b,c,f,g$$

where Ψ_P is flux linkage excited by permanent magnets, ω_m is angular velocity of rotor. The following dependency may be adopted in order to approximate the functions $f_j(\theta_e)$:

(7)
$$\begin{aligned}&f_j(\theta_e)=k_f\big(\sin(\theta_e-i\cdot72°)\big)\quad\cap\quad -1\le f_j(\theta_e)\le+1\\&i=0,\ldots,4\quad\Leftrightarrow\quad j=a,b,c,f,g\end{aligned}$$

where $k_f = 2$ for approximation of trapezoidal EMF with wide trapezoid base (120 electrical degrees), $k_f = 1.2$ for approximation of trapezoidal EMF with narrow trapezoid base (about 60 electrical degrees) and $k_f = 1$ for sinusoidal approximation of EMF.

Output (electromagnetic) torque:

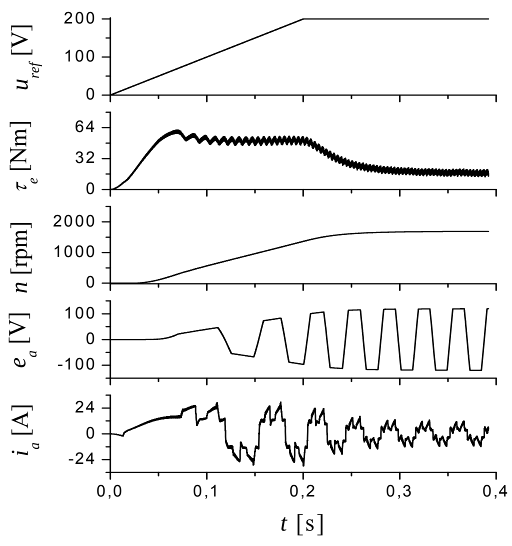
(8)
$$\tau_e=\omega_m^{-1}\sum_j e_j i_j,\quad j=a,b,c,f,g$$

Taking into account Eq. 7:

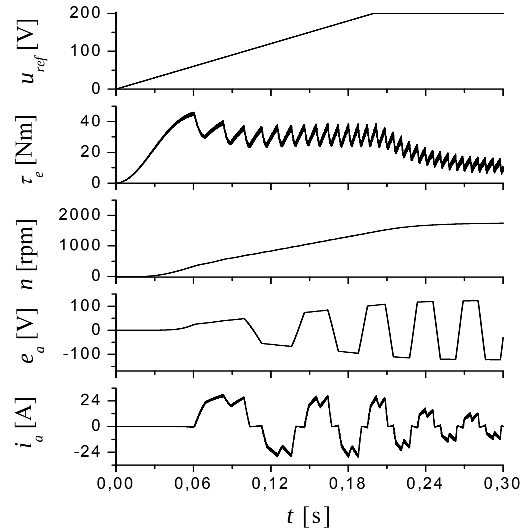
(9)
$$\tau_e=N_p\Psi_P\sum_j f_j(\theta_e)i_j,\quad j=a,b,c,f,g$$

Results of computer simulation

In the model-simulation investigations the following rated parameters of five-phase BLDC motor were taken into account: 6.8 kW, 372 V, 3000 rpm, 20.5 A, 0.04 kgm^2, R_s = 0.5 Ohm, L_μ = 7.4 mH, L_σ = 1.6 mH, $N_p\omega_m\Psi_P$ = 212 V. Also the model-simulation investigations of three-phase BLDC motor have been made for comparison. The following rated parameters of three-phase motor were taken into account: 4 kW, 400 V, 3000 rpm, 11.5 A, 0.025 kgm^2, R_s = 0.5 Ohm, L_μ = 7.4 mH, L_σ = 1.6 mH, $N_p\omega_m\Psi_P$ = 212 V. The carrier frequency of 2 kHz for PWM was adopted in both cases.

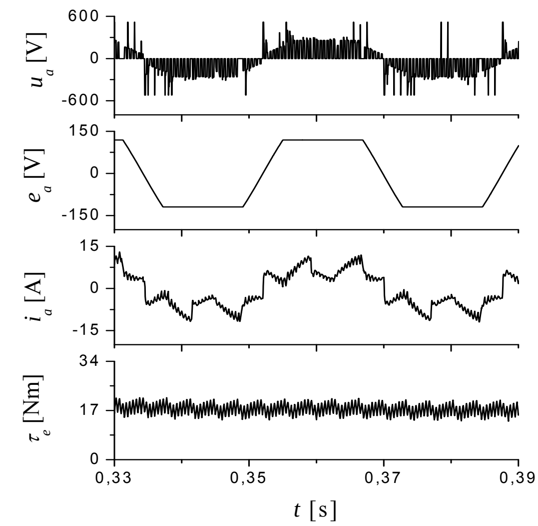


a)

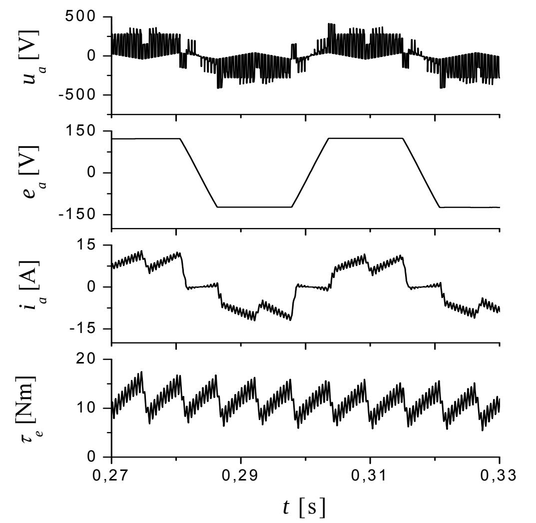


b)

Fig. 4. Reference voltage, output torque, rotational speed, back EMF and phase current during starting the motor: a) five-phase motor 6.8 kW, 0.04 kgm^2, b) three-phase motor 4 kW, 0.025 kgm^2



a)



b)

Fig. 5. Phase voltage, back EMF, phase current and output torque of BLDC motor under load condition: a) five-phase motor 6.8 kW, 0.04 kgm^2, b) three-phase motor 4 kW, 0.025 kgm^2

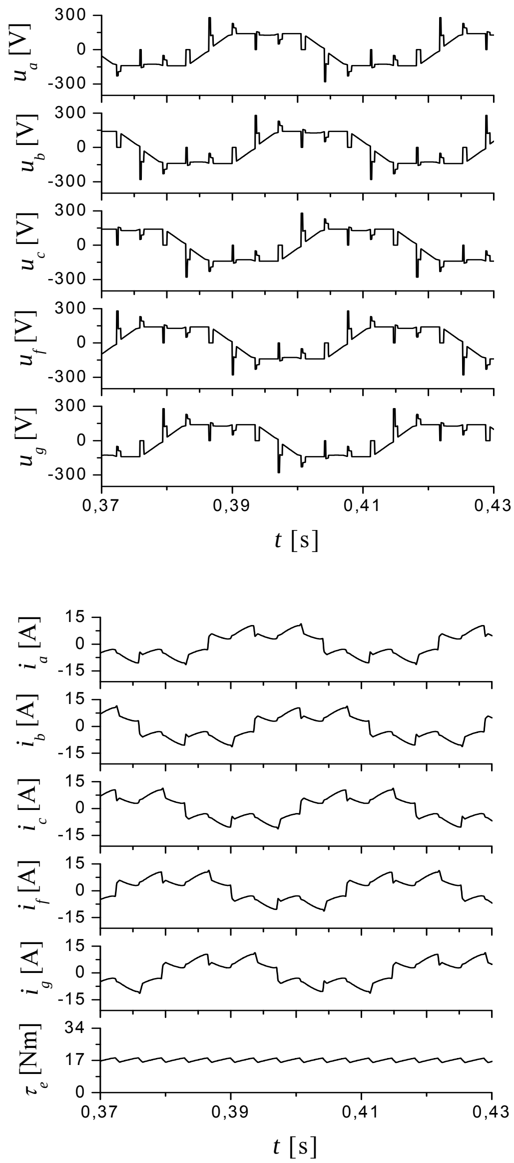


Fig. 6. Phase voltages, phase currents and output torque of five-phase BLDC motor under load condition

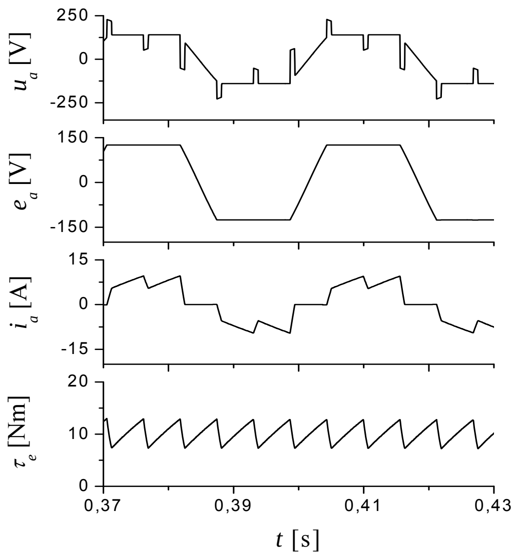


Fig. 7. Phase voltage, back EMF, phase current and output torque of three-phase BLDC motor under load condition

The selected results of computer simulation are shown in the paper:

1. Results of computer simulation of BLDC motor energized by PWM-controlled converter system [1,2] are shown in Figs. 4 and 5.
2. Results of computer simulation of BLDC motor energized by PWM-controlled converter system with low-pass filter LC [1,2] are shown in Figs. 6 and 7.

Discussion of the results

In the computer simulation the same parameters of stator phase windings (R_s, L_μ, L_σ and $N_p \Psi_P \omega_m$) for both compared BLDC motors, i.e. three-phase motor and five-phase motor, and about 5/3 times higher rated power (6.8 kW) and rated torque (21,6 Nm) as well as higher moment of rotor inertia of five-phase motor were taken into account. Also the same relative load torque was considered in computer simulation for both compared motors: 10/12,7 ≈ 0,79 and 17/21,6 ≈ 0,79 of rated torque.

The major differences concern actual electromagnetic torque and actual phase currents: a) the magnitude of torque ripple in relation to the average torque is about four times higher for three-phase motor (compare the second picture in Fig. 6 and Fig. 7), b) the actual phase currents of three-phase motor are discontinuous in contrast to the five-phase motor, however, the current peaks are similar in both cases (compare for example Figs. 5a and 5b or the second picture in Fig. 6 and Fig. 7).

The considered five-phase BLDC motor tolerates well various types of back EMF i.e. induced in trapezoidal fashion with wide and narrow trapezoid base as well as in sinusoidal fashion (these results are not shown in the paper): the magnitude and peak of phase currents are similar, however, the sinusoidal back EMF results in the increased slightly current distortion coming from HES-control (low-frequency harmonics), whereas the torque distortion dependent on PWM (high-frequency harmonics) decreased for sinusoidal back EMF.

Further investigations

In the paper the BLDC motor unit, including five-phase stator winding connected in pentagonal fashion and electronic commutation circuit, was studied. In turn, the equivalent circuit of BLDC motor unit, including five-phase stator winding connected in star fashion and electronic commutation circuit, is depicted in Fig. 8. The following inverter states in the consecutive periods of sequence may be taken into account: a) S1 = S9 = S4 = S6 = on, S2 = S3 = S5 = S7 = S8 = S10 = off, b) S1 = S9 = S6 = S8 = on, S2 = S3 = S4 = S5 = S7 = S10 = off, c) S1 = S3 = S6 = S8 = on, S2 = S4 = S5 = S7 = S9 = S10 = off, d) S1 = S3 = S8 = S10 = on, S2 = S4 = S5 = S6 = S7 = S9 = off, e) S3 = S5 = S8 = S10 = on, S1 = S2 = S4 = S6 = S7 = S9 = off, f) S3 = S5 = S2 = S10 = on, S1 = S4 = S6 = S7 = S8 = S9 = off, g) S5 = S7 = S2 = S10 = on, S1 = S3 = S4 = S6 = S8 = S9 = off, h) S5 = S7 = S2 = S4 = on, S1 = S3 = S6 = S8 = S9 = S10 = off, i) S7 = S9 = S2 = S4 = on, S1 = S3 = S5 = S6 = S8 = S10 = off, j) S7 = S9 = S4 = S6 = on, S1 = S2 = S3 = S5 = S8 = S10 = off.

The abovegiven BLDC motor unit will be the subject of further investigations.

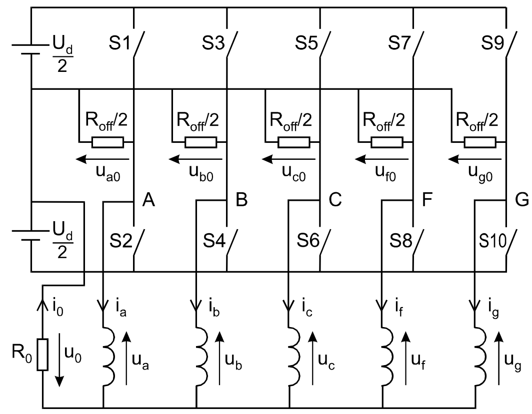


Fig. 15. Equivalent circuit of BLDC motor unit including five-phase stator winding connected in star fashion and electronic commutation circuit

Conclusions

The presented mathematical model and results of computer simulation of five-phase BLDC motor give a good base for design activity dealing with BLDC motor control systems. This model and results may be used to verify various control strategies. Computer simulation based on the presented mathematical model of five-phase BLDC motor unit, including stator winding and electronic commutation circuit, can effectively shorten analysis and development cycle of control systems based on BLDC motor as well as evaluate rationality of control algorithm.

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